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A STUDY FOR AN ACCURACY DETERMINATION PROGRAM FOR THE PSEUDO-DIURNAL TRANSIT TECHNIQUE OF SPACE NAVIGATION

("PDT STUDY")

Honeywell

Military Products Group

AERONAUTICAL DIVISION — ST. PETERSBURG, FLORIDA

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A STUDY FOR AN ACCURACY DETERMINATION PROGRAM FOR THE PSEUDO-DIURNAL TRANSIT TECHNIQUE OF SPACE NAVIGATION ("PDT STUDY")

PHASE I

NASA CONTRACT NAS1-2858

FINAL REPORT

To Guidance and Control Branch,
Aerospace Mechanics Division,
NASA Langley Research Center,
Langley Station,
Hampton, Virginia

By The
Navigation Devices Subgroup
Advanced Technology Group
Florida Aero Division of
Honeywell Military Products Group
13350 U.S. Highway No. 19
St. Petersburg, Florida

ABSTRACT

This document is the Phase I final report to the Guidance and Control Branch, Aerospace Mechanics Division, NASA Langley Research Center, for Contract NAS1-2858; it reports the study work accomplished under that contract by the Navigation Devices Subgroup, Advanced Technology Group, Florida Facility, Aero-East Division of Honeywell Military Products Group in St. Petersburg, Florida, during the period 29 May 1963 to 17 November 1963.

The Phase I study of the Pseudo-Diurnal Transit (PDT) Technique for Celestial Navigation in Interplanetary Midcourse has produced a preliminary mechanization design, and an accuracy analysis based upon that design, with recommendations for experimental confirmation during Phase II.

Basically, the "PDT" technique provides a means for establishing vehicle lines-of-position from near bodies by angular measurements derived from planet and star transits across a pair of reference "great circle" markers which are optically swept across the celestial sphere by the rotation of the mechanism about a single, known axis in inertial space (the "Pseudo-Diurnal Axis"). Direct measurements of declinations are ingeniously avoided. The doubtful accuracy and limited resolution of the human eye are replaced with a high-precision, electro-optical transit detector capable of resolving 0. 168 arc second. The difficulties associated with scale readings are eliminated by application of a Honeywell digital, dynamic goniometer ("DYNAGON") with a 12-inch diameter rotor to the PDT axis, about which all angles are read; this device has a basic resolution of $\pm 0.15^{+}$ arc second. The pseudo-diurnal motion at constant rate, originally postulated, has been replaced by the central rotor motion of the dynagon, whose rate need be constant only during resolution between adjacent 1/8192 th (of a circle) markers. Data processing is automatic. Manual star designation by gross methods suffices. Manual back-up modes with greatly degraded accuracy are provided for, however. In-flight maintenance and repair is provided for by plug-in module replacement.

The error analysis concludes that lines-of-position can be taken to an RSS accuracy of 3.6 arc seconds during interplanetary mid-course.

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SECTION 1
THE PSEUDO-DIURNAL TRANSIT TECHNIQUE

(The general principles and details of study progress.)

1.1 INTRODUCTION

1.1.1 Development of the Pseudo-Diurnal Transit (PDT) Technique

Conceptual development of the Pseudo-Diurnal Transit (PDT) Technique for Celestial Navigation in Space is described by its originator, Vahram S. Kardashian, in References 1.1 through 1.6. Appendix 1-F to this report gives a simplified explanation of the principle. In these studies a thorough analysis of expected instrument accuracy was at the time impractical due to lack of a mechanization model.

1.1.2 Preliminary Mechanization Description

The five-month study reported herein has resulted in a preliminary mechanization description in sufficient detail that design of an operating instrument kit for some specific future application could now be undertaken with confidence, by those well versed in the art. This mechanization model, described in Section 2 and appendices thereto, has been subjected to an accuracy analysis described in Section 3 which shows that the expected error in determination of one line-of-position to a "near body" is ±3.6 arc seconds (RSS), subject of course to experimental verification.

1.1.3 Preliminary Reliability Estimates

Provision is made in the design for inflight maintenance and repair, and all portions of the instrument set can be made accessible for repair or replacement without extra-vehicular capability. Preliminary reliability estimates indicate a system mean time between failures of 6,000 to 10,000 hours. The critical components have been noted and a kit of spares tentatively recommended in Section 4 for a representative manned Mars Mission.

1.1.4 Recommendations

Section 5 lists some recommendations for future effort.

1.1.5 References and Appendixes

Each section of this report has been augmented by separate appendices, appropriately numbered (i.e., 1-A, 1-B, 2-A, etc.), in order to permit preparation of these sections separately as the study progressed. Similarly, separate reference lists are applied to Sections 1 and 2.

Section 1 (this section) is largely derived from a rewrite of monthly progress reports; this was done for sake of continuity and to permit this document to stand on its own as a complete study report; the reader may trace the growth of, and reasoning supporting, the mechanization described in Section 2, without referring to separate monthly reports.

1.2 BACKGROUND

1.2.1 History

During the period 1959-1962, the basics of the pseudo-diurnal technique were presented to various aerospace companies and to all government agencies who had expressed an interest; presentations were made by personnel of Aero-Florida and the Military Proucts Group Research Staff as well as by Mr. Kardashian. On 18 October 1962 an unsolicited proposal to NASA Langley Research Center (Reference 1-7) led to award of this contract on 29 May 1963. The basic paper referred to in the proposal is included in this report as Appendix 1-F, for the convenience of readers who may be unfamiliar with the general method.

1.2.2 Objectives of the Study

The objectives of this study were:

a. To establish the accuracy capability of the Pseudo-Diurnal Transit Technique for Celestial Navigation in Space.

b. To describe a feasible mechanization based upon the general technique.

c. To determine the extent to which a human operator can (or must) participate in the observations necessary for the technique.

These objectives have been analytically achieved, but experimental verification is lacking. This phase of the study did not provide for experimentation or hardware construction, which was deferred to subsequent phases.

1.2.3 Basic Statement of Work

The basic statement-of-work is quoted below for reference:

A. The Contractor shall, (in conformance with the terms and conditions hereinafter set forth) furnish all personnel, services, material and facilities necessary to conduct a study for an accuracy determination program for the pseudo-diurnal transit technique for space navigation as follows:

B. The initial phase of this program has as its goal the determination of the best design for the pseudo-diurnal transit instrumentation. The initial design phase shall be limited to analysis and design rather than including experimental or fabrication efforts. There are two primary areas to be studied in this phase: (1) the study of a suitable design for the instrumentation necessary to generate and measure the precise pseudo-diurnal rotation required (rate-generating apparatus); and (2) the study of a design for the observational instrumentation necessary to time the transits of stars and planets (observational apparatus). In these areas, the following approach is to be used:

1. Definition of operational requirements. In defining the operational requirements of the instrumentation, it will be considered that the vehicle is manned. With respect to the rate-generating apparatus, the effects of changes in the vehicle's moment of inertial due to moving about of the occupants and sloshing of stored liquids should be considered. The use of the occupant's abilities in operating and monitoring the observational apparatus should be given consideration, especially with regard to the problem of star identification.

2. Preliminary performance analysis. There may be several designs of each type of apparatus, rate-generating and observational, which meet the operational requirements. Each of these designs will be given a preliminary performance analysis based on known or expected errors in fabrication.

3. Choice of designs for detailed study. The best of the alternative designs will be chosen for detailed study after consultation with the Contracting Officer. The results of the preliminary performance analysis, together with considerations of reliability and use of human abilities, are to be taken into account in making the choice.

4. Detailed study of chosen designs. The most promising design of the rate-generating apparatus and the most promising design of the observational apparatus will each be subjected to a detailed design study and error analysis, taking into account fabricational tolerances, alignment limitations, observational limitations, and all other factors which have a predictable effect on performance. The result of this study will be a report containing the detailed design and mechanization layout with a thorough error analysis describing the expected performance.

1.3 PERSONNEL INVOLVED

The individuals involved in this study were:

NASA

Mr. Leon W. Fitchett - Contracting Officer

Mr. Patrick A. Gainer - Technical Representative for the Contracting Officer

HONEYWELL

Administrative

* Mr. William H. Snyder - Contracts Administrator

* Mr. Arnold W. Sullivan - Chief Engr. Advanced Tech.

* Mr. Richard W. Lowrie - Navigation Devices Group Supervisor

Technical

** Mr. Vahram S. Kardashian (Resch. Scientist) - Concept Originator

** Mr. Charles W. Benfield (Senior Engr.) - Work Director and principle investigator

** Dr. Irving G. Foster (Staff Engr.) - Physics Consultant, Optical Engr. and mathematical analyst

** Mr. Albert R. Bock (Senior Engr.) - Mechanisms Consultant and design supervisor

Captain P. V. H. Weems (USN Ret.) - Navigational Astronomy Consultant

Dr. Dan D. Epps (Senior Engr.) - Optics Consultant

Mr. Halpert Fischel - Optics Consultant

Mr. Bartley Conroy - Lasers and Electro-Optics Consultant

Dr. Robert W. Long (Senior Engr.) - Dynagon Consultant

Mr. Harry H. Poole (Senior Engr.) - TV and Electro-Optics Consultant

Mr. Mark C. Zeiler (Senior Engr.) - Computer Applications Consultant

** Mr. Charles ("Scotty") Muir - Principal Draftsman

Mr. David Obershain - Draftsman

Mr. Carl E. Jenkens - Draftsman

* Mr. Len Sentowski - Illustrations Consultant

* Mrs. Sandra Kocha - Illustrator

* Mr. Keith Mendenhall - Reports Editor

Miss Joy Henefelt - Computist

Mr. Ronald Overstreet - Computist

The major technical contributors to the study are designated (**); others contributed as their talents were called for. Those designated (*) are involved in general support and administrative functions and did not charge directly to the contract.

RE-CAP OF STUDY PROGRESS BY MONTHS

FIRST MONTH'S PROGRESS

1.4 PROGRESS OF THE STUDY (BY MONTHS)

1.4.1 First Month's Work

The first month's effort was centered around, but not limited to, the first area of the work statement "Definition of Operational Requirements".

The preliminary study included an extensive review of the various ways of using the pseudo-diurnal technique. Stress was put on defining operational requirements in each mode of applying the pseudo-diurnal method, and a choice of the optimum mode was made. Alternate methods for producing and measuring the pseudo-diurnal rotation were considered, with a general analysis of the performance expected.

The navigator's role in the operations of star identification, and transit timing were considered. It has been decided tentatively that star identification and acquisition can be done manually, but that, at least in the primary mode, the transit observations should be done automatically without directly involving the navigator (for maximum accuracy).

Illustrations portraying the basic ideas of the various modes were prepared.

It became increasingly apparent that modes involving rotation of the vehicle itself probably would not be sufficiently accurate; this led to heavier emphasis upon schemes for providing a rotating observation platform referred to a stabilized platform upon a stabilized vehicle, and eliminated for the moment any consideration of disturbing torques occurring within the vehicle due to random movements of personnel, liquids and machinery. However, considerable concern with the relatively low reliability of inertial platforms required consideration of alternate schemes.

Certain tentative but specific conclusions were reached which permitted limiting the number of possible mechanizations. Several preliminary mechanizations were originated. Some of the significant conclusions are listed and supported by discussions in the following paragraphs.

1.4.1.1 Capability Comparison

The first tentative conclusion reached was that the capability of the PDT scheme must exceed that of the various space sextant methods, if it is to survive as a useful scheme for navigation in space where manual operation is involved, unless some other subtle advantages become apparent.

Captain P. V. H. Weems is internationally recognized as having extensive experience with sextants and celestial navigation and for his original ideas regarding manual navigation in space. Accordingly, he was consulted regarding the present and potential capabilities of sextants and other direct, angular measurement systems in space. His experiments have shown that a sextant can easily and consistently be read to an accuracy of ±12 arc seconds, and that lines-of-position upon the earth can be reliably reduced from this data to an accuracy of 0.4 nautical mile under fairly adverse conditions. This error may be reduced by a series of careful sights (made by skilled observers) to about 0.25 nautical mile (or about ±15 arc seconds), using a high quality sextant, fitted with a 20X, 50 mm telescope (Reference 1-8).

Furthermore, a graphical method for reducing star altitudes to latitude and longitude has existed for many years (Reference 1-9). The method has been guaranteed by Captain Weems to be accurate to 2.5 minutes of arc; when properly applied accuracies of 0.4 arc minute are possible. Similarly, the idea of these "Star Altitude Curves" could be revised (by removing refraction corrections and re-plotting) into sets of star co-altitude curves which would permit graphical reduction of a series of space sextant sights to a time history of lines-of-position within similar accuracy per star pair sighting. A geometrical, graphic method suitable to the original "Iso-azimuthal" concept and possibly useful in the back-up manual mode has been described by Kardashian in Reference 1-13 (Appendix 1-D). This method has been modified by Weems, Foster, and Benfield into a pencil and paper plot compatible with the present instrumentation and accurate to ±2 arc minutes with a reasonable volume of star charts (accuracy being limited chiefly by chart size, scale, and area).

Reference 1-12 describes an ingenious graphical reduction method, developed in the 1940's for air navigation, which involves accurately plotted stars upon a metal, spherical surface equipped with micrometer protractor scales. Where space aboard the vehicle permits, this "spherographical navigation" method can yield line-of-position plots accurate to two arc minutes, and requiring absolutely no calculation. The basic device is as applicable to the PDT method as it is to reduction of sextant sights, and can serve as a star locator chart as well as a plotting surface. Increase in accuracy would require use of a larger sphere and much more precise machining of the protractors hence it is regarded as impractical for the primary navigation mode but worthy of consideration for the back-up mode. The device described has not come into general use because of price and lack of availability (four of these plotters exist, one is in the possession of Captain Weems). While a spherical plotting surface eliminates the need for gnomonic charts, it also limits the accuracy due to sheer size as scale increases. The tentative conclusion is that use of plane star charts probably will be dictated by the available interior volume of spacecraft and the accuracies required.

As previously stated, well known means suffice for accuracies on the order of ±0.2 arc minute; therefore, it seems reasonable to conclude that the PDT system must be operationally capable of exceeding ±12 arc seconds accuracy, to be of significant value in space navigation in its primary mode of operation, and at worst capable of ±2 minutes accuracy in its back-up mode.

1.4.1.2 Present Methods in Lunar Flight

Present space sextant design goals are unofficially understood to be on the order of ±5 to 10 arc seconds per sighting. With automatic read-out and filtering, it is conceivable that an order of magnitude greater accuracy might be achieved in reducing a series of 100 or more sightings to a time history of several lines-of-position.

Optimization of the statistical process has been under study by Dr. Richard H. Battin of M.I.T./I.L. His early study (reported in Reference 1-10) assumed a sighting error on the order of 0.05 milliradian rms (a little over 10.3 arc seconds) and used between 22 and 126 sightings per correction decision during a simulated lunar flight. This accuracy was found to be adequate within the context of the assumed mission, although Battin warns that "too great an importance should not be attached to the numerical data presented" (in that further study is required). It is understood that his general method will be applied to Apollo navigation.

The tentative conclusions drawn from the above are:

a. That, very probably, direct angle measuring methods will be found adequate for lunar flight, so that the PDT system study has concerned itself chiefly with applications to interplanetary flight.

b. The design goal for the PDT instrumentation should exceed five arc seconds, and should approach as nearly as possible the "one arc second or less" accuracy postulated by Mr. Kardashian as a result of his extensive analyses (References 1-2 and 1-3).

The analysis of Reference 1-2 assumed no error in sighting of local vertical in view of the limited data available at the time of its preparation. This problem subsequently was treated extensively by Mr. Kardashian in Reference 1-3, in which he indicates that iterative techniques conceiveably might serve to reduce the error in "local vertical" to less than 0.3 arc second above 0.9 planetary diameters altitude. The technique described in Reference 1-3 involves electronic scanning of the planetary image. The effectiveness of manual observations for accomplishing the equivalent of timing a planetary-center-transit or pointing to a planetary center will require human factors experiments involving simulation facilities.

1.4.1.3 Photographic Methods

Very likely no in-space method will exceed the accuracy of the photographic reduction reported by the Russian astronomer, Plyugin (Reference 1-11), who claims that with 32 measurements he obtained a probable error (at earth) in local-vertical-to-the-moon of ±0.1 arc second using data points accurate to 0.33 to 0.64 arc seconds (probable error); this was after the fact by reduction of photographic data.

Certainly, without some iterative technique, the error in "local vertical" will become the dominant one in the PDT system, as it usually is in other celestial navigation systems; indeed, the limiting errors in the use of the marine sextant are largely due to difficulties with such as sea tilt error, horizon resolution, refraction error in horizon, dip correction and deflection of the vertical due to gravitational anomalies (since with proper optics, stars can be "tracked" to a fraction of an arc second, even by manual means).

A non-iterative, manually operated, photographic method is described in Reference 1-14. The accuracy of this method is undetermined, but is estimated at ±1/2 arc minute at best to ±1/2 degree at worst conditions, it requires no calculations whatsoever for determining lines-of-position and is entirely independent from the vehicle's electrical and electronic systems, except for an illuminated chronometer dial (battery operated). The predominate error in that scheme is associated with manual observation of "local vertical" to a "near body".

1.4.1.4 Error Allocations

The angles (α) being measured indirectly by the originally conceived PDT system are determined from angular rate (ω) times interval $t = (t_x - t_0)$.

$$\alpha = \omega t, \quad \frac{\partial \alpha}{\partial t} = \omega, \quad \frac{\partial \alpha}{\partial \omega} = t \tag{1.1}$$

The sensitivity of α to errors may be studied from the partial equation:

$$(\delta\alpha) = \frac{\partial \alpha}{\partial \omega}(\delta\omega) + \frac{\partial \alpha}{\partial t}(\delta t), \quad \text{or} \tag{1.2}$$

$$(\delta\alpha) = t(\delta\omega) + \omega(\delta t). \tag{1.3}$$

If one assumes a pseudo-diurnal rate of 15 degrees/hour, and if $\delta\alpha$ is not to exceed 5 arc seconds, during a 15 minute observation at ω = 15 degrees/hour, then some typical, allowable errors are:

$\delta t \cong 1 \times 10^{-3}$ sec.

$\delta\omega \cong$ 0.0056 arc sec/second.

The error in timing is far less critical than error in angular rate. Indeed, if timing errors are considered to be zero, the required angular rate accuracy still is $\delta\omega$ = 0.00555 degree/hour.

A man can observe the second hand of a watch (360 degrees/minute or 21, 600 arc sec/time sec) easily; perhaps with proper optics he could observe the minute hand (360 degrees/hour or 360 arc sec/time sec). From this, it does not seem reasonable to expect a man to observe rates on the order of 0.00555 degree per hour. From this consideration we must tentatively conclude that:

Conclusion A

Rate measurement cannot be manual, unless long observing times are allowed.

A reasonable assumption for man's ability to time the star transits would be ±0.1 second for each, taking into account his reflex and judgement times.* This error alone, at a 15 arc second/second rate, would account for ±1.5 arc seconds of error. This would assume a constant rate, and any deviation from constant rate would introduce a further error. Since 1.5 seconds of arc exceeds the goal of 1 second by itself, this method probably must be discarded except for back-up operation.

* P. A. Gainer has found the variance in manual transit timing ability to be 27 milliseconds, under the conditions of an experiment conducted with his group at NASA Langley. The figure of 0.1 second includes reflex reaction time to an expected event; this time varies with individuals and training and in some cases can be as large as 1/2 second. Further human factors research is indicated, before serious specification of this manual mode of operation.

1.4.1.5 Inertial Rate Measurement

Present state of the art rate gyros have thresholds on the order of 0.01 degree per second. Even anticipating a future improvement to 0.01 degree per hour we still would have to conclude that:

Conclusion B :

Using present gyros, rate measurement cannot be inertial.

At earth rate (15 degrees/hour) for a 5-minute observation time, a maximum of one second absolute error demands a relative error of $\frac{1}{5 \times 60}$ or 0.0033.. arc sec/sec or 0.0033.. deg/hr. This assumes no timing error.

Since the above requires a rate threshold which is a factor of about 10^4 better than the present state of the art, inertial rate sensing appears to be impractical for the desired accuracies (see also paragraph 3.5 of Section 3).

1.4.1.6 Preliminary Error Analysis

The use of diurnal rate dictates that this rate be exact or constant so that between any two instants of time, the exact angular displacement be known. Ideally, the accuracy of the angular displacement would be limited only by the ability to measure time (5×10^{-10} parts/day)* <u>if the rate were constant.</u>

It is assumed that rate generation techniques should be constant within 0.5 sec of arc absolute at any 5×10^{-10} sec of time. This assumption allows 0.5 sec of arc error in transit definition to comply with a total system error budget of 1 arc sec in the determination of λ_p in $\lambda_p = \omega t$, where λ_p is pseudo-right ascension angle correction observed.

The error in ability to measure time is practically insignificant. The value measured would actually be ΔT of three events, and would include either the error incurred by the human observing transit of stars or of some automatic or semiautomatic device. Reasonable assumptions for these sources of error would appear to be 0.1 sec of time for human, and 0.02 sec of time for an automatic photo-electronic sensor. This error would be magnified by any lack of stability of the vehicle. Assuming that the vibration

* Claimed stability of Hewlett Packard 103AR digital, electronic clock oscillator (however, resolution of read-out is ±1 millisecond!).

level remains relatively constant, the expected error would be minimized and would not contribute more than 0.01 sec of time since the measuring direction is not reversible in a given observation.

The lack of stability at low frequency in vehicle stabilization will be corrected by star tracker photo multiplier tubes pointed at a pole star and a star about 90 degrees from the pole star. These star trackers will serve to correct (the transit angle from planet to star or star pairs) for motion about any axis. An 800 cps sweep should yield an error of no more than 0.001 sec of time.

The total error induced by the factors above are:

Semiautomatic	0.1
	0.01
	0.001
	0.111 sec
Automatic	0.02
	0.01
	0.001
	0.031 sec

Next consider the error incurred from the lack of a constant rate. With the selection of a gear train for a rotational force it is improbable that the deviations from a constant acceleration would be of a high frequency due to the inertia of the train. Therefore if an angular bit measuring system is used, the rate would only need be constant for its use in interpolation during the last bit. The smaller the bit the higher the frequency of the disturbance would have to be to measurably disturb the accuracy. The smallest angular bit size in a commercially available device is 12.8 sec in "MIDARM".* With this bit size the rate would only need be constant to 0.1 percent for an error of 0.01 arc sec. (A recent Honeywell development "DYNAGON" has a gross, bit size of 0.35 degree counted down by rate interpolation to 1.24 arc seconds. This can be improved in larger sizes to 0.044 degree counted down to 0.16 arc second.)

System design assumptions (for beginning design study) were:

1. 12-inch diameter fluid hydrostatic compensated bearing - axis wobble less than 0.1 arc sec - constant friction.

2. Photo-electronic collimator resolution of point of transit to 0.1 arc sec.

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*Appendix 1-C.

3. Angular readout device (MIDARM) 12.8 arc sec prime pulses repeatable to 0.02 arc sec - absolute to 0.05 arc sec (interpolation between bits to be discussed later).

4. Star trackers (photomultiplier tubes) resolution 0.5 arc sec 800 cps.

Errors	Due to: Effect on ω	Effect on t
1. Bearing	negligible	negligible
2. Collimator	0.1 $\widehat{\text{sec}}$	negligible
3. MIDARM	0.05 $\widehat{\text{sec}}$	2 (0.01)*
4. Star Tracker	2 (0.5) $\widehat{\text{sec}}$	negligible

Diurnal λ_p error = 0.1 + 0.01 + 0.05 + 0.01 + 0.1 = 0.27 sec

Space stabilization (Star Tracker) = 1.0 sec

It follows that for this mode (and with these simplified assumptions) the maximum error in λ_p is 1.27 arc sec.

Additional errors are to be discussed later in this report. Having found this rather encouraging possibility, the study proceeded on a note of optimism to investigate alternative mechanizations for this general mode of operation.

1.4.1.7 Altitude Limits

Conclusion C

Planet observations are limited to some minimum distance from each planet.

As the vehicle approaches or departs from a planet, the apparent angular diameter of this planet varies (approximately) inversely with the distance of the vehicle from it. At some minimum distance the image will be so large that it completely fills the telescope. Whether one tries to locate the planet center visually or by some more sophisticated manner involving

* Rate need only be constant to 0.1 percent during the interpolation of beginning and starting bits.

the observation of points on the limb, the result can be inaccurate or meaningless unless one can observe at least one-fourth or one-half of the disc for a time in the order of a minute (while using the PDT technique).

If one arbitrarily chooses a one degree field of view for the telescope, we may assume that the planet image must be smaller than one degree in diameter if the above condition is to be fulfilled. The minimum distance of approach varies with planet diameter from 124,000 miles for the moon to 450,000 miles for the earth.

1.4.1.8 Discussion of Planetary Angular Diameters

The apparent angular diameters of Venus, Earth, Moon, Mars, Jupiter and Saturn from various points in the solar system are of the order of one minute of arc or less, except on approach to the planet itself.

Planetary Positions

For Julian Day 2438240.5 (30 July 1963) at 0^h E.T. the celestial coordinates of the planets are:

		Heliocentric, Earth - Equatorial Referenced						
	Dia.	Right Ascension			Declination			Distance from sun
	(miles)	deg	min	sec	deg	min	sec	R (A.U.)
Venus	7,570	106	52	30	24	12	22	0.718917
Earth	7,890	308	19	29	-18	47	9	1.015440
Moon*	2,160	308	19	29	-18	47	9	1.015440
Mars	4,230	211	17	41	-12	9	36	1.58868
Jupiter	88,400	7	40	21	1	53	1	4.97660
Saturn	75,000	321	54	17	-16	7	34	9.88171

* R and coordinates for earth and moon are values for earth-moon center of mass.
AU = abbreviation for "Astronomical Unit" or earth's mean altitude over the center of the sun ("heliocenter").

Angular Diameters

Simple arithmetic leads to the following results, expressed as seconds of arc in each case.

Saturn

As seen from Mars orbit	15 sec. - 20 sec.
As seen from Earth orbit	15 sec. - 19 sec.

Jupiter

As seen from Mars orbit	30 sec. - 58 sec.
As seen from Earth orbit	33 sec. - 50 sec.

Mars

As seen from Earth orbit	4 sec. - 17 sec.

When Mars has an apparent angular diameter of one degree, the observer is 0.00263 AU $\simeq$ 240,000 miles away.

Moon

From Earth	~ 30 min.
From Mars orbit	2 sec. - 9 sec.

When the Moon has an apparent angular diameter of one degree, the observer is 1.35×10^{-3} AU $\simeq$ 124,000 miles away.

Earth

From Mars orbit	7 sec. - 31 sec.

When Earth has an apparent angular diameter of one degree, the observer is 0.0049 AU $\simeq$ 450,000 miles away.

Venus

From Earth orbit	10 sec. - 57 sec.
From Mars orbit	7 sec. - 20 sec.

When Venus has an apparent angular diameter of one degree, the observer is 0.0047 AU $\simeq$ 435,000 miles away.

Conclusion

If the observing instrument has a one degree field of view, and the pseudo-diurnal rotation is at earth rate (15 deg/hour) then the whole disc of a planet (except the one being approached) will be in view for four minutes of time.

If one can determine the planet center for a maximum apparent angular diameter equal to the field diameter, then the pseudo-diurnal technique is applicable up to such approach distances as are noted above.

1.4.1.9 Single vs Pairs of Stars*

Conclusion D

Individual star observations are preferable to pair observations.

This conclusion is fully discussed in Appendix 1-A. The important points are:

a. As the telescope rotates about the pseudo-polar axis, an imaginary great circle passing through the optic axis point on the sphere will, perhaps, transit two stars simultaneously. These two stars are an isoazimuthal pair. In any given area of the celestial sphere a certain number of such pairs will be found, each on some one of the great circles through the zenith.

   On the other hand, there are many more stars which are nearly paired and which will give non-simultaneous transits across the same great circle. These latter observations are completely

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* Kardashian's original concept was based upon simultaneous transit of identified star pairs across reference reticle lines representing some great circle upon the celestial sphere. The equations of this observed great circle then being known at that time from the cataloged stellar coordinates, it could be "swept back" (mathematically) in psuedo-right ascension, to the time of planet crossing.

adequate in this method (four stars can be "swept-back" as easily as two pairs, if the same "great circle" lines of reference are transited individually).

b. The actual technique of observation calls for keen judgement on the observer's part in trying to determine both the great circle position and the instant of transit. It is questionable whether a man could make these judgements with the necessary accuracy. If the transit is determined automatically it is necessary to apply a fairly complex correction to the measurements since transit across a small circle normally is obtained.

1.4.1.10 Star Recognition

Conclusion E

Star recognition is to be done by the navigator.

The navigational technique will make use of only 57 (or fewer) of the brightest stars (Appendix 1-B) and certain planets. The equipment proposed in the various star recognition techniques would greatly add to the total equipment required by the pseudo-diurnal technique. There are no difficulties inherent in recognition by the navigator, particularly if he is supplied with a convenient set of star maps. In general, star and planet recognition is a task for which the human brain is admirably suited, and to date, relatively unchallenged by mechanisms.

1.4.1.11 Choice of Psuedo-Poles

Conclusion F

Angular Separation of pseudo-pole and planet should be about 90 degrees.

There are three basic reasons for this demand, which is to be understood as an optimum condition. It is more reasonable, probably, to say that this angle should be between 60 degrees and 120 degrees, or 70 degrees and 110 degrees. Any great circle arc through the zenith will sweep an area on the celestial sphere whose size depends upon two things; the angle between psuedo-pole and planet and the orientation of the great circle with respect to the equatorial plane of the pseudo-diurnal rotation. Instrumentation considerations almost certainly will limit the length of great circular arc along which stars can be imaged, therefore one should choose that configuration in which the greatest area of the celestial sphere is made available. This is ideal for an angle of 90 degrees.

In the second place, one wishes to observe in the telescope field, star transits across the great circle images as nearly at 90 degrees as possible. It can be shown that this will depend upon the position of the star being observed as well as the angle between the planet and pseudo-pole. In general, the most favorable situation is that of the 90 degree separation of planet and pseudo-pole. This is a fairly difficult geometrical problem in optics involving the projection of planes upon each other. It arises because of the tentative (but probably firm) conclusion that the star images must be brought into the field by auxiliary plane mirrors, if direct, optical observation is involved (as opposed to remote, electro-optical observation).

Finally, the measurement of psuedo-diurnal rate perhaps may be simplified by the same choice of 90 degree separation (if rate is indeed to be observed from star transits).

1.4.1.12 Vehicle Attitude Stability

Conclusion G

Vehicle Stabilization is to be within ten minutes of arc.

This figure is somewhat arbitrary but after consultation is deemed a reasonable assumption of the vehicle's stabilization controls. A further assumption is made that the natural frequency will be low (something less than one cycle per second).

1.4.1.13 Rate Generation Accuracy

Conclusion H

Rate generation (at 15 deg/hr) must have a relative error less than 2×10^{-4} arc sec/sec to yield less than one sec of arc error in absolute angle with no error allowance for time (assuming 83-1/3 minutes to sweep over an angle of 20.8 degrees).

In order to maintain constant rate, friction, thermal stability, and input torque must be constant. In order to erect and maintain a rate of this character, a method of measurement would have to be derived. Therefore, it is apparent that this measuring system could be used to measure directly the absolute angle in terms of its smallest absolute increment and that the constant rate assumption would only be necessary for interpolation of the starting and ending bits. For example, if a bit was ten sec of arc and the rate ten sec/sec then for a reading of five min. duration, the error from rate constancy assumption would only apply to that part of the first and last bits, a period of less than two seconds of time. It is therefore not necessary to measure the rate over the whole 300 second period.

To repeat, one measures the coarse angle using any accurate, (ten arc seconds) more or less conventional means, and the fine angle by PDT interpolation to get from ten arc seconds down to one arc second.

Inherent in this and preceding discussions is the tentative assumption that all problems of rate generation and stabilization are less if the required time for which rate must be maintained is kept to a minimum, in accordance with equation (1.3).

RE-CAP OF STUDY PROGRESS BY MONTHS

SECOND MONTH'S PROGRESS

1.4.2 Second Month's Work

Summary

Work during the month of July was largely devoted to a study of alternative designs for the navigation equipment required. In almost every instance, at least two possible designs were considered. Known or expected errors were considered where appropriate and trade-offs were noted. Recommendations for best design have been made. The role of the navigator was given particular attention.

The computer program for transforming star coordinates was completed, and the resulting data used to provide a number of star charts. More specific limitations on angular search requirements of the observing telescope could also be made using this data, and it became possible to limit the choice of polestars to two or four.

Careful consideration of the various ways of obtaining pseudo-diurnal rotation led to the fundamental decision of the month's work. This decision not only affects much of the design, but also the basic mode of obtaining the navigation data. It is therefore discussed in detail in Paragraphs 1.4.2.1 through 1.4.2.6.

A number of illustrations and diagrams indicating the possible mechanizations of the various elements of the pseudo-diurnal technique were prepared for oral presentation on flip charts. These illustrations are included at appropriate places within this report.

A verbal presentation of operational requirements and preliminary designs subsequently was made on 8 August 1963 at NASA Langley Research Center, Hampton, Virginia. Those present were:

NASA Langley:

Mr. Patrick A. Gainer	- Guidance and Control Branch, SMD (Contracting Officer's Technical Representative).
Mr. Wilbur Mayo	- Astrodynamics Branch, SMD
Mr. Henry A. Pearson	- Flight Mechanics Branch, SMD
Mr. Harold A. Hamer	- Flight Mechanics Branch, SMD
Mr. William M. Adams	- Flight Mechanics Branch, SMD
Mr. C. Ray Davis	- Procurement
Mr. Kenneth Garren	- Guidance and Control Branch, SMD

Honeywell:

Dr. I. G. Foster

Mr. A. R. Bock

Mr. V. S. Kardashian

Mr. C. W. Benfield

The mechanization approach informally agreed upon during the meeting was approved by NASA, according to letter NAS1-2858 (CRD) of 28 August 1963 to Mr. W. H. Snyder (Honeywell Contract Administrator) from Mr. Leon W. Fitchett (NASA Langley Contracting Officer).

A two month extension of the contract at no cost to the government was requested and granted. This made certain key persons available on a part time basis.

1.4.2.1 Operational Requirements

Some aspects of operational requirements were briefly discussed in paragraph 1.4.1. There is a small amount of repetition of this material in the following paragraphs where clarity makes it desirable.

1.4.2.2 Rate Versus Angle Measurement

The Kardashian proposal for the pseudo-diurnal method of navigation had as its basis an attempt to measure angular separations in some manner more accurate than the static method of a star sextant. As he envisaged this, a telescope was to rotate at known rate $\overline{\omega}$ about a fixed axis, and stars or planets would thereby transit across the field of the telescope. If $\overline{\omega}$ were known with accuracy, both in magnitude and direction, then by simple multiplication (ωt), the period of time separating two transits would provide the required angle. It is the contention of the pseudo-diurnal technique that these instants of transit can be measured with great accuracy, so that the angle of separation (ωt) can also be obtained with great accuracy.

It has always been understood that the method as originally conceived depends on the ability to obtain a constant $\overline{\omega}$ or on one's ability to determine with great accuracy the instantaneous value of $\overline{\omega}$, from which functional dependence of $\overline{\omega}$ on time an integration would produce the given angle. There are two possible ways of achieving $\overline{\omega}$. The first would fix the telescope in the vehicle and rotate the vehicle. The second would stabilize the vehicle and a rate table within it, (carrying the telescope), would rotate. In the first case, rate gyros could perhaps measure the angular rotation rate. It is shown in Paragraph 1.4.1.5 that the rate gyro cannot measure ω to the precision required. If direct measurement of rate is not possible, one must measure angle and time to obtain it. Here, at once, the minimum angular bit measurable puts a limit upon our knowledge of rate during the time taken for this angular motion to occur. Perhaps even more important is the obvious fact that we are measuring θ, converting it to ω, and recomputing θ. Why find ω at all, then? Before pursuing this thought, let us briefly consider the rate table method.

It is appealing to suppose that by using a frequency controlled motor, or a carefully designed gear train, a table might be made to rotate at a constant rate with respect to the vehicle. Only small energy output is required. On the other hand, the table with its relatively low moment of inertia is subject to many perturbing forces and torques, so that it is hardly possible to accept a nominal rate, perhaps established before the flight, as the value of ω. A self-checking rate, or at least a table whose speed can be checked, is almost certainly going to be required. Here again, we are back to the fact that an angle must be measured before ω can be found.

Then why measure rate, since angles must be known anyway? Paragraph 1. 4. 2. 6. 5, discusses several angle readout devices with accuracies of the order of magnitude of 0. 1 seconds of arc. With such a device on the "rate" table another look at the basic method is warranted, but, emphatically, the fundamental value of the transit technique is still retained for this study.

It was tentatively concluded that the following mechanization represents the best method to follow: The vehicle is stabilized and the relative angle of rotation of a table with respect to the vehicle is measured. This table carries the observing equipment and is oriented directly or indirectly by a star tracker to the polestar. The nomenclature already adopted is retained by calling a rotation about the axis from vehicle to polestar "pseudo-diurnal rotation" about a "pseudo-diurnal axis", but this does not imply any kind or degree of uniform angular velocity. On the contrary, pseudo-diurnal rotation means only an angular displacement of the table (and telescope) about the pseudo-diurnal axis, a displacement which is to be recorded by some angle readout device. The time required for this angular motion is of little importance, nor is the device which produces it. But, and this is most important, the determination of the angle scale readings at the planet center, and at the stars, will still be made dynamically. A detailed discussion of the required measurements, defined as scale readings θ_0, θ_1, θ_2, is in Paragraph 1. 4. 2. 4. 2.

These scale readings are made as the table is turning, and the planet or star is drifting across the telescope field. When the star or planet transits the "reticle line" the readout now gives angle directly rather than time. It is still a dynamic rather than a static reading, but has been made independent of a constant rotation rate.

The mechanization studies described in the following sections of this report accept this basic fact, that angular rotation rate will not be established or measured, and go on to describe alternate methods of operation with this in view.

1. 4. 2. 3 Functions of the Navigating Instrument and Operations of the Navigator

The navigating instrument must be used to locate a planet center at a given time, and to record transit times of four stars. The sequence of necessary operations and the role of the navigator are discussed below.

1. 4. 2. 3. 1 Planet Search, Acquisition, Observation ("Local Vertical")

By using auxiliary star charts (discussed in Paragraph 1. 4. 2. 4. 2) referred to the chosen pseudo-polestar, and knowing the approximate vehicle location, one chooses the planet to be observed on the basis of the star field in the planet neighborhood. An auxiliary or finder telescope is required to locate the planet approximately. The field of the finder is equipped with a reticle which indicates the extent of the field of the observing instrument. The finder and the observing instrument, hereafter called the telescope, must have two degrees of freedom about the pseudo-diurnal axis. OA (Figure 1-1) is the optic axis of the telescope, and it must be free to move through angles ϕ and ψ. These angles need not be known, nor need the axes of the motion $A\phi$ and $A\psi$ be accurately perpendicular. In fact, OA need not even pass through the PD axis. What is required is that the orientation of OA with respect to PD axis, whatever this orientation is, be rigidly maintained once the telescope is clamped. Note that the motion in ψ is about the PD axis and the pseudo-diurnal rotation of the platform may be used for the search function. Some limitations on maximum ϕ and ψ can probably be established, but in theory at least ψ = 360 degrees, ϕ = 180 degrees.

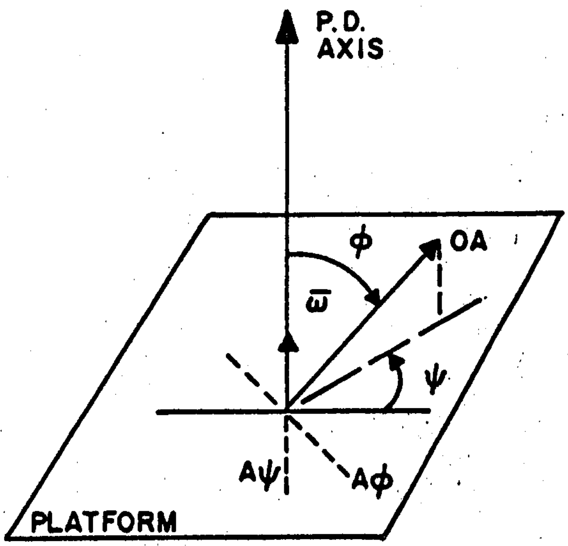


FIGURE 1-1

These rotations enable the navigator to put the planet first in the finder field, then in the telescope field. By properly adjusting the telescope in ϕ and ψ the image can be moved to such a position in the field that, with the telescope clamped, the PD rotation will carry the image through or near the center of the field.

The search for and recognition of planets must be carried out by the navigator. No reasonable mechanization presently known could provide for the automatic search of an object whose apparent position on the sphere depends on the motion of itself and on the position of the vehicle.

Once the planet has been acquired and is drifting across the field, the function of the instrument is to indicate the instant at which the center of the planet transits the center of the telescope field. It is assumed that the center of the field coincides with the optic axis of the telescope and is clearly defined as the intersection of rectangular crosshairs or reticle lines. In the simplest sense, this transiting adjustment can be carried out manually by the observer. This involves a visual judgment in regard to the location of the center of the planetary body and then a second judgment as to the instant of coincidence of this center and the center of the field. At this instant, a record of time is made by the observer. No planet appears as a clean circular disc of uniform brightness--there are shadings and markings, and the planets will also show phases. These factors probably rule out the possibility of manually locating planetary centers with enough accuracy to make visual observation feasible, with a simple telescope viewer.

Even if the planets were clearly defined discs, it would still be necessary to adjust the telescope in ϕ and ψ to assure a true center transit. This would mean continuous corrections until the instant of transit, at which time the telescope would have to be clamped to the platform, rotating with it at PD rate thereafter. The path traced out on the celestial sphere by the telescope optical axis extended is called the pseudo-diurnal circle.

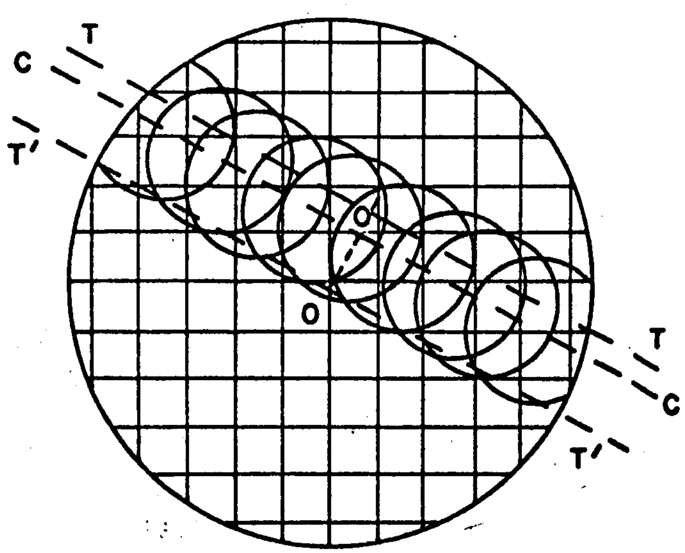


FIGURE 1-2

A method for locating the "local vertical" or planet center more accurately is this. Some point on the limb of the planet image becomes tangent successively to a series of parallel lines in the field of view, as shown in Figure 1-2. At the same time a different point on the planetary limb, 90 degrees from the first, is successively a point of tangency with a second set of parallel lines perpendicular to the first. TT and T'T' represent the lines defined by these tangent points; 0 is the center of the field.

The instant at which the planet center is at 0', the instant of closest approach, can be computed if one knows the equations of TT and T'T' in which t is a parameter. An iteration process will then determine the angular distance 00' from which one can determine the coordinates of 0' from the coordinates of 0. This iteration process is an integral part of the computation leading to a vehicle fix.

An alternate method of observation requires only two tangency point readings, one at a vertical line and one on a horizontal line. These two lines form a single orthogonal pair at the center of the field. The only difference from the previous operation is the smaller statistical accuracy obtained.

It is possible that the sequence of time measurements required for the tangent point method could be carried out by the navigator unaided by instrumentation. This involves a series of judgments as to the instants at which tangency occurs. A reticle with a series of rectangular lines may be necessary and for easiest observation this reticle ought to be rotatable so that the disc moves across the field at about 45 degrees to the lines. Note that the manual timing problem places a limit upon accuracy here.

On the other hand, no difficult judgments of planet center positions are needed in this method, and one may consider an instrumental approach. Among the possibilities for such instrumentation are Vidicon, photo-sensitive strips, and optical fiber networks using multiple, solid state sensor arrays.

The first phase of the navigation observation is completed when the planet center is located and the telescope is clamped to the platform. The data recorded in this stage includes the planet's name, a single instant of planet center transit, or a sequence of position vs. time readings (if the tangent point method is used).

Reference 1-4 discusses a pyramidal mirror method for observing local vertical at low altitudes.

1.4.2.3.2 Star Search, Acquisition, Transit Observation

The telescope, now clamped to the platform, swings along the pseudo-diurnal circle. The navigator refers again to the appropriate starcharts, and makes a choice of the four stars whose consecutive transits must be recorded. The finder telescope is again used for this purpose.

First, it is necessary to locate a specific great circle passing through the zenith of the telescope. In addition, the navigator must be able to choose any of the possible great circles whose common point of intersection is the zenith point, and a line must be inserted in the field of the telescope to represent the image of this great circle. It is across this "great circle" that the navigation star actually transits, so the navigator must see this occur in the field of the telescope. This is not actually a difficult requirement to meet. Figure 1-3 is a schematic diagram of the telescope and auxiliary observing mirror.

The mirror M is supported on an axis AA normal to the great circle plane. As the mirror rotates about AA, it will reflect into the telescope at 0 stars at various points on the great circle. On the mirror surface is a line L or an illuminated cross hair, perpendicular to the axis direction, and hence lying in the plane. This line, if imaged in the field, will coincide with the image of the great circle and clearly indicate its position. It should be noted that if L lies in the plane and not just parallel to

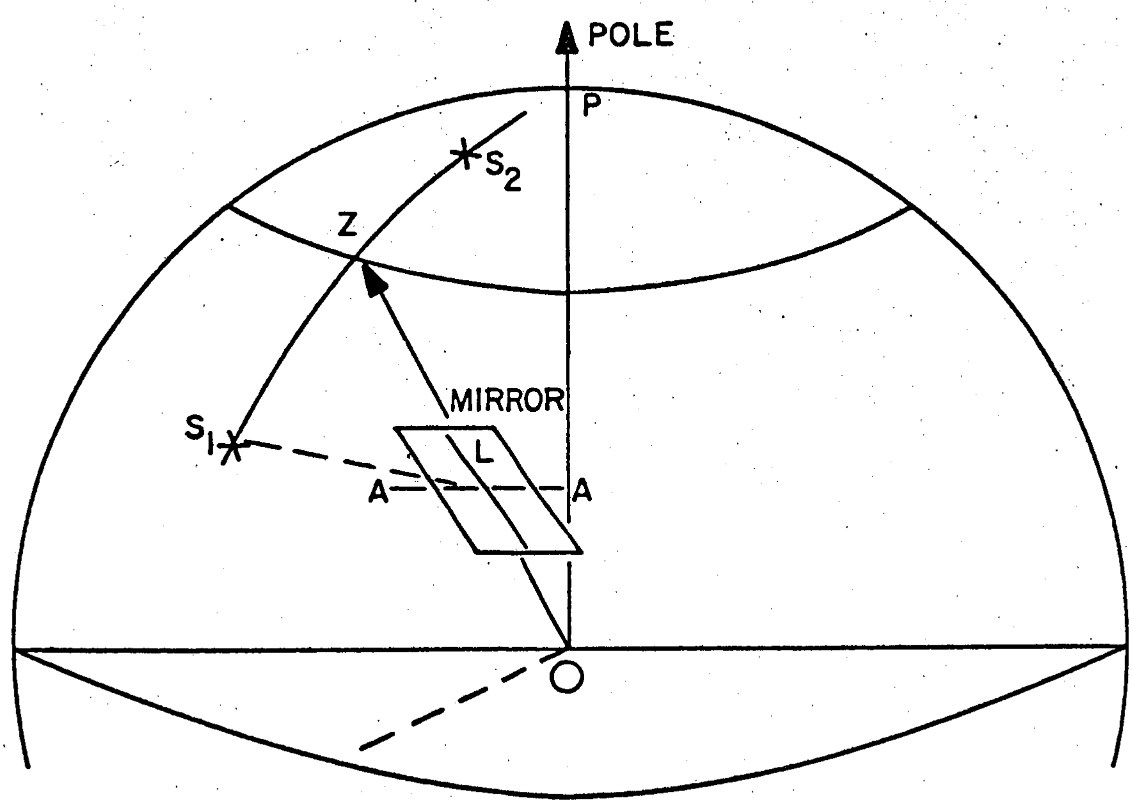


FIGURE 1-3

it, its image will pass through the center of the field, the optic axial point. In this discussion, the words "mirror" and "illuminated cross hairs" are used to help describe a necessary function of the telescope, which probably will be carried out using other optical elements.

Now two stars lying close to a great circle through Z are chosen. As the telescope swings along the PD circle, first one and then the other star transits the great circle line. Using star charts, making a choice of these stars should be reasonably quick and easy. It is most unlikely, and from a geometrical point of view undesirable, that the second star of the pair be close enough to the first to be seen at the same mirror setting. Therefore, if Figure 1-3 shows the mirror orientation for star one (S_1), the mirror must be rotated about AA in order to see S_2. During rotation, axis AA must be held as closely as possible to its original orientation.

After noting each transit, the navigator turns M to a second great circle position and repeats the process. Note that this demands a rotation of M around the optic axis OZ of the telescope. Again, it is strictly required that this rotation not displace the image of the great circle; this image must still pass through the optic axial point in the field.

It has been stated that there are rigid requirements on the rotations of the auxiliary mirror. It is possible that mirror orientation cannot be held to the required accuracy. In this case there must be a capability for realigning the mirror after each change in its position.

So far, it has been implied that the transit observations have been done visually by the navigator. This is a possible operation, and one easier to carry out than the planet center observation, involving only the determination of the instant at which the star image crosses a well defined line in the field. But again, it may be considerably more accurate to do the transit timing with instruments-- a photocell sensor in the field, for instance. This device may be combined with or be a part of the same instrumentation used for planet center location.

The second phase of the navigation operation is completed when the star transits have been noted. The name of the stars, and the instants of transits are recorded.

Figure 1-4 is a schematic representation of the mechanization of the pseudo-diurnal method, but it is in no way to be understood or viewed as a diagram of proposed or actual pieces of hardware. The basic characteristics and capabilities demanded by the method are included. Even though the final configuration may be quite different, it must include these characteritics and capabilities. For example, some of the four motions 1, 2, 3, 4 may be redundant or be combined in the final design.

1.4.2.3.3 Other Instrumentation

For clarity, Figure 1-4 indicates three necessary instruments in addition to the telescope. First, the platform rotates with respect to the vehicle, and this relative rotation is determined by the angle measuring device. In turn, this angle must be referenced to the celestial sphere to get absolute pseudo-diurnal rotation. Not only must the angle be measured, but the direction of the pseudo-diurnal axis must be established and maintained. This is the function of one of the star trackers. Again, one must not look upon these devices as actual pieces of hardware, but only as representations of functions.

1.4.2.3.4 The Navigator's Role

The navigator perhaps might visually and manually carry out every operation needed in the pseudo-diurnal method. It may be true that the actual transit recording would be more accurate if done automatically, but no device could do a better nor more rapid job of recognizing and acquiring planets and stars than the navigator, if he has proper help.

First, he needs some general acquaintance with the distribution of the navigational stars on the celestial sphere as well as the apparent positions of the planets at the epoch in question.

Second, a set of proper star charts must be provided. For each possible polestar there is a set of charts with the stars plotted in terms of the coordinate system associated with that particular polestar. The telescope itself perhaps is equipped with scales of longitude and latitude, associated with motions 1 and 2, respectively. Simply setting the proper coordinates on these scales puts the star in the finder, if the zero mark on the longitude scale is properly established.

Third, the vehicle is in some predetermined flight path within small limits, so the navigator can also be provided with additional charts indicating the apparent planet positions at certain times. Again, such information will help him choose a useful polestar and rapidly locate a suitable planet.

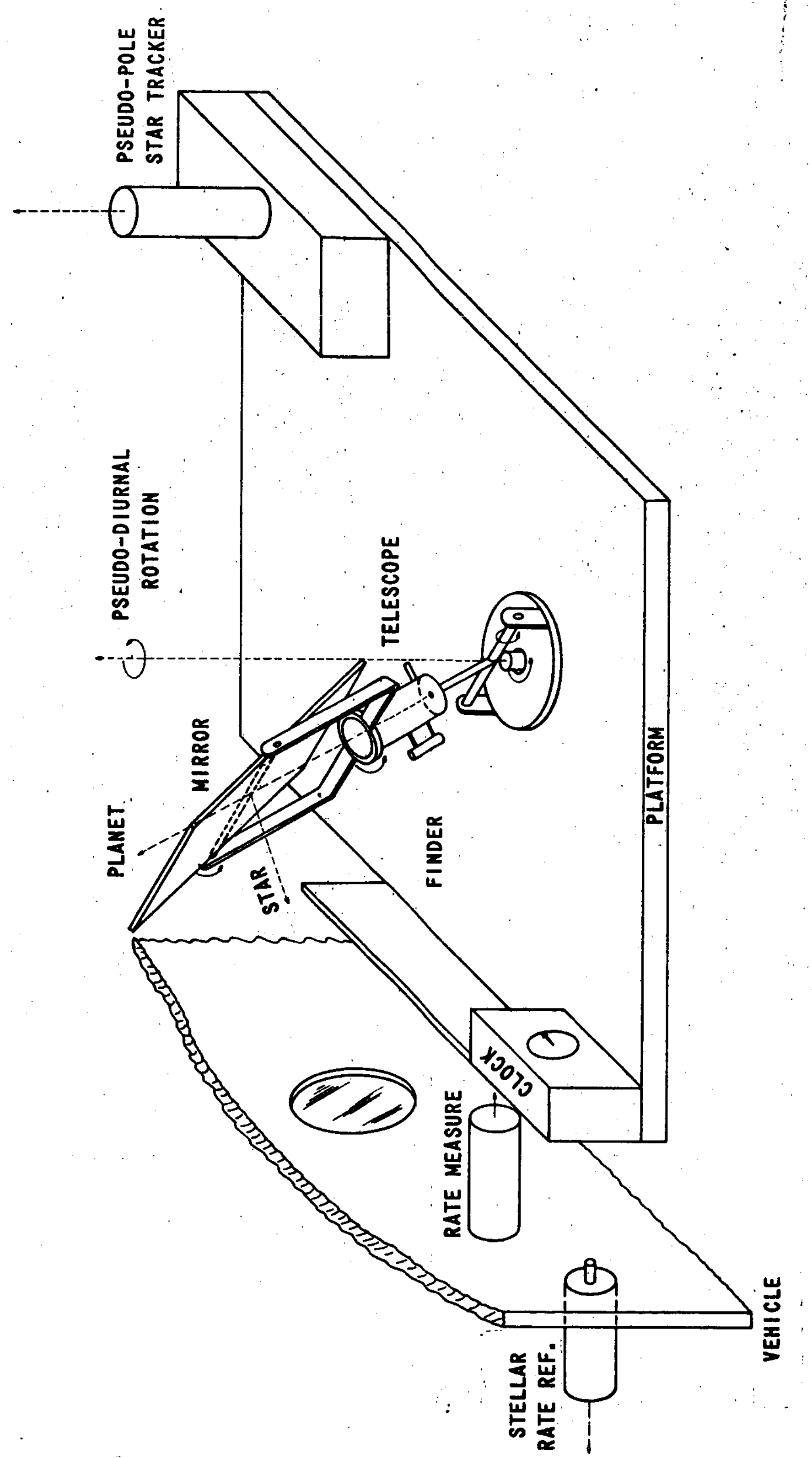


FIGURE 1-4
FUNCTIONAL REQUIREMENTS

In addition to the task of locating stars and planets, the navigator should also carry out such simple operations as orienting reticle lines, clamping the telescope to the platform, and realigning the auxiliary viewing mirror after rotation. In other words, the only non-manual, non-visual operations are the actual recording of the transits. The rest of the procedure is strictly within the province of the navigator, albeit, calculations might better be done by a computer.

1.4.2.4 The Effect on Instrumentation of Navigation Star Distribution and Choice of Pseudo-Polestar

The observing instrument will be used to sight planets at 90 degrees ±20 degrees from the pseudo-diurnal axis. The navigational stars to be used lie in a 30 degree band about the coordinate system's equator. Charts showing these stars are given for each of four chosen polestars, and a gnomic projection of a 30 degree area of the celestial sphere is shown with an example of the navigation calculation.

1.4.2.4.1 Choice of Pseudo-Polestar

The fundamental demand of the pseudo-diurnal navigation technique is the choice of an axis of pseudo-diurnal rotation. This axis is established generally as a line extending from the vehicle to a known star. It is not the purpose of this subsection to discuss the mechanical requirements of vehicle stabilization or star tracker accuracy needed.

In choosing a polestar, there are several criteria which must be considered. The telescope must sweep across a planet to define the angle between the optic axis of the telescope as a reference direction and a chosen pseudo-diurnal axis. Figure 1-5 illustrates this point, where σ is the defined angle. The planets (and the vehicle) will always be within a few degrees of the ecliptic plane. There is no fundamental need that σ be 90 degrees, but further considerations indicate the wisdom of such a choice.

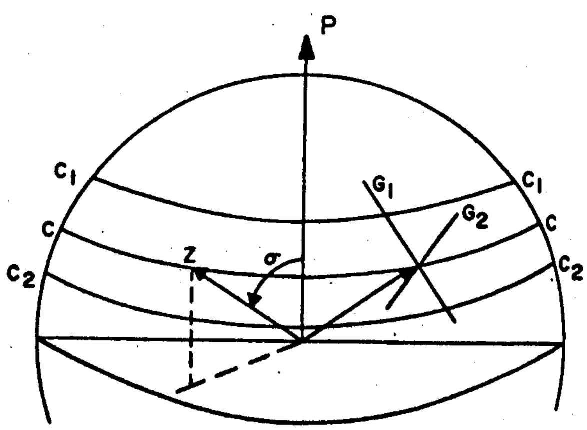


FIGURE 1-5

First, four stars could be chosen to provide the iso-azimuthal pairs* eventually required for a line-of-position. In doing this the telescope

*Actually, two stars are sufficient.

is rotated about the PD axis, along the small circle CC and searches a band centered on this circle, shown as limited by the circles C_1 C_1 and C_2C_2. It is obvious that if σ = 90 degrees, the band will include the greatest area and, therefore, the greatest number of navigational stars are available for use. If instrumentation allows the telescope to search for stars in a band 30 degrees wide near the ecliptic, a sufficient number of stars is available.

Secondly, the observing of transits, either visually or instrumentally, depends on the passage of a star across a line. Such a transit is more sharply defined where the angle of crossing is large, 90 degrees being the optimum condition. This angle depends upon the orientation of the great circle chosen for transit and the location of the star in question. For a simpler instrumentation the great circles are restricted to two in number. These circles (passing through the optic axis intersection with the celestial sphere and moving with the pseudo-diurnal rotation) make 45-degree angles with the great circle containing the optic axis part and the pseudo-pole. This is shown in Figure 1-5, with great circles marked G_1 and G_2. This means that with Z on the equator crossings occur at 45 degrees, even for stars close to the equator. As we use stars further away from the equator, at higher latitudes, the transit angle becomes more shallow, becoming zero at 45 degrees latitude. If the planet is well above the equator, this effect seriously narrows the choice of stars which make well defined transits and are still within reach of the 30-degree search pattern of the telescope.

Finally comes the question of resolution. Let us suppose that ϵ represents the smallest angle which can be resolved by the telescope. This is quite independent of telescope orientation, as shown in Figure 1-6. Either on the equatorial circle EE or the small circle SS, two points or two events are indistinguishable if they are separated by an angle less than ϵ. But the uncertainty in the pseudo-diurnal angle which is measured in the equatorial plane is also ϵ and can be defined as the angle between the two great circles 0_1P and 0_2P. Quite clearly the angle of uncertainty along SS marked δ is greater than ϵ and the angle of uncertainty along EE. Hence, greater errors in determining positions will occur when the objects viewed are at higher latitudes.

The results of these considerations is obvious. Polestars close to the north or south ecliptic poles should be chosen to establish the angle σ at approximately 90 degrees.

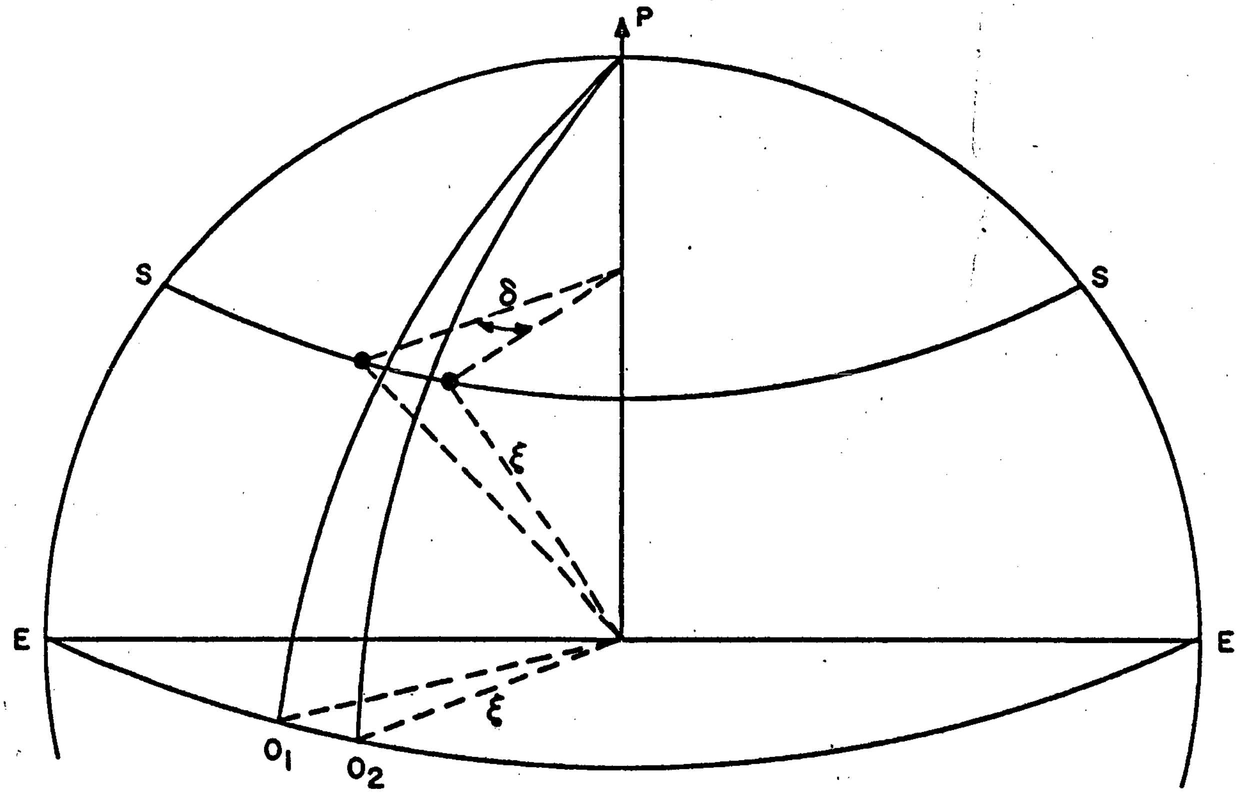


FIGURE 1-6

There are four stars, two near each ecliptic pole, which can serve as polestars. They are:

	Star Name		Celestial Equatorial Coordinates	
a.	β UMi	Kochab	222° 41' 54.0"	74° 18' 24.0"
b.	α Cep	Alderamin	319° 25' 27.0"	62° 25' 42.0"
c.	β Car	Miaplacidus	138° 12' 3.0"	-69° 33' 53.0"
d.	α Car	Canopus	95° 46' 57.0"	-52° 40' 30.0"

In the mathematical process of reducing the observations, it becomes necessary to transform the coordinates of the iso-azimuthal stars into a system defined by the chosen pseudo-polestar. In Reference 1-2 (Honeywell MPG Document R-ED 6206, "A Self-Contained Celestial Navigator for Interplanetary Travel," by V. S. Kardashian) it is shown that the proper transformation is:

$$\begin{pmatrix} \cos\beta \cos\lambda \\ \cos\beta \sin\lambda \\ \sin\beta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\delta' & -\cos\delta' \\ 0 & \cos\delta' & \sin\delta' \end{pmatrix} \begin{pmatrix} \sin\alpha' & -\cos\alpha' & 0 \\ \cos\alpha' & \sin\alpha' & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{pmatrix} \quad (1.4)$$

which gives

$$\tan\lambda = \frac{\cos(\alpha' - \alpha)\cos\delta\cos\delta' - \sin\delta\cos\delta'}{\sin(\alpha' - \alpha)\cos\delta} \quad (1.5)$$

$$\sin\beta = \cos(\alpha' - \alpha)\cos\delta\cos\delta' + \sin\delta\sin\delta' \quad (1.6)$$

where $(\alpha'\delta')$ and $(\alpha\delta)$ are the celestial equatorial coordinates of the pseudo-polestar and the star to be transformed, respectively.

The new system of coordinates has its pole at the pseudo-polestar and the zero of its longitude scale at a point $(3\pi/2 + \alpha')$ around the celestial equator in an easterly direction. The equatorial plane of the new system intersects the celestial equatorial plane at this point.

More important for present purposes is the fact that the result of this transformation, when applied to all the other navigational stars for the case of a particular polestar, gives coordinates which can be plotted to show star distribution in the desired 360 degree band about the new equator. Since each of the four pseudo-polestars is near an ecliptic pole, the equators of the systems will be quite close to the ecliptic.

These transformations were carried out on the Honeywell 800 computer and the results are given in Tables 1-1 to 1-5 of Appendix 1-B. A more informative presentation is in the following star charts which show the stars within ±30 degrees of the equator for each of the four cases. It can be seen at once that in every case the number of stars available is quite sufficient for navigational purposes. The planet positions have not been shown, although their coordinates are given in Table 1-5. The apparent position of the planets quite obviously depends upon the vehicle position in space, and is within 20 degrees of the equator.

1. 4. 2. 4. 2 Star Charts and Navigation Procedure

The charts marked 1-A, B, C, and D display simple rectangular coordinates, and therefore do not represent the correct spatial relationship of the stars, although the distortion is not great since the charted areas center about the equator, the zero latitude circle. To show the stars in their true relationship, or if one wishes to discuss a graphical solution of the navigation problem, some other representation is required. Possible graphical solution methods are described by V. S. Kardashian in Appendix 1-D.

An example of a gnomic projection chart is shown as Chart 1-E. It covers a field of view of 30 degrees diameter, using a one degree grid. It can be read to 0. 1 degree, even on the scale used. Larger charts to the same scale, or charts covering smaller areas on a larger scale, are quite possible, of course.

The two stars shown (on Chart E), γ ORI and α ORI, are plotted in the system for which β CAR is the polestar. A planet is assumed to have the apparent position shown, although the coordinates of this position are unknown. The following sequence of observations is required. Figure 1-7 is given as an aid although a discussion of this instrumentation is deferred to a later subsection. A rotation of 360 degrees about the PD axis and a rotation of ± 20 degrees about the search axis allow the telescope to be oriented and then clamped so that a planet transit is made. The plane of the rotating ring is thereafter normal to the initial planet direction. Ring rotation about an axis at 0 is therefore rotation about planet direction. Finally, the telescope rotates ± 15 degrees about the great circle axis in one of the two positions allowed by the stop. These rotations, as discussed in paragraph 1. 4. 2. 3. 2 provide, in fact, the four basic motions required for the observing instrument. Note that the ± 15 degrees refers to altitude change, so that the angle about a great circle axis must be ± 22 degrees.

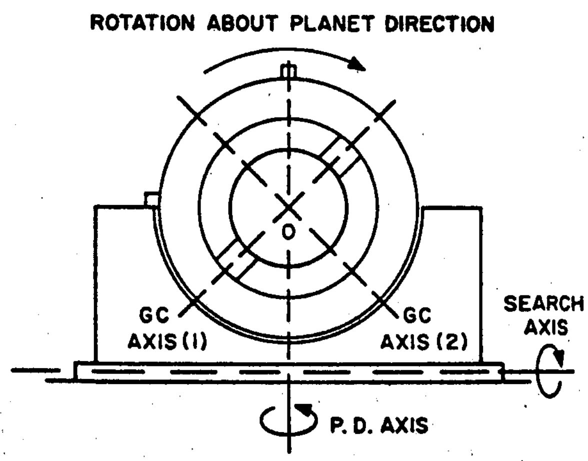


FIGURE 1-7

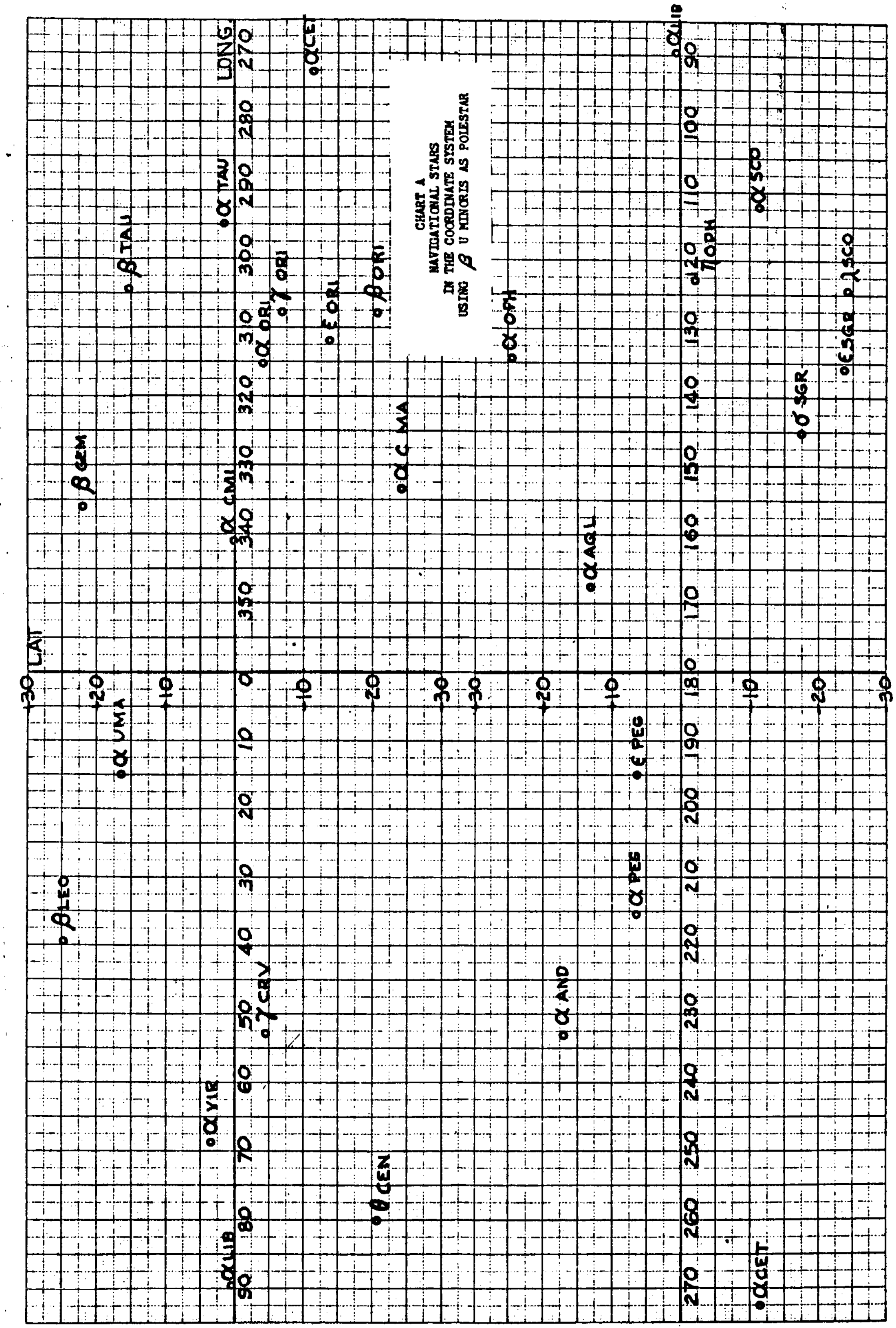


CHART A
NAVIGATIONAL STARS
IN THE COORDINATE SYSTEM
USING β U MINORIS AS POLESTAR

CHART 1-A

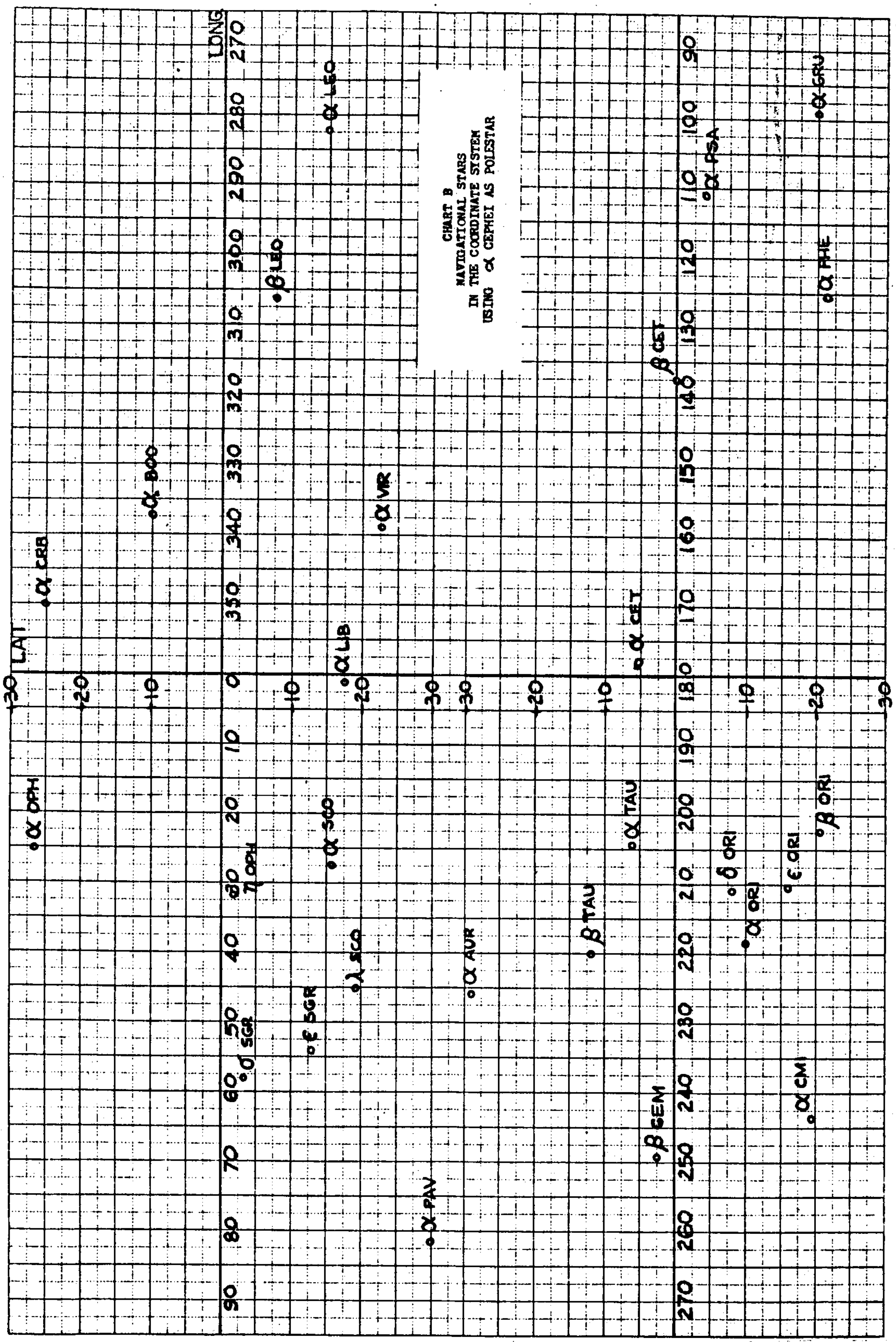


CHART B
NAVIGATIONAL STARS
IN THE COORDINATE SYSTEM
USING α CEPHEI AS POLESTAR
LAT
LONG
α LEO
β LEO
β CET
α GRU
α PSA
α PHE
α BOO
α VIR
α CRB
α LIB
α CET
α OPH
η OPH
α SCO
λ SCO
ε SGR
σ SGR
α AUR
β TAU
α TAU
β GEM
α PAV
δ ORI
α ORI
ε ORI
β ORI
α CMI

CHART 1-B

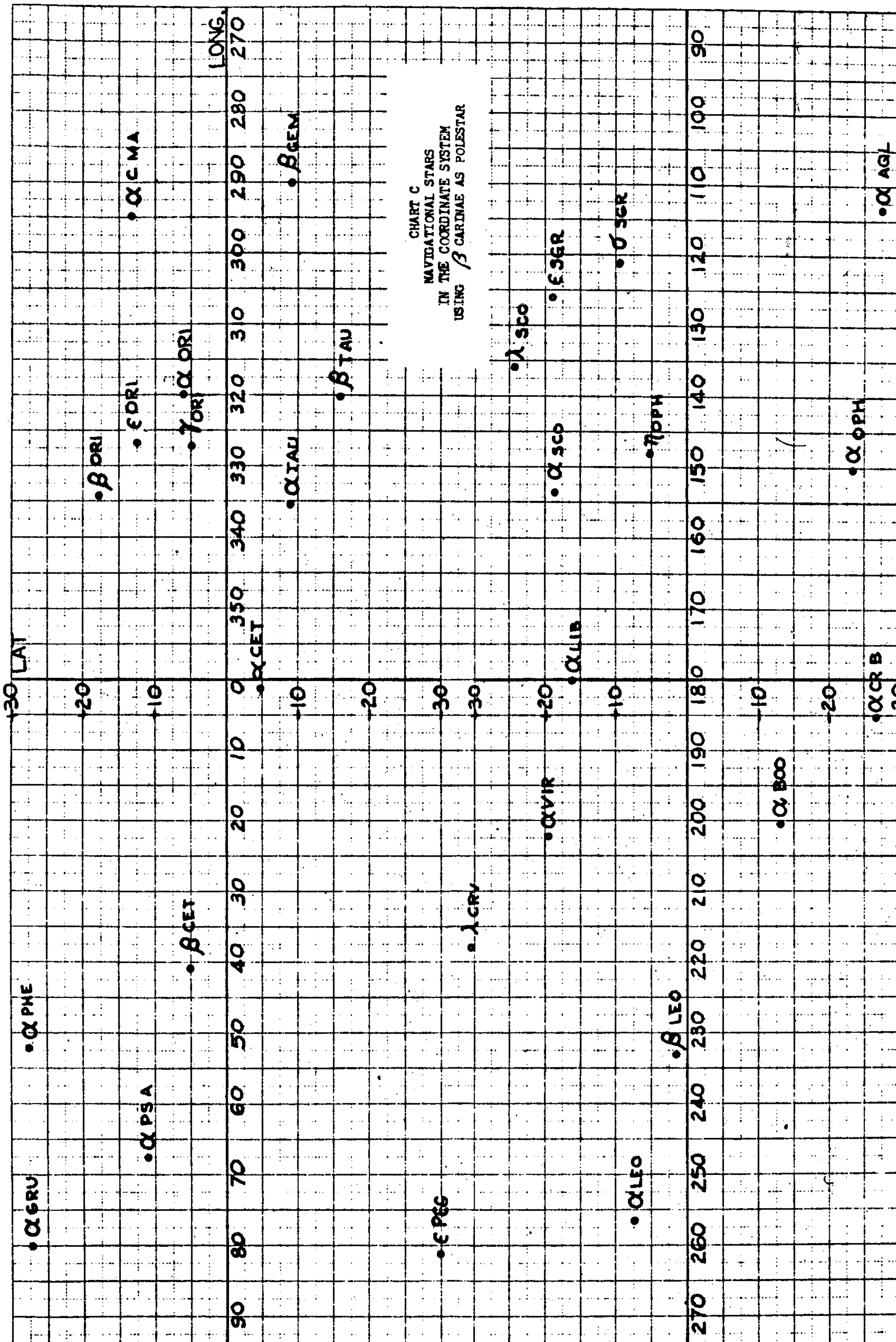


CHART C
NAVIGATIONAL STARS
IN THE COORDINATE SYSTEM
USING β CARINAE AS POLESTAR
LONG.
LAT

CHART 1-C

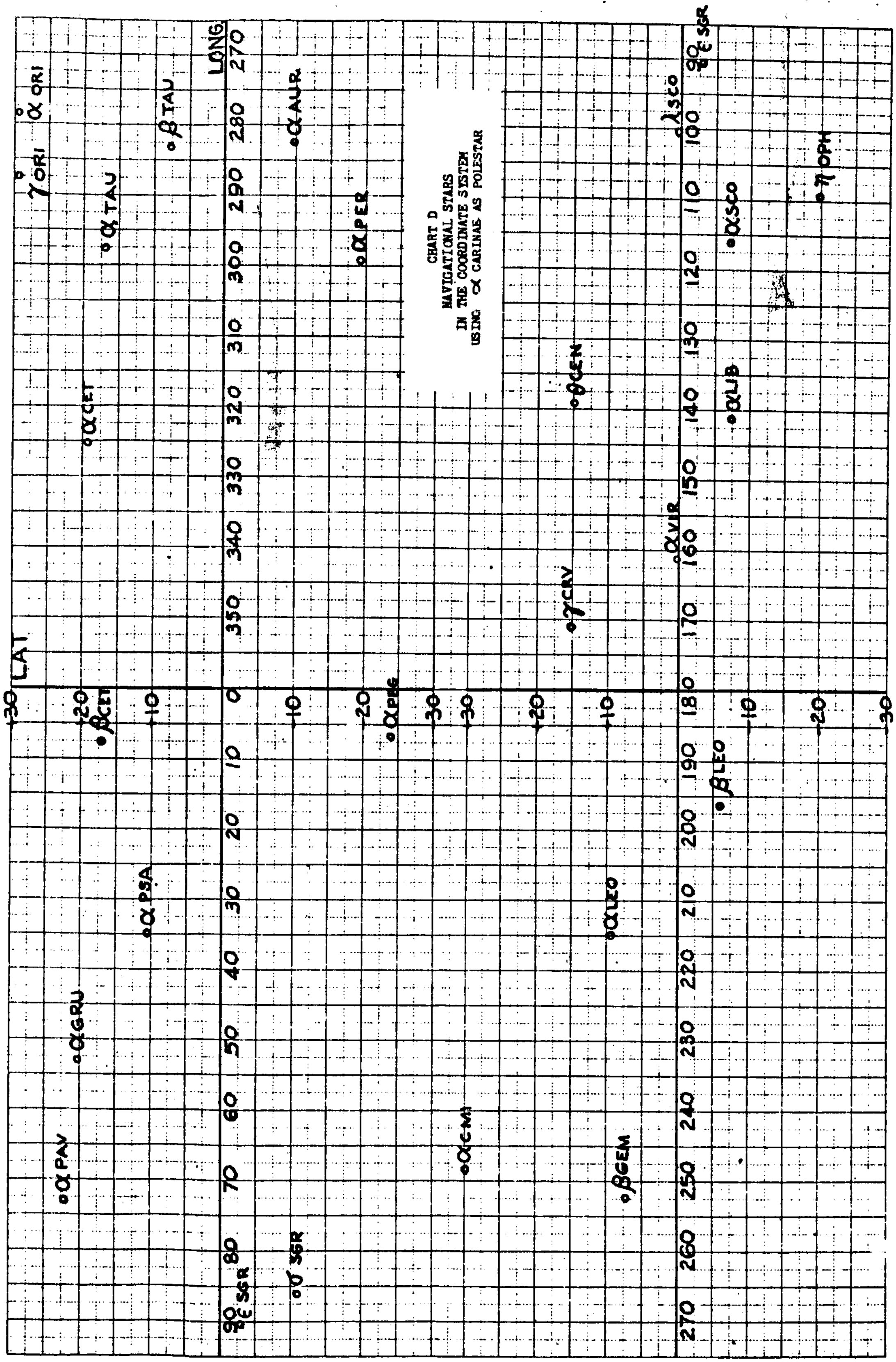


CHART D
NAVIGATIONAL STARS
IN THE COORDINATE SYSTEM
USING α CARINAS AS POLESTAR
LAT
LONG

CHART 1-D

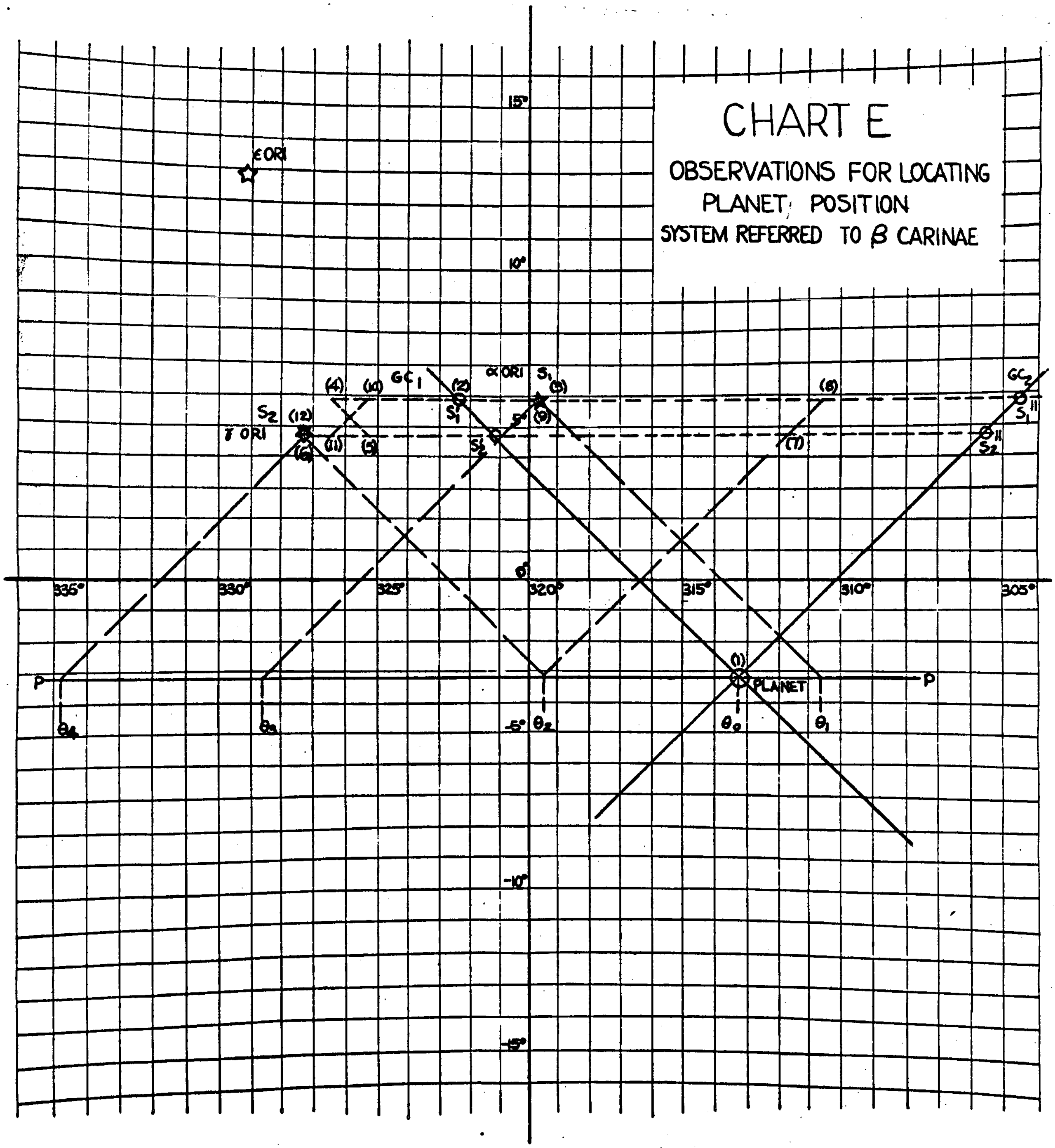


CHART E
OBSERVATIONS FOR LOCATING PLANET POSITION
SYSTEM REFERRED TO β CARINAE
ε ORI
α ORI
γ ORI
PLANET
15°
10°
5°
0°
-5°
-10°
-15°
335°
330°
325°
320°
315°
310°
305°

CHART 1-E

Now return to Chart 1-E. Once the planet is acquired, no further rotation about the search axis is required or allowed. Pseudo-diurnal rotation moves the telescope along the path PP on the celestial sphere, and no motion of the telescope will affect this angle. Note in passing that angle circles can be associated with the search axis and the great circle axes as an aid to star location. The main angle scale is already measuring the PD angle. The angles θ_0, θ_1, etc. are indeed measured on this main scale. This schedule of observations now follows:

1. The planet is acquired and θ_0 is recorded as the initial or reference angle in longitude. It is only a scale reading and has no absolute significance.

2. The telescope rotates about GC_1 to the "latitude" of S_1. PD rotation moves the telescope across S_1 and the transit angle θ_1 is recorded.

3. The telescope is moved along the latitude circle of S_1 until it is near S_2, it is adjusted to the S_2 latitude and the transit angle of S_2, θ_2, is recorded.

4. The telescope is rotated to the other great circle sweep position, either remaining at the latitude of S_2 or at any other latitude desired.

5. The whole sequence of events is now repeated either using the same two stars, as in our example, or using two others, or one new and one old one.

6. A motion about the PD axis gives a transit of S_1 and θ_3 and then a further motion and adjustment in latitude gives θ_4 as the transit of S_2.

NOTE: The numbers in parentheses represent the sequential positions at which the telescope is pointed.

Since the only motions which are measured are those about the PD axis (along circles of latitude), we may correct the known star longitudes to obtain the fictitious stars S_1' S_2' S_1'' S_2''. These corrections are

$$\begin{aligned} \lambda_1' &= \lambda_1 - (\theta_1 - \theta_0) & \lambda_1'' &= \lambda_1 - (\theta_3 - \theta_0) \\ \lambda_2' &= \lambda_2 - (\theta_2 - \theta_0) & \lambda_2'' &= \lambda_2 - (\theta_4 - \theta_0) \end{aligned} \qquad (1.7)$$

It is to be noted that we have assumed a capability for rotation in either sense about the PD axis. If this is not desirable, one need only choose to record transits in a different order, or choose other stars. The stars need not be as close to the planet as shown here and it would be better if they were of greater latitude separation.

Finally, it is quite clear that if one now plots the fictitious coordinates on the chart, the intersection of the two great circles thereby established (by $S_1' S_2' S_1'' S_2''$) is the position of the planet.* The accuracy of a graphical determination is limited only by the magnitude of the graphical scale, if no observational errors are assumed.

The example chosen above must not be understood to set up a rigid pattern of telescope motions. Once the planet is acquired and the ring locked into position, the subsequent order in which stars are observed is not important and will be determined by their spatial distribution.

1. 4. 2. 5 Recommended Configuration

Many possible choices exist for almost any given component of the total system required by the pseudo-diurnal technique. A detailed discussion of each component in its various forms is given in Paragraph 1. 4. 2. 6. From these detailed considerations, a preferred configuration can be recommended, and this follows.

1. 4. 2. 5. 1 Vehicle Stabilization

The vehicle will be stabilized about the pitch, roll, and yaw axes to within ±10 arc minutes during transiting times. The deviation from nominal will be read out by a star tracker and form a correction in the computer that will result in a true star-referenced angle. Two star trackers will be necessary for this correction, one to track the selected pole star, another to track a star at approximately 90 degrees from the pole star. Stabilization of the vehicle is held to a 20 arc minute error circle in order to obtain a resolution of displacement angle to 0. 2 arc second. Star trackers have been built having a resolution ability of 0. 5 second of arc with a 1 degree field as discussed in Paragraph 1. 4. 2. 6. 5. The reduction in field from 1 degree to 20 arc minutes should make feasible a resolution accuracy to 0. 2 arc second. Both the correction errors recorded will apply at all transit points, since all transiting is done about 45-degree lines in reference to the pseudo-diurnal axis.

*At the time of noting θ_0.

Error Contribution

The error discussed above (0.2 arc second) will range for any measurement of θ (the angle measured in the equatorial plane between stars or planet and stars) from 0 to 0.8 arc seconds, depending on the degree of compensation by the vehicle motion about a transit point and between transit points.

1.4.2.5.2 Measurement of Angle

The measurement of θ (the angle in the equatorial plane of the selected pole star between stars or planet center and stars) can be accomplished within the accuracy limitations by either of two methods which are, in summation, of equal desirability:

a. Interferometry Method

The incremental bit size is 0.1 arc second and the total angle would be a summation of these bits. A counting rate of 1,000 bits/second has been achieved, which does not necessarily represent the ultimate limitation. A counting rate of 1,000 bits/second, a measuring rate of approximately six times earth rate, gives an average reading time of 20 minutes for approximately 30 degrees of range. Power failures or the like would negate the measurement proportionally. Since the bit size is an order of magnitude smaller than any other method of measurement, the system represents a forward step in the "state of the art". With possible improvements in stabilities and frequency of fiduciary reference, the method probably presents the most satisfactory approach for future angle measuring problems. Advances in coherent optics may make it even more attractive.

b. Dynagon Method

On instrumentation under construction, the bit size is 1.24 arc seconds but by increasing the diameters of the digital discs to 12 inches, in order to obtain more graduations on these discs, and by improving the bearing by a fluid suspension, the bit size can be reduced to 0.16 second of arc. The basic advantage of this device is that for each revolution of the spinning disc (1 - 2 revolutions per second) there would be a whole number readout referenced to the starting point. Power shutoff or instabilities occurring in the period between time 0 and the time of final transit minus 0.5 - 1.0 second, would not affect the accuracy of readout. (Note: RSS error of the 12 inch dynagon is ±0.25 arc second; the basic resolution is ±0.16.)

c. Error Contribution

The minimum expected error for each θ determination will be equal to one-half of the bit size for each transit or a total for each transit of 1 bit which is 0.1 arc second for the interferometer or 0.16 arc second for the Dynagon.

1.4.2.5.3 Transit Measurement and Image Transfer

There are two considerations involved in discussing optical equipment. First is the method of determining the instants of star and planet transit. By using a photosensitive surface divided into two parts along a line in the surface, essentially a photosensitive "reticle line" is produced. Experience with this device indicates that angle indicating accuracy on the order of 0.05 - 0.10 arc seconds is attainable. A suggested mechanism for this is shown in Figure 1-18.

It is required that the images found by the finder and telescope be transferred to an eyepiece or screen and there viewed by the navigator. No question of accuracy is involved, but without this visual information the navigator cannot perform his functions. Two general solutions are possible. A train of optical elements probably passing through the vehicle wall, and articulated to compensate for the observing instrument rotations is one possibility. The other is the use of one or two Vidicons at the observing instrument and the electronic transfer of the images to the navigator. The Vidicon method seems likely to be preferable, if the resolution is great enough, and if in-flight replacement can be provided for.

Figure 1-8 is a diagram of this preferred system, showing the many components in their relationship to each other and to the vehicle. The mechanization of Figure 1-8 would result in the navigation procedure illustrated in Figure 1-9. The sequence of observations from beginning of planet search to completion of final star transit can be followed in the order they are carried out. The finder and telescope fields in their indicated motions and observation positions show what the navigator would be seeing.

1.4.2.5.4 Optical Instrument Axis Errors

It has been stated that the axes of the four necessary rotations have requirements as to accuracy. The pseudo-diurnal axis has been discussed above. The search axis poses no problems other than the need that it can be firmly clamped into a fixed position and retain this position. The motion of the telescope in its great circle axes requires very accurate rotation along these two directions. It is also necessary to have a fixed position of the telescope which puts its optic axis normal to the

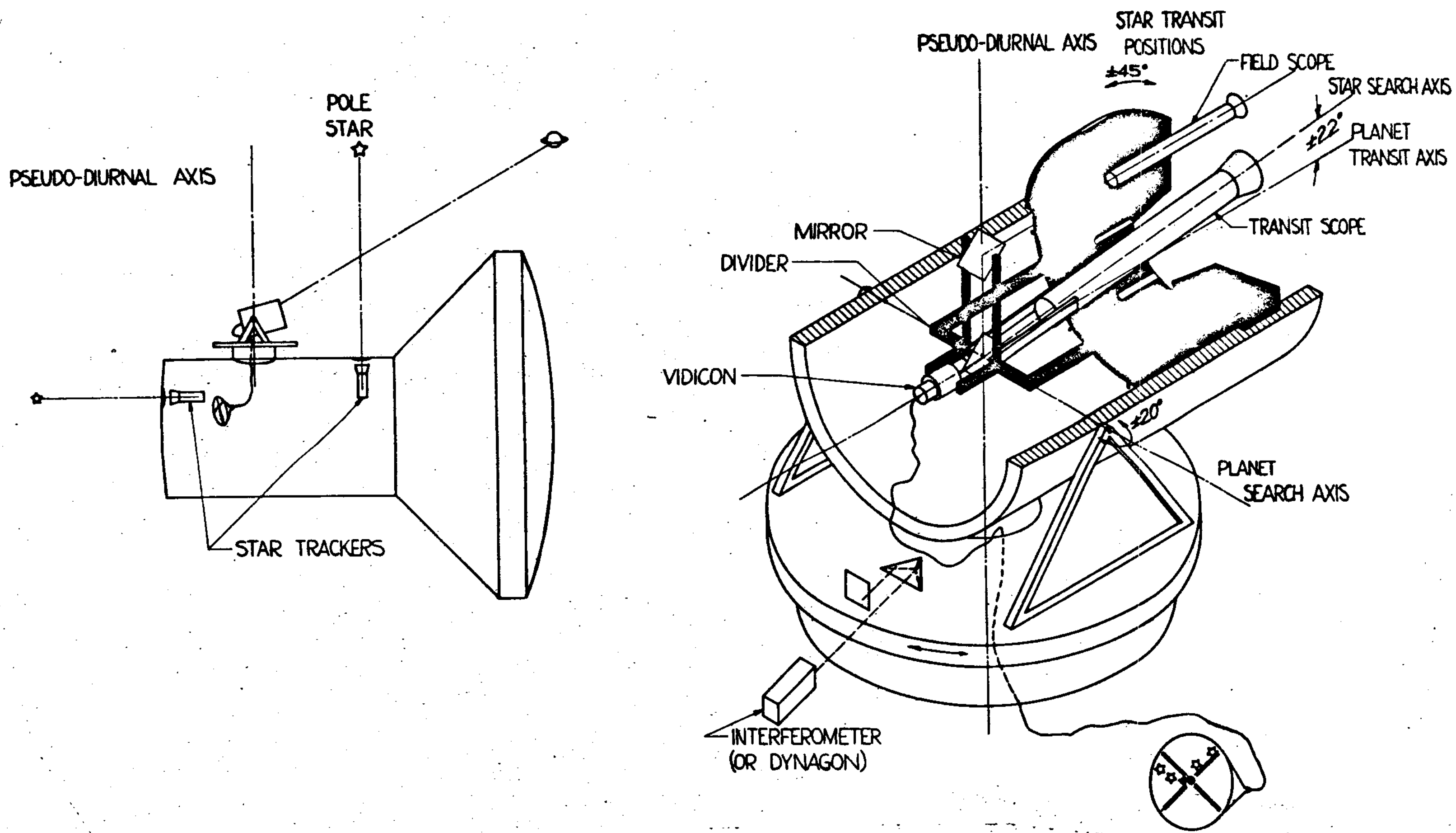


FIGURE 1-8
PSEUDO-DIURNAL TRANSIT
PREFERRED CONFIGURATION

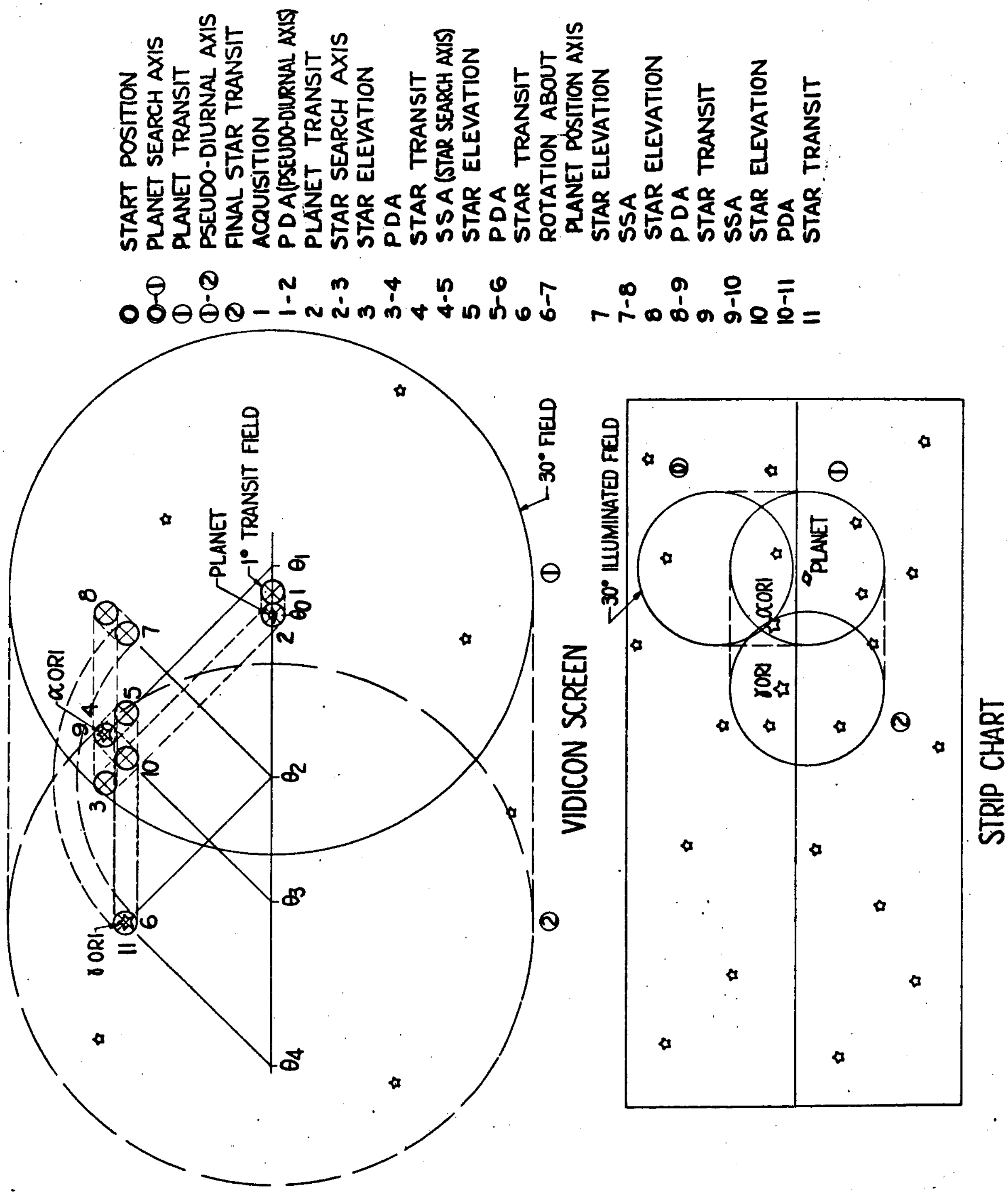


FIGURE 1-9
NAVIGATOR OBSERVATIONS

plane of the ring. Finally, as the ring rotates, its plane must remain fixed in the clamped or planet orientation. The errors introduced into the position fix are not necessarily proportional to the errors in axis orientation. Furthermore, these axis errors can be circumvented by calibration of the instrument. Regardless of the details of mechanization, errors in the four rotation axes will exist and are considered in the detailed analysis of Section 3.

1. 4. 2. 5. 5 Computer

The mathematical computations involved in reducing the simple data of the observing method to a position fix are straightforward applications of spherical trigonometry and analytic geometry. There are many such calculations, particularly in the iteration method used to circumvent the difficulties of determining planet center transit. The navigator, with a slide rule and a set of Math tables could carry out the necessary work, but this would be a lengthy job with many opportunities for error.

State of the art computers seem to be well able to handle the problems of the pseudo-diurnal system. In view of the time required for a set of observations (some 20 minutes), the data becomes available piecemeal. In any midcourse maneuver, there is no short-time course correction required. It follows that required computer speed is of a low order. Some information needs to be stored, a brief ephemeris for example, but this could be put on tape. For the calculations involved, a memory in the order of 1,000 words is sufficient. An ordinary computer will give seven or eight place accuracy, at least an order of magnitude greater than needed. Finally, neither input or output rate is at all high. Present on-board computers easily fulfill these requirements, with storage capacities of 3,000 to 30,000 words.

1.4.2.5.6 Preliminary Estimate of Total Error

In terms of the discussion above, and other sources of error inherent in the method (but not yet discussed in detail), it was estimated that total error in a single measurement will be from 1.5 to 3.0 seconds of arc. Section 3 presents the complete error tabulation (showing 4 arc seconds in a line-of-position determination).

1.4.2.6 Alternate Mechanizations

Many different ways of performing the functions demanded by the pseudo-diurnal technique have been considered in arriving at the preferred configuration. Most of these alternate schemes are discussed in the following paragraphs.

In the original proposals for the pseudo-diurnal technique the optical instrumentation was very complex, particularly the mirror arrangements used to search the great circles. The relaxation in the demands for large angle rotations about the various axes and the possibility of limiting the great circle motion to two, 90 degrees apart, makes such complexity unnecessary. In comparing the alternate schemes the principal criterion of judgment is simplicity. The fewer the optical elements and the smaller the angular rotations needed the fewer errors will arise out of the mechanization itself.

1.4.2.6.1 A Basic Arrangement

Figure 1-10 shows what must surely be the simplest possible mechanization model for the pseudo-diurnal technique. The navigator orients his vehicle about some pseudo-diurnal axis and observes the sky visually through a porthole. As his vehicle rotates, his simple viewer is directed at a band on the celestial sphere which contains both a planet and two pairs of stars. He observes and times the planet and star transits on the crosshairs, and from these data computes his position. The viewer field need not contain both planet and star pairs simultaneously. Vehicle rotation will bring such pairs into the wide field of the viewer eventually. An outboard mirror provides for observation of the pseudo-polestar.

All of the fundamental processes of the method under study here are displayed, and the four required rotations are available in at least a rudimentary fashion. Quite obviously the accuracy obtainable would be very poor and the star pairs available might entail long waits between planet and star pair transits.

1.4.2.6.2 Star Search Mirror Arrangements

The realization that the iso-azimuthal mode, which in its original form requires simultaneous transit of the stars of a pair, can equally well be applied to consecutive, non-simultaneous, transits of the two stars, in itself provides a real simplification. Since only one star at a time must be seen in the telescope field, only one auxiliary search or viewing mirror is required. Figure 1-4 illustrates the kind of instrument which could be developed along these lines. There are two grave difficulties however. First, a mirror surface in space is probably subject to rapid deterioration. The other difficulty is the necessity of holding a very strict orientation of the mirror surface or axis in respect to the telescope itself, and of providing "reticle lines" for planet transit separately from those used in star transits.

Several schemes for mounting such a star search mirror are possible, but the same difficulties are involved in each.

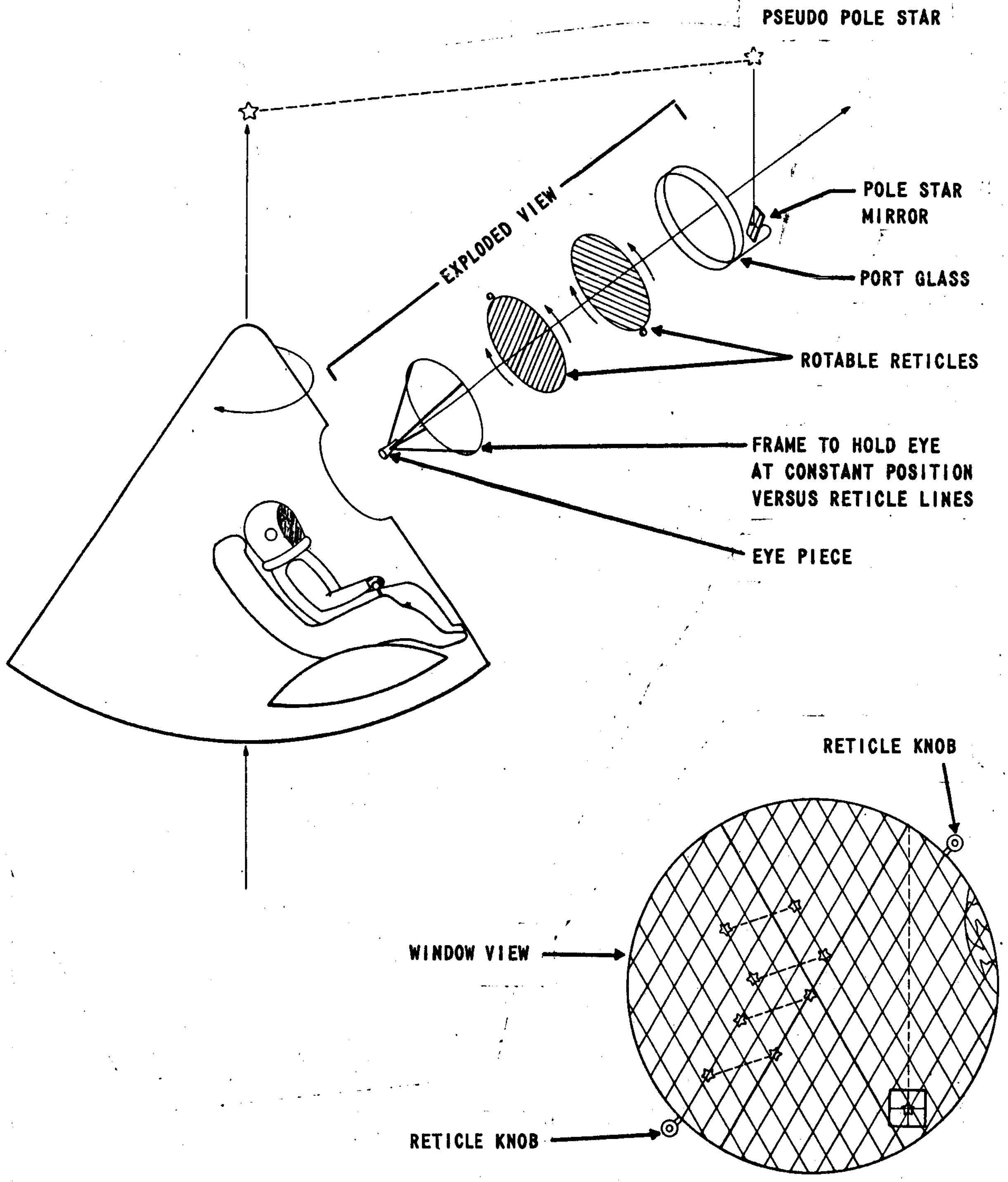


FIGURE 1-10
ISO-AZIMUTHAL METHOD (PORT HOLE)
SIMPLEST CONCEIVABLE CONFIGURATION

1.4.2.6.3 The Rotating Telescope Mechanization

In order to avoid the use of a search mirror, one must provide for a motion of the telescope which enables it to search the great circles for stars. This is possible, as shown in Figure 1-8.

Rotation about the pseudo-diurnal axis, motion No. 1, is a motion of the supporting table with respect to the stabilized vehicle. Motion No. 2 is rotation about the planet search axis and is restricted to ±20 degrees. The whole optical system is suspended on this axis so that both finder and telescope move together in this rotation. The optical system is moreover mounted on a disc and the disc itself can be moved into one of two positions, 90 degrees apart. This motion No. 3 takes place about the planet direction axis or planet transit axis which is very accurately normal to the plane of the disc. Both the finder (field telescope) and the telescope (transit telescope) optical axes are also strictly normal to the disc, the finder axis permanently, the telescope axis locked into this position while planet search and transit occurs. After planet acquisition and locking of the planet search axis, the telescope is unclamped from the disc and begins star search. In this process the telescope rotates ± 22 degrees about the star search axis, which is coincident with the planet search axis, and is motion No. 4. Note that the telescope alone moves in this last described operation. The telescope can traverse along either of two great circles that intersect at a point on the celestial sphere defined by the optical axis of the finder, or of the telescope in its original planet transit position.

The advantages of such a system are obvious. The auxiliary mirror is not necessary, and therefore no alignment problem involving it exists. A single telescope makes all the observations, and the resulting optical system is much simplified, as discussed in Paragraph 1.4.2.6.6.

1.4.2.6.4 Pseudo-Diurnal Axis Stabilization

The problem of obtaining a stable axis of rotation in space was examined in several possible mechanization schemes. In examining these schemes the effects or limitations about the three primary axes are considered.

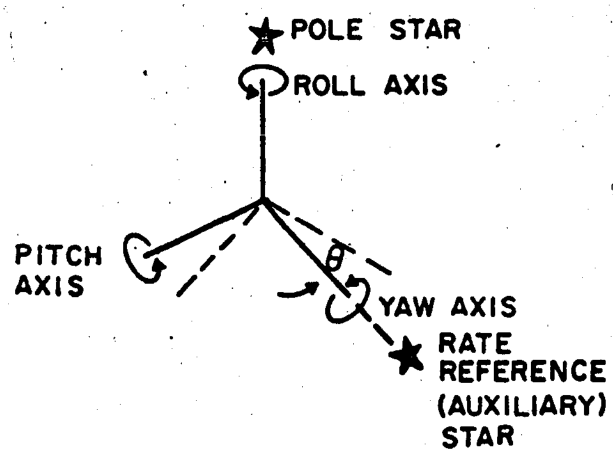


FIGURE 1-11

The major sources of instabilities within a space vehicle are assumed to be the vehicle's propulsion system, stabilization system, occupants' random movements, sloshing of liquids in tanks, and mechanical vibrations due to rotating machinery and transformer laminations. Magnitudes of the instabilities are discussed in the following paragraphs.

Vehicle Propulsion

If the vehicle is propelled by one of the presently conceived methods, the vibration effect can be safely ignored only since it generally would not be in use during navigational observations. It usually would be necessary to use propulsion only after trajectory determination to alter the ballistic trajectory. (Inertial dead-reckoning probably will be used during thrusting.)

Vehicle Stabilization

Gas jet controls should not contribute high frequency vibrations of sufficient amplitude to materially affect the instrumentation. There will be created a low frequency 1-2 arc min/sec correction rate within a maximum stabilization system dead zone dictated by gas consumption economy. The maximum deviation compromise for presently planned missions is ±1 degree. However, this could be easily decreased to ±10 arc minutes at the expense of increased rates of attitude reaction mass consumption.* It does not appear unjust to assume that an improvement will occur in control stabilization energy economics by the advent of interplanetary missions; probably some control methods such as inertial gyro vectoring will allow an assumption of ±10 arc minutes maximum deviation in pitch, roll, or yaw. This deviation in any case would be corrected by the use of star trackers aligned on the pseudo polestar and a star at 90 degrees to the pseudo polestar. At the instant of transit the plane coordinates of these star positions would be read out automatically, and a computer correction would then be applied to correct the nominal positions of those stars. The resulting error would be dictated solely by the resolving power of the star tracker which for a one-degree field would be 0.5 arc second at each point of transit. If the field were decreased to 20 arc minutes the error would correspondingly decrease to approximately 0.2 arc second and vice versa.

Passenger Random Movements and Fuel Sloshing

The inertial magnitude of these motions must be equal to or less than the attitude correction dead zone capability. Otherwise, the vehicle would not be in control. If this becomes impractical, there would have to be restrictions on the passengers' movements and trapping or baffling of the fluids. Assuming this, the effects of these inertial movements would appear only as changes in rate of the drift occuring in the two-degree or 20-minute stability circles. This would not affect the transit angle (θ) measurement, since these deviations at the instant of transit are being read and corrected. Reference 1-15 discusses the indeterminate dynamics of this situation.

Mechanical Vibration Due to Electrical Inputs

These disturbances will lie relatively high in frequency, 60 to 400 cps or higher with a corresponding decrease in amplitude. The error to be expected because of these vibrations is computed in the following way:

> θ error = ωt where ω is the vibration speed and t is the resolving time of the tracker.

Assuming an amplitude of 0.5×10^{-4} inches at the objective end of the tracker telescope and a focal length of 20 inches:

$$\omega_{\text{average}} = \frac{1 \times 10^{-4}}{\frac{1}{60} \times \frac{1}{2}} \times \frac{1}{20} = 6 \times 10^{-4} \text{ rad/sec}$$

*Or the dead zone may be narrowed by using a reaction wheel (or gyro precession torquer system) as a Vernier within the dead zone of the gas jet system, during observations.

Since $t = 1 \times 10^{-3}$ for a 1,000 cps star tracker, then

$$\theta \text{ error} = 6 \times 10^{-4} \times 1 \times 10^{-3} = 6 \times 10^{-7} \text{ rad/20 in}$$

or

$$\theta \text{ error} = 0.12 \text{ second of arc}$$

For the telescope, which is observing transits, the resolving time is at least an order of magnitude less than the resolving time in the above error computation. Hence this error will be considered negligible for the transit telescope. There will be, however, an error of the backlash type which should be for the above assumed conditions $\frac{1 \times 10^{-4}}{20}$ or one arc second. This error would not be induced if all transits were taken in the same direction, or if we deal with a relatively simple motion (a steady state condition) so that it could be calibrated and compensated for.

Axis Bearings

Since the rotational axis must be of the state of the art type in freedom from wobble and any other displacement during rotation, the best approach would be a fluid bearing. Liquid bearings with their absolute compensation ability also have a quality of high damping and would tend to reduce any high frequency vibrational error of the transit scopes.

1.4.2.6.5 Possible Mechanizations of the "Rate" System

Vehicle Rotation

A roll rate could be introduced into the vehicle itself. By timing the transit events and multiplying by the known rate, θ is found. θ measurement would have to be a summation of instantaneous rates by their appropriate time intervals in order to compensate for disturbances in roll rate caused by occupant movement, fuel sloshing, and other vibration sources. One method of doing this is by use of the rate gyros which are discussed later in this paragraph. These are not accurate enough now, nor will they be in the foreseeable future. Another method would be to monitor a star in reference to the roll angle or θ. This would necessitate a monitor (star tracker) located on such an axis that the angle could be read. A regulated roll maneuver of the vehicle is also required.

Gimbaled Platform Within Vehicle

This mechanization was considered, and by introducing gimbal torque loops the static stiffness obtainable would be on the order of five to seven arc seconds and with a frequency of 30 to 35 cps. The error in θ would be directly proportional to static stiffness on all axes, and therefore is intolerable.

Diurnal Axis Table

In any case except that in which the vehicle itself is rotating, a table in the vehicle is required. This table must be suspended on a most precise fluid bearing. It has been concluded that the preferred configuration is such a table equipped with an accurate angle readout device and space-stabilized through the use of star trackers. Figure 1-8 illustrates these points; Paragraphs 1. 4. 2. 5. 1 and 1. 4. 2. 5. 2 discussed these ideas.

Star Tracker

Section 2 (Paragraph 2. 2. 6) describes a star "tracker" based upon an I. T. T image dissector photo-multiplier tube, which is claimed to be capable of ±0. 5 arc second resolution within a 1 degree field of view or 0. 2 arc second resolution within the 20 arc minute (±10 arc minute) dead zone assumed for the vehicle attitude control (see Appendix 2-A).

Reference 1-21 reports an extensive star tracker system study done at Honeywell Aero-Florida in 1962 for USAF ASD.

Angle Measurement

Angles may be measured by a summation of the instantaneous values of rate multiplied by the time increments at which they occurred or by a direct angular readout device. A gyro enclosed by a torquing loop is a means of reading out rate. Three types of gyros are discussed: GG159, an air bearing gyro; ESG, an electro-static gyro, and the Laser Gyro. The latter is a ring type; a light source within the ring emits beams in opposite directions. Rotation about an axis in the plane of the ring produces a difference in frequency of the light beams which is a measurement of the angular rate of rotation. Also discussed are three direct readout devices, Midarm, Dynagon, and an Interferometer method.

Inertial Rate Measurement

All gyros are susceptible to certain errors of drift. For a laser gyro the following errors are presently expected:

Laser Gyro - θ error = long term drift plus random drift

θ error = 0.001 deg/hr x 1/6 hr + 0.001 deg/hr

(Perhaps 3 years in future) = 0.0012 deg in 10 min

= 0.0012 deg x 3600

= 4.3 sec in 10 min

The ESG and the GG159 Gyros fall generally within the same accuracy parameters (for the purpose of this examination), as those shown for the Laser Gyro and would be subject to the additional errors caused by the necessity of torquing loops which are not required by the Laser Gyro.

The potential advantages of using a constant pseudo-diurnal rotation measured by a rate gyro are as follows:

1. A star reference or other auxiliary equipment is not required.
2. Bearing quality requirement is reduced.
3. Probably entails lower over-all cost.

The disadvantages of using a constant pseudo-diurnal rate are these:

1. A rate gyro is a dynamic device subject to instabilities of time, temperature, and power supply.
2. Constant power is required.
3. The accuracy required is several years away for all types of gyros.

Direct Angle Measurement

Three methods of high precision angle measurement are compared for accuracy, followed by a detailed description of each, in the following paragraphs.

Comparison of Methods

1. Midarm[1] - The smallest angular bit that can be measured is 12.8 arc seconds. Any bit has an absolute accuracy of 0.05 arc second. However, for measurement within the bit a constant rate would be required, and the added inaccuracy would be determined by the constancy of the rate and the increment of time. The magnitude of this error would be 0.22 arc second as discussed in paragraph 1.4.1.6 of this report. Total error contribution is 0.27 arc second. Versions of this instrument have been produced.

2. Dynagon[2] - The smallest angular bit is dependent on the angular rate of measurement. However, at the rates envisioned, the accuracy of hardware in production should be 1.26 arc seconds. Introducing a fluid bearing and 12 inch diameter digital disc will provide an accuracy of 0.16 arc second. This device has a basic advantage in that power failures would not interfere with readout accuracy except for the last 0.5 - 1.0 seconds previous to each transit point. There is a whole number readout for each 0.5 - 1.0 second of time. Total error contribution is 0.25 arc seconds (estimated).

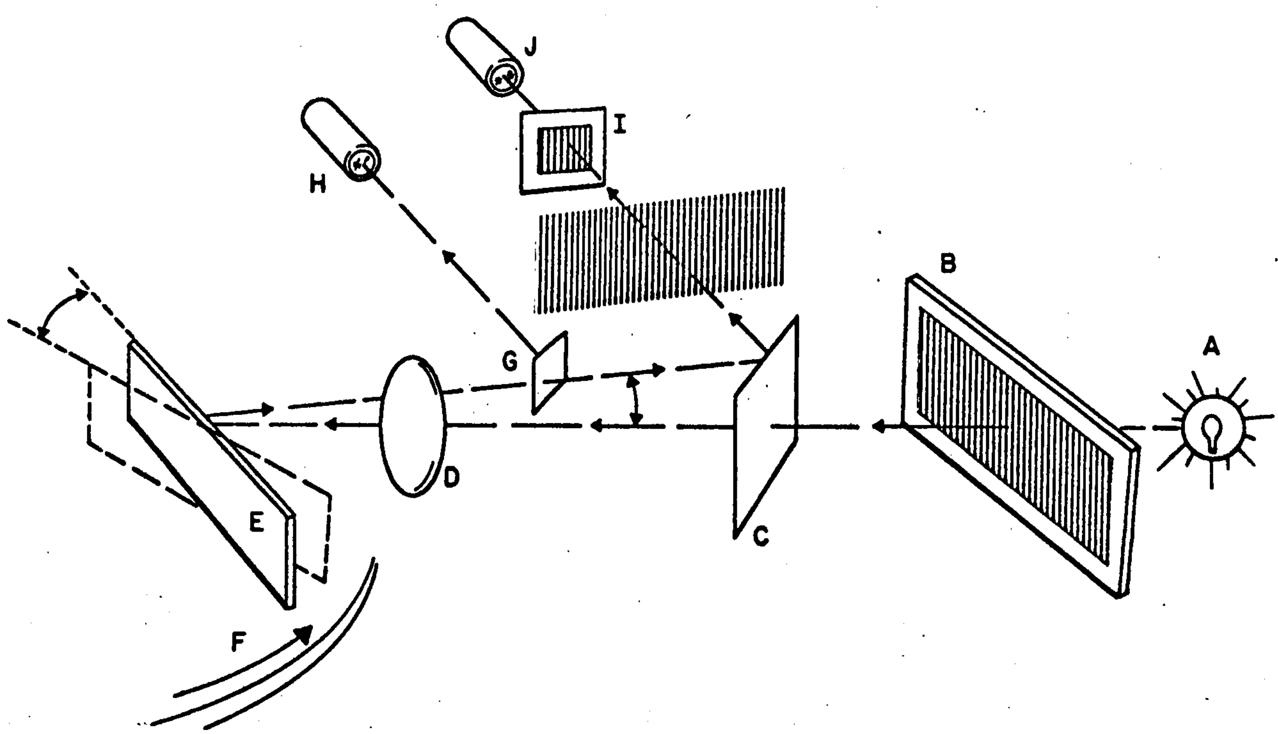
3. Interferometer Method - This measuring principle would have the smallest angular bit; 0.1 arc second, and an absolute error of 0.05 arc second. There has been at least one such piece of laboratory equipment built. This equipment represents the smallest known incremental bit size but it is in the least developed state at this time. There would be no whole number readout except as given by the addition of the total number of bits during the transiting time, which is imagined to be approximately 10 - 20 minutes. Momentary power loss could invalidate a series of sights.

Direct angle measuring is preferable to rate measurement. It requires no constant rate generation and provides larger numbers of available stars in a shorter period of time. Constant rate on a relatively low inertia device with many rate disturbing motions would be very difficult. Midarm requires constant rate for a very short period. Either Dynagon or the Interferometer method is recommended. Dynagon is preferable for its whole number read out and state of production, the Interferometer method for its state of the art bit size. It would probably take a year or more to develop the interferometer apparatus. The interferometer is discussed in References 1-16, 1-17 and 1-18.

[1]Razdow Laboratories - Newark, New Jersey
[2]Minneapolis-Honeywell Regulator Company - St. Petersburg, Florida

Midarm

A schematic representation and simple explanation of this device is shown in Figure 1-12. It is discussed in greater detail in Appendix 1-C.



MIDARM: Microdynamic angle and rate monitoring system developed by Razdow Laboratories, Inc., operates by light from monochromatic source (A) passing through grid (B), beamsplitter (C), collimating lens (D) and striking mirror (E) mounted on rotating specimen to be tested (F). Image is reflected back into system, where it is directed by another beamsplitter (G) to reference photosensor (H). Image is also reflected by the first beamsplitter through a second grid (I) to control photosensor (J). Output voltage of the photosensor has period of 12.8 arc-seconds. Analog voltage output is 30v per arc-sec.

FIGURE 1-12
MIDARM

Dynagon Principle

The Honeywell Dynagon (Dynamic Goniometer) is a spinning rotor device for measuring angles. It provides a digital angle pickoff. The version of the device for the PDT instrument is described in Section 2. The basic development is fully discussed in References 1-24 and 1-25.

The function of the digital angle pickoff is to precisely measure platform angles and star tracker angles and transmit the data to the computer in digital form. The device must work properly when subjected to the environmental conditions and angular rates of a craft in deep space. Because the space craft may be rotating, the encoder must deliver a timing pulse to indicate the precise instant at which the angle is measured.

The Dynagon angle-measuring device is an outgrowth of laboratory experiments that have defined and solved problems. It is basically an "angle-mark" counter with the "angle-marks" precisely and accurately located on a disc such that simultaneous "reading" of all the angle-marks occurs. This process averages the errors of position of individual marks as well as compensates for eccentricities of bearings. (It is as if one were able to "read" a theodolite angle with many pairs of reading microscopes simultaneously.) See Figures 1-13A, B and 1-14A, B.

Since the angle marks are sensed by changes of impedance on a dynamic basis to provide electrical signals, interpolation between the physical angle-marks is accomplished by converting the space marks to time marks, but only for the appropriate interval requiring interpolation. Thus an angle measurement is obtained by a moving observer (detector) that starts a counter at the reference fiducial and adds up the number of angle-marks required to reach the input fiducial. If this latter fiducial were positioned somewhere between the basic angle-marks, interpolation within the interval is required. If the observer (detector) can start a clock upon reaching the last angle mark prior to the input fiducial, the time to reach this input fiducial can be determined as well as the total time to traverse the space interval between angle-marks. With reasonably constant speed during only this interpolation interval, the ratio of times immediately gives the fraction of an interval corresponding to the location of the input fiducial. The total angle has then been determined uniquely as well as the instant in time of its determination.

The digital goniometer functions exactly in this manner.

The unique identification of the input and reference fiducials is accomplished by the use of a once-per-revolution signal that indicates which one of the many angle-mark configurations is to be used. Logic circuits define the operation quite simply.

Physically, the device consists of a motor driven disc having appropriate patterns on its face to generate impedance changes with respect to the two similar discs mounted on each side. These two discs have their particular patterns and constitute the input and reference members for angle measurements. Each member generates a signal with identical frequency but differing in phase for any one angle setting. Any angular rate between the input and the reference members changes the relative frequencies generated, such that rate information as well as angle information is contained in the generated signals. With proper processing of these signals angular rate and angles can be read out. Electrically, the device generates the physically-dependent electric signals and, by use of timing and logic circuitry, processes these signals to provide the counting pulses that determine the angle. Three count-registers are filled corresponding to the data of the physical count of angle-marks and the time data

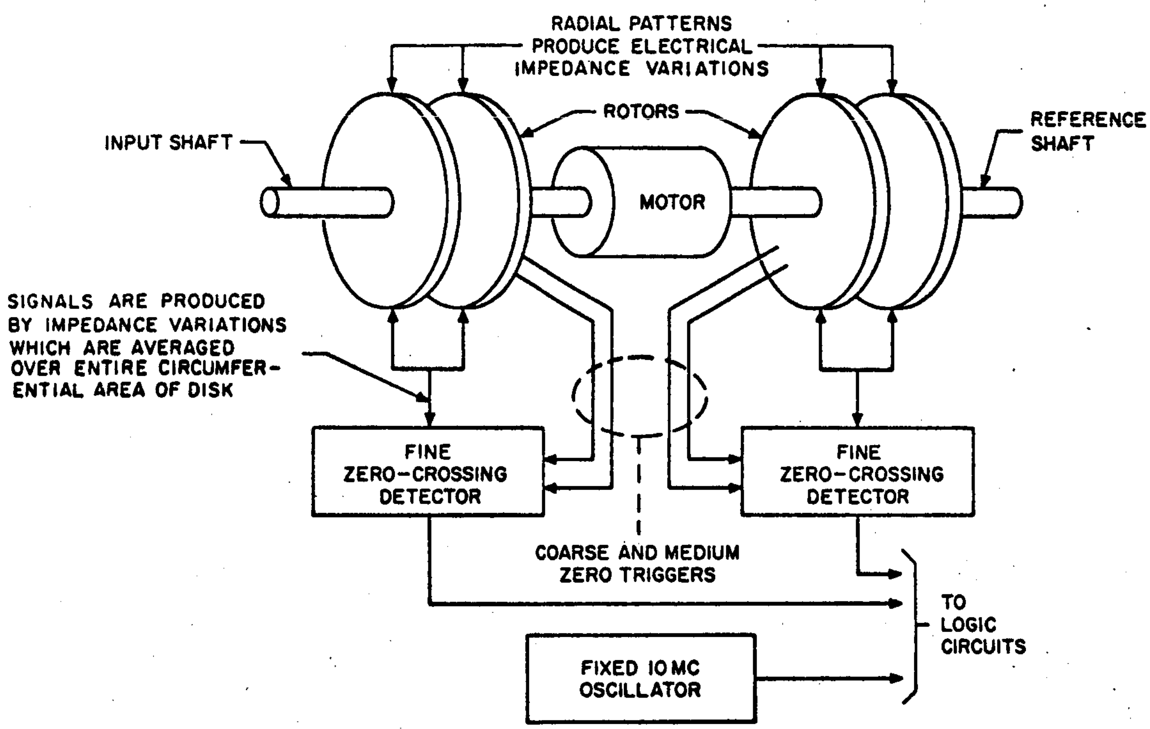


FIGURE 1-13A
DIGITAL ANGLE PICKOFF PICTORIAL DIAGRAM

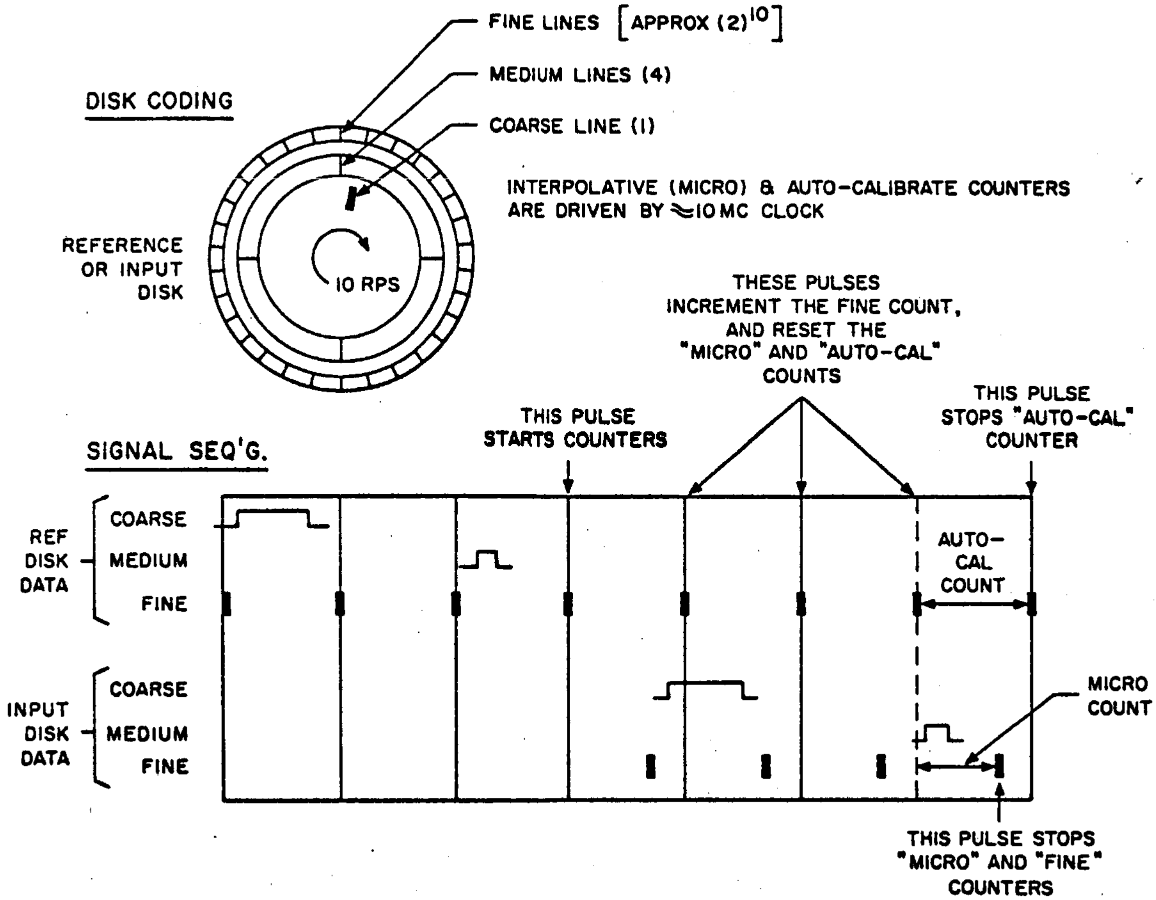


FIGURE 1-13B
DIGITAL ANGLE PICKOFF CODING

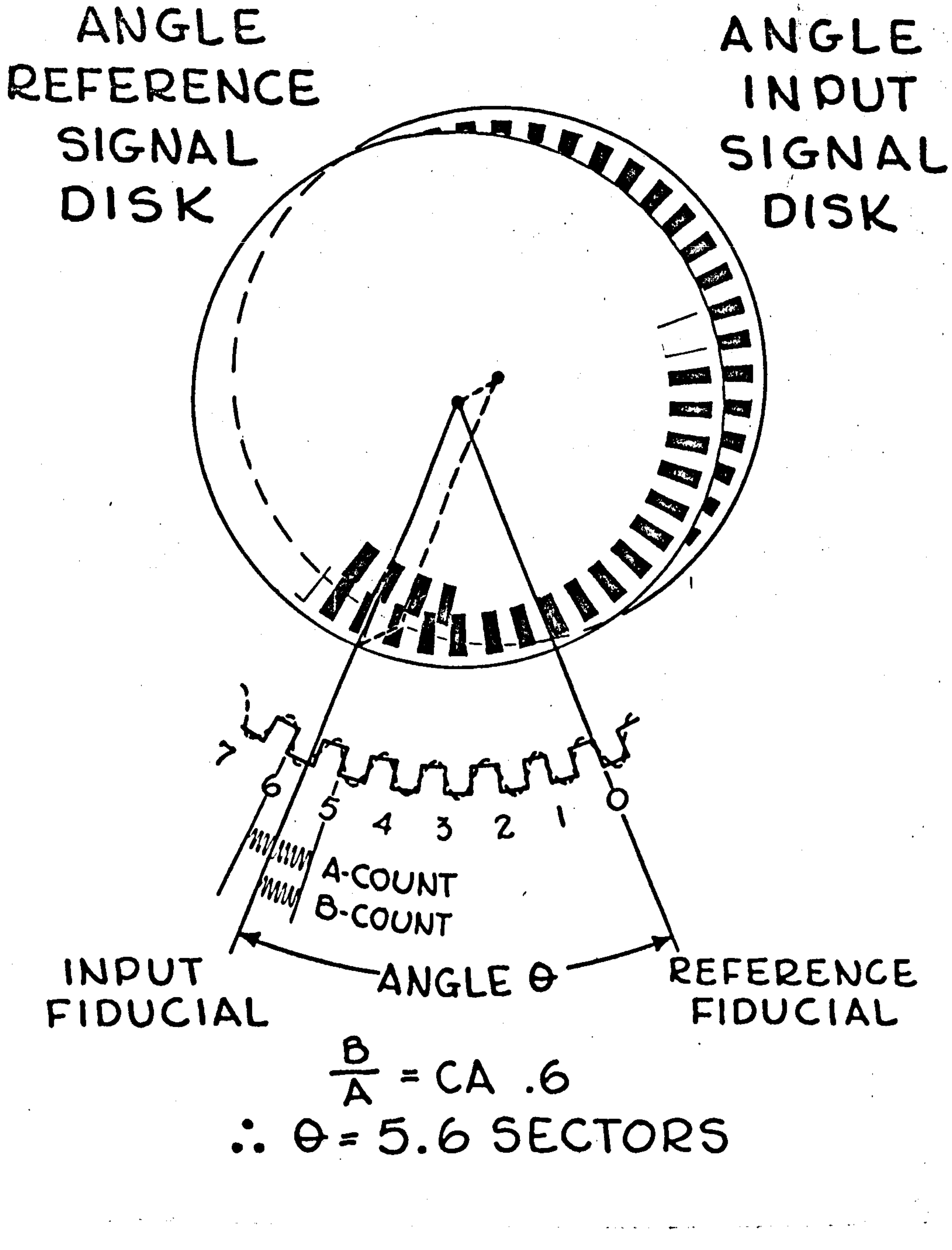


FIGURE 1-14A
SCHEMATIC OF PRINCIPLE OF DYNAGON

1-14A

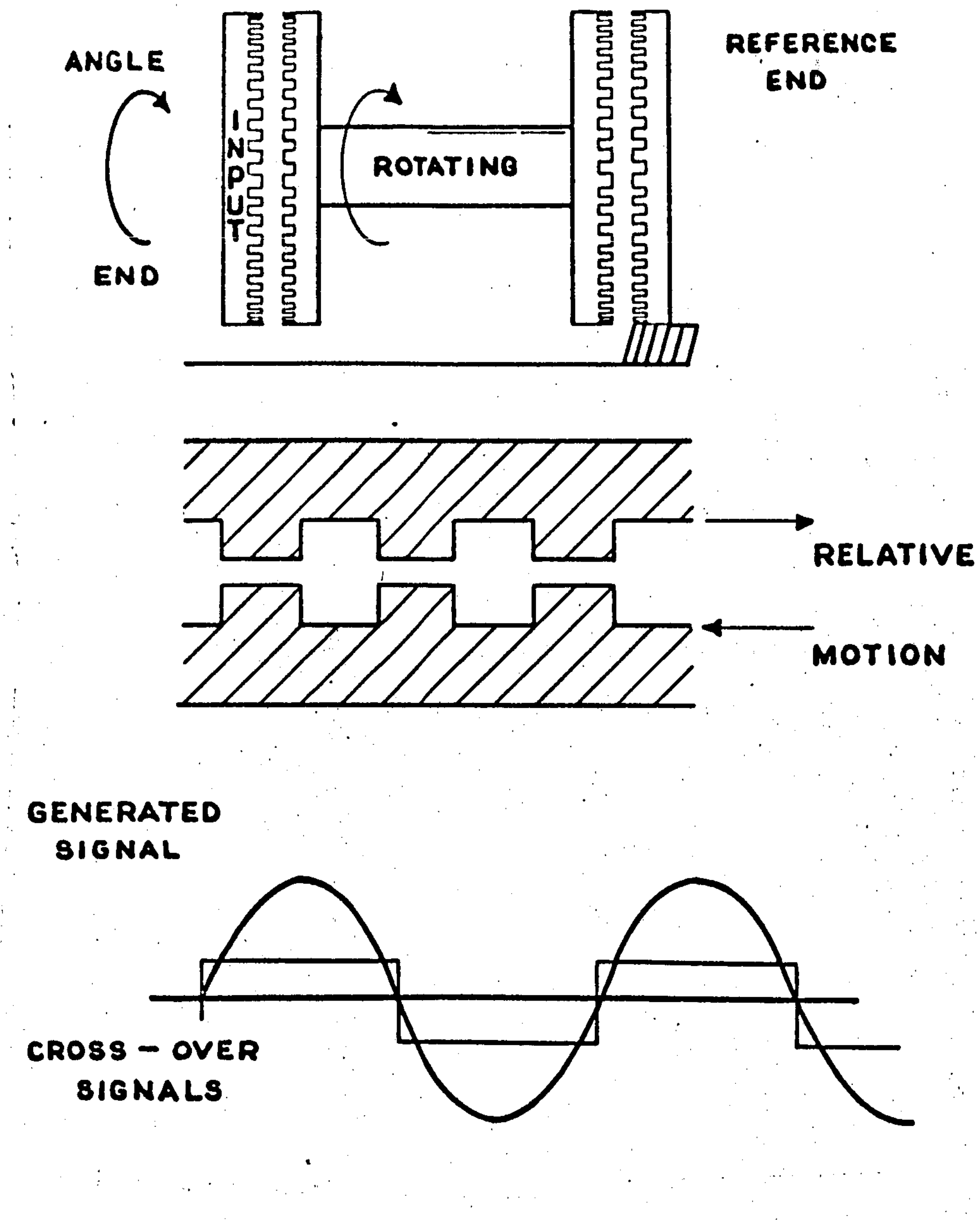


FIGURE 1-14B
SIGNAL GENERATION CONCEPT

for interpolation interval. A computer can readout in parallel the data of each counter. Note that the constant speed requirement for the rotor motor is considerably reduced by this ratio interpolation on only one of the many intervals per revolution. It becomes a self-calibrating interpolator.

The rate information can be extracted by standard heterodyne techniques where the difference frequency is directly proportional to the angular rate.

The device uses a tiny magnet and a magnetic pickoff to generate a once-per-revolution signal which enables the logic circuit to choose a particular crossover signal as a prime fiducial.

Electrostatic field depends on surface shape and conduction only, whereas magnetic fields are volume dependent requiring control of surfaces, shapes, and volume properties. The electric currents involved react with the fields to disturb the relationship of physical surface contours to the effective current path, i.e., alignment difficulties arise which must be satisfied by trial and error. The size of the magnetic coupling is small resulting in small signals. And, also, the delineation between conductor and non-conductors is greatest for electrostatic fields. Thus, electrostatic fields are more sharply controlled with less fringing effects for the same field size-ratios; as a consequence more elements can be crowded into the same length (for any given gap size) which increases resolution. Finally, shielding of extraneous sources is much more complete electrostatically than magnetically.

For the same principle configuration, an optic "field" has no advantages and in fact the higher resolution (lower fringing fields) will introduce more harmonic distortions in the generated waveform. Also, the optical version requires two non-uniform intermediate transducers, (1) electrical to photo (lightsource), (2) photo-to-electrical (detector). But the electrostatic pump directly moves charges producing a current that is amplified by transistor circuits-- a most direct coupling and suitable since the waveform generated has low harmonic content. The reliability of the direct-coupled electrostatic system is demonstrably higher than any other system presently in use.

The increased resolution inherent in an optical "field" required careful consideration. If mechanical bearings are used, then the Dynagon principle can be used up to the limitations of such bearings. The run out-- whether from change in loading, defects, or play, defines the smallest sector-element that can be used, whether optical sector-elements or electrostatic. Practically, however, the extra requirement of controlled gap spacing for electrostatic fields would appear to make the optical version somewhat better; but the non-uniformity of light sources and detectors affect this small advantage, since electrostatic fields can average uniformly completely around the subdivided circle. The Dynagon also incorporates a feature of a second, displaced pattern which is utilized in a bridge circuit to reduce the effects of gap variations.

Obviously, fluid bearings (whether gaseous or liquid) would be a better approach to bearings; also spacings can be maintained accurately and consistently by the fluid Bernoulli effect. Whenever the fluid-field state of the art becomes advanced enough, the whole goniometer concept could be executed without electronics.

For the incremental or nonspinning rotor-type goniometers, field control is just as exacting. For the optical version (which here differs generally from the other field approaches), the only means available to get high resolution is to use a large number of subdivisions of the circle which in turn requires extraordinary bearings. For the magnetic and electrostatic versions, the exact field shape is essential to compare the two out-of-phase driving frequencies needed. That is, the angle information is dependent on the phase relationship of the complete waveform rather than on the phase of cross-over points alone needed by the DYNAGON approach. Then, too, these static versions are incremental only and require elaborate means to identify the particular increment being monitored; either multiple coarser incremental portions are needed to define the complete angle, or an up-down counter which loses all with a power failure.

Honeywell has considered a static version also, even though it has the drawbacks mentioned, since it was discovered that waveform control could be obtained in a new and unique manner. The unavoidable "fringing" fields (a function of spacings) could be used to provide an impedance variation closely sinusoidal. But, the attractive features of high resolution for angles and angular rates, both inherent in the Dynagon concept, led us to abandon (temporarily) the "static" version which can never quite equal the performance of the dynamic devices.

With current state of the art, it is believed that a resolution of 2^{20} (one and 1/4 arc seconds) can be obtained with reasonable size discs having 2^{10} (1024) angle marks or sectors, each of which can be electrically subdivided into two equal parts with time interpolation further subdividing these parts by 2^9 (512). The concept is not limited to this resolution, and the 12 inch device being considered has a basic resolution of 0.16 arc seconds.

Engineering Prototype

The engineering prototype currently being tested embraces all the results of the experimental and analytical program. In brief, it has a pattern with 1,024 sectors (3.6 inch diameter) which, when averaged with its complementary pattern, will produce a resultant impedance variation in the fraction-of-an-arc-second range. The electronics package will process the 10 kc/s fundamental frequency from the sectors and provide a 10 mc/s time interpolation corresponding to a resolution of one and one-quarter arc second and an accuracy goal of one and one-quarter arc second. Logic circuits direct the signals unambiguously to storage binary registers from which the information can be "read-out" in parallel by a computer. The mechanical design has emphasized the stringent requirements of maintaining the small gap that determines impedance variations, while also defining concentricities of elements to tight tolerances without resorting to adjustment means.

Even though this is called an engineering prototype, certain features were retained which are completely experimental. Particularly, provision has been made (and the master pattern already has it) to have one extra band of sectors at a smaller radius than the two main bands of sectors. The middle band of sectors is physically displaced peripherally relative to the inner and outer bands which are in sector alinement. For the bridge compensation system, either the outer pair or the inner pair of bands is needed; consequently, if the spacing to the adjacent plate with its pattern is adjustable to preserve the desired ratios of dimensions, then two different sized goniometers are possible. Tests of each configuration can determine the critical parameter and the extent of extrapolation of data for design modification.

The engineering prototype has a unique ball-spacer system which lends itself readily to the above changes of spacing by merely changing sizes of the balls. Since the electronics does not "know" what is feeding the raw signal, a completely flexible experimental unit results. It should be pointed out that the electronics only determines the interpolation between the sectors and that the number of sectors around the circle determines the resolution. The accuracy of the sectors and the interpolation determines the accuracy of the finest resolution.

Bearing limitations generally define the smallest sector size usable and hence the resolution for any chosen size of circle to be subdivided. For a particular bearing and, in turn, particular sector, a larger circle with large numbers of sectors would allow higher resolution with no increase of accuracy expected except that the increased number of sectors are also averaged to INCREASE the accuracy as well as resolution of the larger device.

Error Analysis Synopsis

For the pattern system and mechanical configuration of the Honeywell engineering prototype of Dynagon, an error analysis has found that the error contribution to the raw electrical signal is about 1/2 arc second. The resultant pattern system error contributions and the resultant mechanical error contributions are approximately equal for typical values encountered from experimental data. Data on the over-all mechanical-electrical system of the feasibility model exhibit a stability to 1 part in 2,500 (effectively 1/2 arc second) consequently the electrical portion must contribute a smaller amount. Thus the Honeywell prototype DYNAGON appears to be capable of 1-1/4 arc second accuracy.

1.4.2.6.6 Optics of the Observing Instrument

Two independent questions are considered here. The first involves the internal optics of the telescope and finder and the means of determining transits, the second is concerned with the means of transmitting the image fields to the navigator.

Optics of the Telescope and Finder

Whatever the method used to transfer the desired images to the navigator's view, these images must first be brought out of the telescope. The optical arrangement to accomplish this is illustrated in Figure 1-15.

The telescope and finder are mounted on a ring R. The direction of the finder optic axis is normal to the plane of R, and is fixed in this orientation. The optic axis of the telescope in its normal or reference position is parallel to that of the finder. The telescope can, however, be rotated about axis AA, as shown, or about BB if the ring is rotated 90 degrees about 00. Planet search is accomplished by rotation about HH, in which case the two optical axes stay parallel.

Search for and acquisition of stars or planet will be easily accomplished as follows. The finder has a wide angle field of view, preferably about 30 degrees. The much smaller image field of the telescope, about one degree, will be superimposed on the image field of the finder. At planet transit the field of the telescope will be centered in the field of the finder.

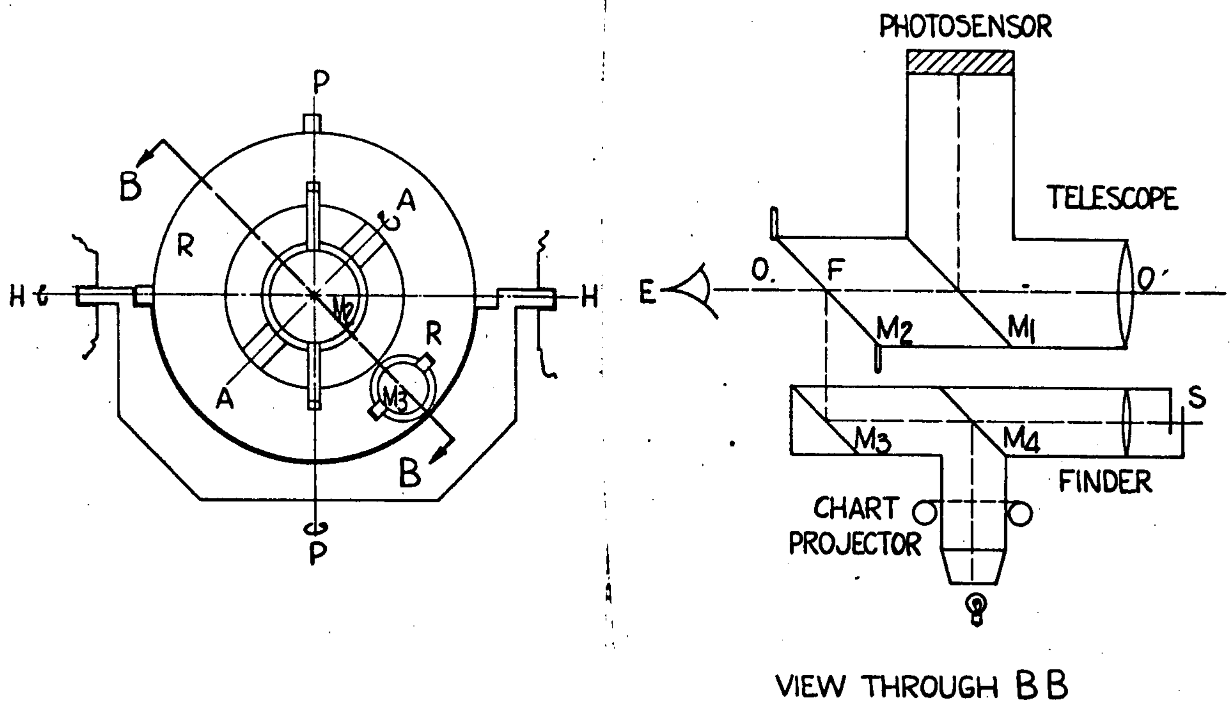


FIGURE 1-15
BASIC OPTICS
OF THE OBSERVING INSTRUMENT

1-15

Figure 1-16 illustrates this schematically. As the telescope now searches along one of the two great circles (independently of the finder) the field of the telescope moves along one of the tracks shown in Figure 1-16. The final images seen by the navigator optionally can be arranged so that the finder field remains stationary and the telescope field moves across it, or the finder and telescope fields may move together. In this manner the navigator can recognize the stars or planet in the large field of the finder, and by moving the telescope he readily points it at the required object. Furthermore, he can observe the transit of star or planet visually, knowing when transit is imminent and when it is completed. In an emergency he could time these events on the basis of his visual observations alone. Generally however, the display of the field is not designed for the purpose of visual determination of transit, but only for recognition and acquisition. Hence there are no stringent requirements upon the display optics other than that the telescope's one-degree field be superimposed on the finder field so that the coincident portions of the fields represent the same point on the celestial sphere.

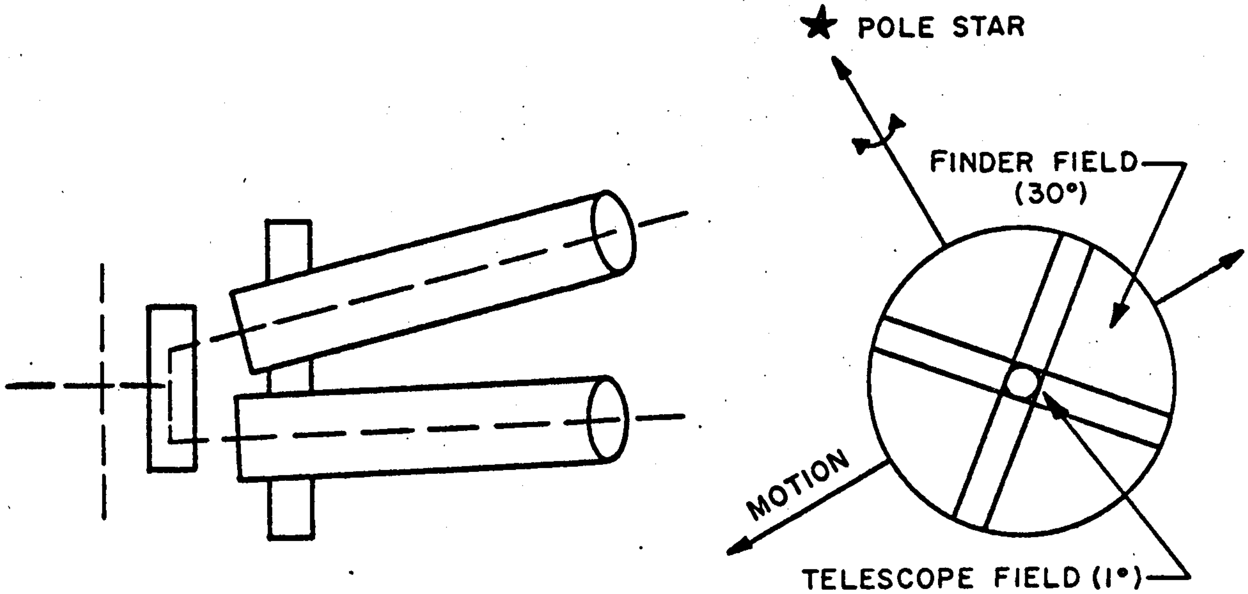


FIGURE 1-16

On the objective end of the finder is a variable filter S for filtering out all stars below a chosen magnitude. This may be controlled by the observer, manually. The objective image field of the finder is at F and the image can be examined by an eyepiece or can fall on a translucent screen if bright enough. Mirror M_2 is not fastened to the telescope tube, but to the ring R.

Light entering the telescope strikes the partial reflector M_1. The reflected beam falls on C which is a photosensor for recording transits. The transmitted beam forms an image at F, and this image is also observed at the eyepiece E by the rays transmitted by M_2. Ordinary methods of optical construction will be quite good enough to give superimposed fields in which the images of the planet coincide as long as the two optical axes are parallel.

Finally, as the telescope is now unlocked and rotated about AA, the image it forms in the plane F also moves along a direction BB. When the ring is rotated to the 90-degree position the mirrors also rotate but the direction of the incident beam is quite unchanged, and the image will now move along the direction of AA in the field. It is important to place the mirror M_2 in such a position that the normal to the mirror surface shall lie in the plane defined by the sweeping optic axis.

It may be advantageous to provide the navigator with a star chart. This is shown schematically as a simple projector on the side of the finder. The image of the chart appears in F and is observed at E. Its brightness can be easily controlled by varying the illumination in the projector. If the navigator wished, he could vary the S setting and the chart brightness to about the same intensity and match the chart against the star field. This would require the preparation of star charts as defined in Paragraph 1.4.2.4.1. The charts in question, showing star distribution in a band 30 degrees wide centered on the equator, could be positioned in the projector to center on that latitude circle at which the finder is directed.

The presentation of the chart need not be done in this fashion of course. It could be projected into the plane of the final image that the navigator sees rather than at the beginning of the optical system. If the image transfer is carried out electronically, as discussed later, the chart may have to be displayed above or below the viewing screen rather than on it.

These considerations outline the recommended optical system for the telescope and finder. The process of transit recording is discussed later.

<u>Transfer of Image to Navigator</u>

From point E in Figure 1-15 the images formed by the telescope and finder must be transferred to a point where they are seen by the navigator. There are two methods possible: an optical train, or electronically, using a Vidicon and Kinescope.

Transfer of Image to Navigator by an Optical Train

The optical train must be able to transmit the light beams emerging at E to some other location, some distance from the telescopes. If these telescopes are outside the vehicle the optical train must pass through the vehicle shell. The first consideration of such a system involves the loss of light intensity on reflection from mirrors or transmission through lenses. At E the light from either the finder or the telescope has passed through two partially reflecting mirrors. In addition, there is a loss of light by absorption at each surface, commonly taken at five percent average by optical designers. Hence the intensity of the finally transmitted beam, the intensity at the end of the optical train, is given by

$$I_T = I_0(0.5)(0.5)(0.95)^{N+4} \quad \text{for the telescope} \tag{1.8}$$

$$I_F = I_0(0.5)(0.5)(0.95)^{N+6} \quad \text{for the finder} \tag{1.9}$$

where N is the number of optical surfaces beyond E.

Less then half of the navigational stars have magnitude less than 2. For the normal eye a 6th magnitude star is the faintest star visible. A standard astronomical relationship is

$$I_1/I_2 = (2.512)^{m_2 - m_1} \tag{1.10}$$

where I_1, I_2 are the intensities of two stars and m_1 and m_2 are their magnitudes. A 2nd magnitude star will appear like a 6th magnitude star if its intensity is reduced in the ratio

$$I_1/I_2 = (2.512)^{-4} = 0.025$$

Hence we set

$$0.025 = (0.25)(0.95)^{N+6}$$

$$0.1 = (0.95)^{N+6} \text{ or } N=40$$

This is the maximum number of surfaces allowed in the train beyond E, or the number of elements is 20. This, of course, is a maximum number and 15 elements would be a more reasonable expectation. Since the navigator must be able to see the image finally transmitted to him it is altogether probable that the intensity may be so low that his eye must be dark adapted. This is an added inconvenience.

There are two other motions of the optical block which are not yet accounted for in the system defined in Figure 1-15, rotation about HH and the pseudo-diurnal rotation about PP. If the final presentation of the image to the navigator is to be stationary, compensatory rotation of some of the elements of the train is required.

In the third place, one must remember the demand for coincidence of the image fields of the finder and the telescope. That is, the segment of the finder field upon which is superimposed the one-degree telescope field must indeed be the image of the same point on the celestial sphere which the one-degree field exhibits. Remembering Figure 1-16, this can be said another way. The track of the one-degree field as it moves across the finder field during the search of one of the 45-degree great circles must be made up of two straight lines through the field center, these lines crossing at 90 degrees.

Suppose some mirror in the optical train is oriented in such a fashion that the plane defined by the beam from the telescope, and the normal to the mirror, is not itself normal to the mirror surfaces. As the telescope tracks along one of the 45-degree great circles this skew ray when reflected does not track along such lines. This skew ray effect can be avoided if mirror surfaces are properly used in pairs.

In Figure 1-17 the ray marked PLANET lies in a plane normal to the mirror surface and including the normal. The two rays STAR NO. 1 and STAR NO. 2 do not lie in planes normal to the mirror surface, and represent two positions of the telescope as it searches along a 45-degree great circle. The plane AA'Z' contains these two rays and is the great circle plane. The reflected rays are not drawn, but we assume that they intersect the viewing plane at points 1 and 2, according to the following argument.

Ray No. 1 and the normal to the surface of the mirror determine a plane $P_1P_1'Z$, in which the reflected ray lies. The trace of this plane in the viewing plane is the line P_1P_1. Therefore, the reflected ray must intersect the viewing plane at some point on this line. Since Ray No. 1 makes only a small angle with Ray ZZ', point 1 is a small distance from Z on the viewing plane. By a similar argument point 2 is on the projection of $P_2P_2'Z_2$ as shown. In the limit, as the incident angle of the skew ray becomes greater, the traces in the viewing plane rotate towards coincidence with AA'.

It follows that although the telescope tracks along a great circle path AZ'A', the image in the viewing plane moves along a curved path which is, of course, not a straight line. It is this distortion we want to avoid. Only when the direction of the incident ray is in the plane 0Z'0' will the reflected ray move on a straight line in the viewing plane, 00'.

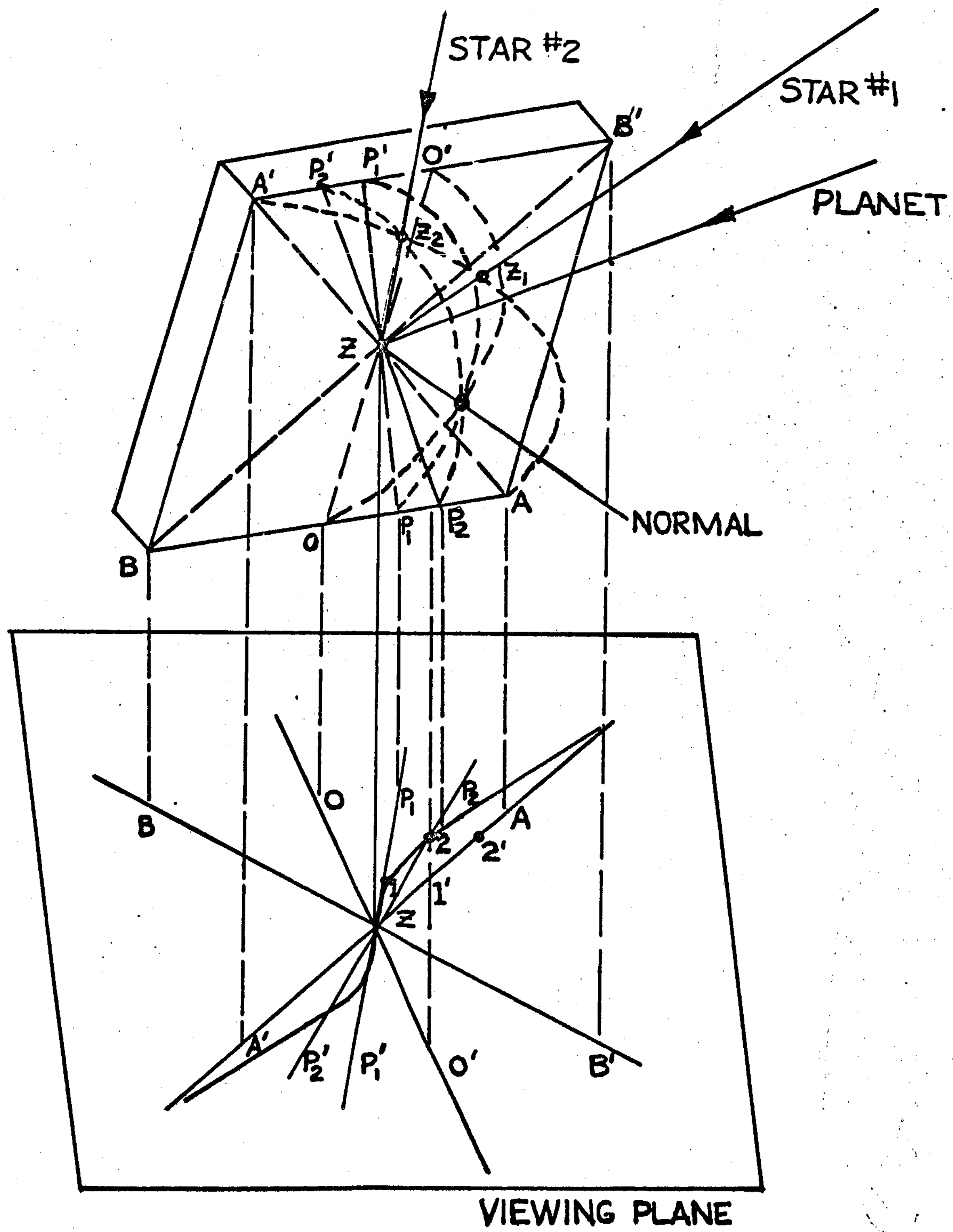

STAR #2
STAR #1
PLANET
B'
A'
P2'
P1'
O'
Z2
Z1
Z
A
NORMAL
B
O
P1
P2
O
P1
P2
A
B
2
2'
1
1'
Z
A'
P2'
P1'
O'
B'
VIEWING PLANE

FIGURE 1-17

An optical train probably can be adapted to the spatial requirements of the vehicle and navigator if the number of paired elements is less than 15.

Transfer of Image to Navigator by Closed-Circuit Television

Many of the problems associated with the optical train will be avoided if the image at E is transferred by means of a Vidicon tube at the position E in Figure 1-15. The Vidicon's coated face has a usable aperture of approximately five-eighths inch diameter. With proper optical design of the finder, a field of view of 30 degrees should be quite possible. The most pessimistic estimate of resolving power gives a value of about D/100, where D is the tube diameter. This gives an angular resolution of about 18' = 0.3 degree. Since the angular separation of any pair of navigational stars is never less than several degrees, this resolution is more than sufficient for star recognition and acquisition.

Star tracker studies have indicated that the Vidicon can be used to sense star images, and that these images can be amplified both in intensity and size for presentation on a kinescope. Present state of the art in the manufacture of Vidicon tubes results often in small areas of the photo-conductive surface which give false signals. Some care must be taken in presenting the image on the viewing screen so one recognizes such signals for what they are. Since their location is fixed while the star fields change as the telescope moves, this is not a difficult problem.

It is obvious that the use of a Vidicon also allows a display which need not be observed through some viewing instrument by a dark adapted eye. This in itself is a considerable easing of requirements.

There is one demand on the viewing system which a single Vidicon is not able to fulfill. It has been implied if not stated in everything so far discussed that there will be a great difference in the actual physical size of the two images (telescope and finder) finally displayed for the navigator's use. If the 30-degree field of the finder must be accommodated on the five-eighths inch diameter Vidicon face, the physical size of the one-degree telescope field would be proportionally smaller. For a resolving power of D/100 for the whole Vidicon area there would be a resolving power of D/3 within the telescope field, which would certainly give poor information to the navigator.

It is true that the navigator is using this display only for search and acquisition of a planet or a star. It is only necessary that in the telescope field he be able to judge whether the star or planet is on one side or the other of a reticle line, so that he will be able to produce the transit by a proper rotation about the pseudo-diurnal axis. If he can do this, the instrument will serve its purpose. The resolving power, stated to be D/100 can quite probably be improved by a factor of five to an angular resolution of four minutes of arc. In a one-degree field the improvement would be great.

A possible way around this difficulty would be to use two Vidicons, one for the finder and one for the telescope. Then it would be necessary to transform the mechanical motion of the telescope into a corresponding motion of the telescope image in the viewing field. Now the resolution of the telescope image would be as good as in the finder field, even though the linear size of the images are in a ratio of 30 to 1. A different solution of this problem is to mount the finder permanently to the telescope, so that there is no relative motion between fields. If we use two Vidicons we can achieve good resolutions in each, at the expense of the appealing features afforded by a visual confirmation of the motion of the telescope across the celestial sphere.

Until a detailed design was begun it was possible to choose between the strictly optical method of image transfer and the electronic method using a Vidicon. The important tradeoff involved resolving power on the one hand, and mechanical and optical design complexity on the other. In flight Vidicon replaceability would appear to be a mandatory design feature.

Transit Detection

Potentially, one of the greatest sources of error in the pseudo-diurnal technique lies in the detection of the instant of transit. Two general approaches to this problem exist.

Transit Detection - Visual

The most important function of the optical system is the detection of the transit of stars across the "reticle line" which represents the great circle in the field. Similarly, one records the passage of the planet limb across a "reticle line". It has been implied throughout this discussion that this will not be carried out visually but electronically. When the navigator does this visually, much more stringent demands on the optical transfer train are present. The Vidicon method is not usable because of its relatively poor resolving power. In addition, there is unwanted motion of the star image due to vibration or limitations on vehicle stability, which make a judgement of transit difficult, because these vibrations effectively blur out the star image by the integrating effect of the human eye. If the vibration were always identical, this effect might be minimized, but this is not likely. In general, the so-called time constant of the eye is 0.1 second; over this interval the eye will integrate radiant energy coming to it. Changes taking place in times much below this are not perceived separately.

There is one possible observing method which allows a man to improve over this fundamental limiting factor. If one times periodic events of constant frequency of recurrence, experiments have indicated an ability to time such events with a repeatable accuracy of 0.01 second, or approximately an order of magnitude improvement. Imagine a reticle of many parallel lines in the field of view of the telescope, across which the star image is moving at a regular, constant rate. These are the conditions described above, and the timing of the transits might be done very accurately. However, the demand for constant rate is a demand for constant pseudo-diurnal rate and we have considered that within the present state of the art a rate of the desired constancy is not attainable. Hence, to the timing errors still present we would add the errors of the nonconstant pseudo-diurnal rate.

These factors, coupled with the discussion of the following paragraphs, point up the undesirability of attempting a visual observation, except in a back-up mode in which degraded accuracy is preferable to no information.

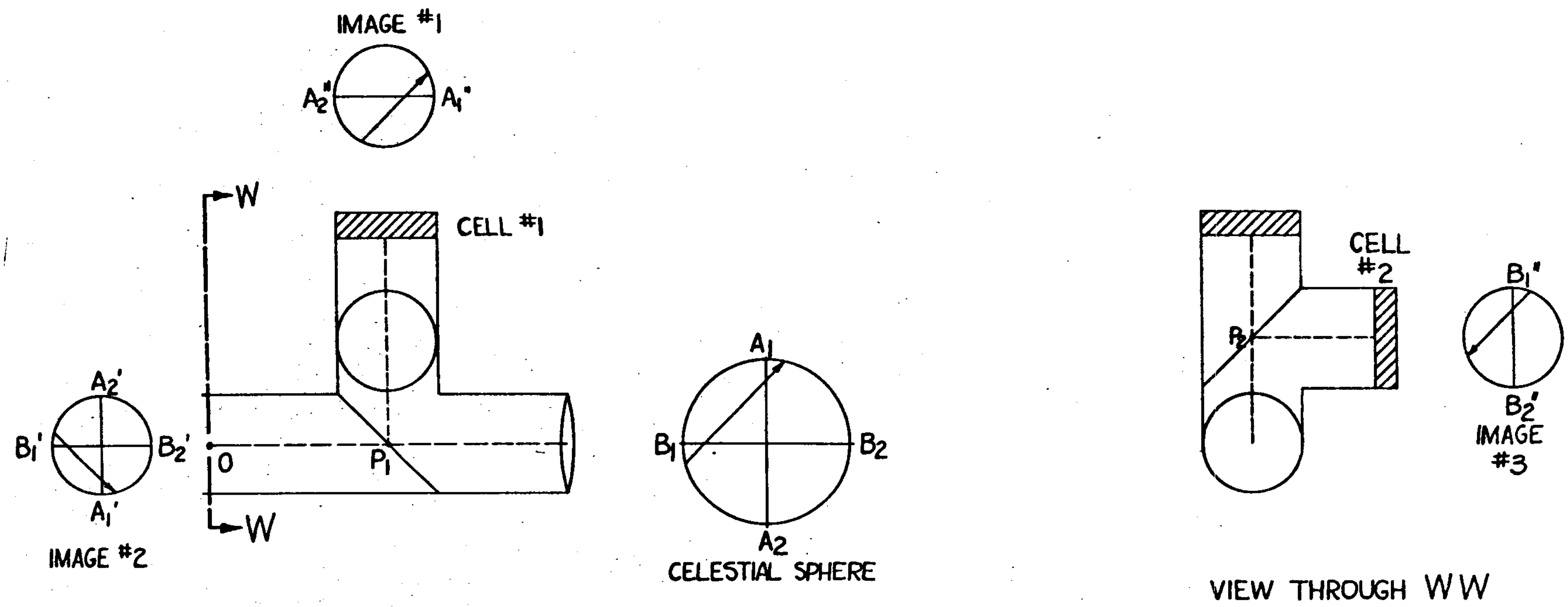
Transit Detection - Photoelectric

It was suggested by Kardashian in Reference 1-3 that a Vidicon tube could be used to record a succession of tangent points of a planet limb, or a succession of passages of a star across a series of "lines". Using an optimistic figure for resolution (600 lines), he obtained a seven arc second resolution for a single reading and by totaling a large number of readings the statistical accuracy can be reduced to 0.2 to 0.3 arc second. Essentially, the same demand is here as above--the necessity for a uniform pseudo-diurnal rate.

The reason for making a number of transit or tangency readings is to increase the statistical accuracy (or to decrease the probable deviation) of "local vertical" over that possible for one reading. If a single reading could be made with an accuracy of 0.1 - 1.0 arc second then such repeated readings would not be necessary.

Figure 1-18 schematically portrays a conceivable arrangement for observing transits.

C_1 and C_2 represent two identical photoelectric detectors. In principle, $A_1''A_2''$ represent a narrow slit in an opaque mask covering the detector. In practice, the photosensitive surface might be split along this line, for example, and the two halves insulated from each other. As the star image passes across this "slit" the signal from one half dies away to zero and that from the other half grows from zero to a maximum. It is possible to use the null or minimum signal as the transit signal desired. Similar



IMAGES #1 AND #2 AND THE CELESTIAL SPHERE ARE SHOWN AS SEEN FROM POINT P_1. IMAGE #3 IS SHOWN AS SEEN FROM POINT P_2.

FIGURE 1-18
TRANSIT ANGLE
DETECTION INSTRUMENTATION

devices are used in such instruments as the Hilger-Watts collimator, in which an angle setting accuracy of 0.05 arc second is obtained.

Cell C_1 is sensitive along a line $A_1''A_2''$ which is coincident with the image of the great circle A_1A_2; cell C_2 is sensitive along the other great circle B_1B_2. Only one of the cells is needed for a star transit, in the figure this is C_1, if the rotation axis of the telescope at 0 is normal to the plane of the paper. In determining planet transits we must use both cells, taking one tangency point from C_1 and the other from C_2. It has been calculated by W. O. Unruh of Aero-Florida in an unpublished memorandum that occultation by a sharp edge would be determined by a simple photosensor to an accuracy of 0.1 arc second. This is a further indication that very small uncertainties are possible using relatively simple electronic components and circuits. Therefore, it would seem unnecessary to go to the more complicated devices like the Vidicon for sensing transit events.

One difficulty with the use of photosensors may be the lack of intensity resulting from the use of two partial reflectors. If a Vidicon is used for image transfer, the first mirror might be 60-40, or 75-25, with more light directed to the sensors. Another possible approach would be to use dichroic mirrors and color-sensitive cells.

RE-CAP OF STUDY PROGRESS BY MONTHS

THIRD MONTH'S PROGRESS

1.4.3 Third Months Work

Summary

Having a chosen preliminary design configuration, the refinement of this design into workable hardware concepts began seriously during the third month. Material derived from this effort has contributed to Section 2 of this report, mostly in the form of drawings.

A fundamental decision was made to use one of the "freon" compounds as the working fluid in the PDT bearing table.

Some ray tracing calculations and study of optical problems was accomplished by Foster and Fischel. Dimensions of a dual-concentric spherical dome were established.

The criticism of the Ultradex manufacturer and a fluid bearing house was obtained to check for inconsistencies in the planned applications.

Studies of graphical methods of data reduction were made.

1.4.3.1 Pseudo-Diurnal Table Bearing Selection

The selection of a hydrostatic bearing has been based upon the stability requirements of the pseudo-diurnal axis. Deflection of the axis in pitch or yaw will result in directly proportional errors and must be held to 0.1 arc second or less. A liquid bearing has been selected rather than a gas bearing since a liquid bearing is completely compensatable. A gas bearing is only partially or proportionately compensatable due to compressibility of the gas. Small tolerances in gas bearings are very difficult to achieve. Orifice diameters and clearances are an order of magnitude less for the same load capabilities. An oil bearing which would require a radial clearance of 5×10^{-4} inches is equal in support capacity to an air bearing (with the same pressure drop) having a clearance of 5×10^{-5} inches. The oil bearing would have twice the stiffness with the same load supporting characteristic, due to the oil's incompressibility. Stiffness is the primary concern in an essentially zero gravitational environment.

The oil hydrostatic bearing would create a problem of oil recovery and the possibility of astrodome fogging by capillary action; there would not normally be a gravitational force to retain the oil as a singular entity. This could be solved by the introduction of an air pressure pump to seal the oil from unwanted distribution onto the optics. This, in turn, would create a necessity for a centrifuge or similar device to separate the oil from the air. Power requirements would then have increased without any useful work being accomplished.

The proposed bearing would use a refrigerant fluid such as ($C\,CL_3\,F$) Tricholoromonofluoromethane. The bearing would be run in its liquid state and the recovery would be done by re-compressing the gas. The compressor would be an air pump.

The advantages of using a refrigerant are:

1. Liquid bearing
2. Gas recovery
3. Uniform temperature of optical environment possible
4. Inert environment (non-toxic, non-corrosive, odorless, dielectric).
5. No clouding of optics.

The disadvantages are:

1. Pumping power higher than pure liquid.
2. Some loss of refrigerant in repairing instrumentation. (The dome would be vented overboard, then repressurized with cabin air. After repairs, the housing would be re-sealed, its air vented overboard, then the bearing and dome re-filled with fluid from a tank.)

This choice of fluids appears to be logical. The fluid in the bearing would be kept in a liquid state, but the fluid which leaks into the astrodome, expands to the gaseous state and is transparent, free from moisture, non-corrosive and non-conductive.

Experiments may be required, during Phase II of this study to prove the workability of the refrigerant bearing; advice is being solicited from those with experience in this area; however, no precisely comparable application has been found.

The expansion of the fluid from the liquid to the gaseous state will cause cooling as in any refrigerator cycle; but the extent of this cooling is estimated to be relatively small.

The support housing is being designed so that after voiding the refrigerant and repressurizing the astrodome with air, the entire housing together with the bearing and all optics can be unbolted from the vehicle frame and brought down into the vehicle for easy access to all parts of the mechanism. This capability will allow for major repairs to the instrument in flight. The potential danger of explosive decompression of the cabin (due to astrodome fracture) during this circumstance makes it advisable to use a double astrodome, and to provide a temporary, astrodome-shield-cover which may be bolted on in place of the dismounted instrument; the bottom cover of the table is suitable for use as a shield cover.

An access (arm hole) air lock also was provided to permit minor adjustments and repairs (such as vidicon replacement) without dismounting the entire instrument.

The capability for in-flight repair is considered to be of paramount importance in view of the duration times of missions for which the PDT instrument is being considered, and in view of the instrument complexity found necessary to meet the operational requirements.

RE-CAP OF STUDY PROGRESS BY MONTHS

FOURTH MONTH'S PROGRESS

1.4.4 Fourth Month's Work

Summary

Mechanical and optical designs were continued during the fourth month. Preliminary design of the television subsystem began.

It was decided to specify a dichroic coating on the outer spherical dome to reduce infrared transparency, permitting easier thermal stabilization.

Some results of the months optical and electro-optical studies are detailed in the following paragraphs.

1.4.4.1 Theoretical Considerations for Video Subsystem

Data on 34 stars of magnitude 2.2 to 1.8 indicate that the average magnitude 2 star has an output of 2×10^{-13} watts/cm^2. With a 3 inch collecting aperture

$$\frac{\pi}{4} (3^2) (2.54)^2 \times 2 \times 10^{-13} \text{ watts/cm}^2 \text{ or } 91.36 \times 10^{-13} \text{ watts}$$

are collected by the optics of the assumed configuration.

Since the focused spot is smaller than the limiting resolution of the Vidicon, this limiting figure must be used in determining system performance.

With a Vidicon target of 0.62 inch diameter, and an assumed limiting resolution of 100 lines, spot size on the Vidicon is thus 0.0062 inches or 0.01575 cm. The illumination on the Vidicon is thus

$$\frac{91.36 \times 10^{-13} \text{ watts}}{\frac{\pi}{4} (.01575 \text{ cm})^2} = \frac{91.36 \times 10^{-13}}{19.5 \times 10^{-5}}$$

$$= 4.68 \times 10^{-8} \text{ watts/cm}^2$$

since there are 680 lumens/watt (valid only at 555 mμ)

$$= 3.18 \times 10^{-5} \text{ lumens/cm}^2$$

converting to lumens/ft^2

$$= \frac{3.18 \times 10^{-5}}{cm^2} \times \frac{6.45\ cm^2}{in^2} \times \frac{144\ in^2}{ft^2} = 2.95 \times 10^{-2} \text{ ft. candles}$$

If optical loss, including the partial reflecting mirror is held to 50 percent, 1.5×10^{-2} foot candles incident light may be expected on the Vidicon.

As an example, consider a Z5294 image orthicon which has a sensitivity of 16.5 magnitude stars with 16.5 inch collecting optics. The limiting sensitivity of this tube is about 10^{-6} foot candle. For this program, it is desired that a magnitude 2 star be viewed by the Vidicon. This is a reduction in sensitivity of 14.5 magnitudes or approximately 10^{-6}. Thus a magnitude 2 star should present approximately 1 foot candle to the Vidicon with a 16.5 inch collecting optics. With a 3 inch lens, $\left(\frac{1}{5.5}\right)^2$ of the light is received; or 3.3×10^{-2} foot candles. Finally, utilizing the expected 50 percent loss due to optics, 1.65×10^{-2} foot candles incident light may be expected at the Vidicon.

1.4.4.2 Vidicon Characteristics

1.4.4.2.1 Size

Normal Vidicons are 6 inches long with a maximum diameter of 1-1/4 inches. However, required deflection, focus and alignment coils will increase the diameter to close to 2 inches. Smaller Vidicons (1/2 inch) are available, however, these have poorer resolution and sensitivity, so that the conventional size should be utilized, if possible. Room should also be provided for any lenses or shutters required.

The usable photo cathode diameter is designed for 16 mm photographic lenses, and is usually 5/8 inch. However, some Vidicons have slightly larger usable diameters.

1.4.4.2.2 Sensitivity

Limiting sensitivity of a Vidicon is a function of both the desired contrast and desired resolution. Thresholds of about 10^{-3} foot candles are quoted for the best Vidicons, however, this is typically at contrasts of 10 percent of maximum and resolution of 100 lines. If 10^{-2} foot candles incident light is available, maximum contrast sensitivity is available from the tube, while if illumination is further increased to 10^{-1} foot candles, resolution improves to about 500 TV lines. Typical tubes include the 4915, 4401, 2048 and 7263.

Based upon the required sensitivity of about 1.5×10^{-2} foot candles, developed in paragraph 1.4.4.1, it appears that the better Vidicons have a safety factor of 10 for this application, since 100 line resolution is acceptable.

1.4.4.2.3 Resolution

TV resolution is commonly quoted as the number of separate lines which may be resolved vertically in a televised test pattern. Due to the 4:3 aspect ratio, diagonal resolution is 5/3 times this figure. Finally, since optical resolution refers to line pairs, rather than lines, equivalent optical resolution of a television camera is 5/6 of quoted resolution. Resolutions in the order of 1000 lines have been quoted for some experimental Vidicons, but these require extremely high operating potentials and complex dynamic corrections. 800 lines may be considered a more practical maximum, with typical figures of 500 lines. Minimum resolution occurs with maximum sensitivity of the tube, and is usually about 100 lines.

1.4.4.2.4 Vidicon Replacement

If necessary, the Vidicon may be replaced in flight. When a new Vidicon is inserted in the video system, the system controls must be adjusted to match the tube characteristics. These adjustments involve "TARGET", "BEAM", and "OPTICAL FOCUS" controls, and possibly the magnets and deflection yoke.

1.4.4.2.5 Video Bandwidth

The bandwidth required is a function of the number of scan lines, the desired horizontal resolution, and the repetition rate, as well as certain other, less important, parameters. In general:

$$B\,W = \frac{A\ R_H\ Nf}{2 \times 0.9} \qquad (1\text{-}11)$$

where A = Aspect Ratio (4/3 in conventional television)
R_H = desired horizontal resolution (equal area)
N = number of scanning lines
f = frame rate, or picture repetition frequency

Vertical resolution may be computed by

$$R_V = .95 K_1\, K_2\, N \qquad (1\text{-}12)$$

where K_1 = 0.75 for random interlace, 1.0 for full interlace
K_2 = .7 (Kel factor)

Consider this example:

Let $f = 20$ cps as minimum frame rate
$R_V = R_H = 400$ lines
$K_1 = 0.75$ (using random interlace)
$A = 1$

therefore $N = 800$ scanning lines

$$\text{and } BW = \frac{400 \times 800 \times 20}{1.8} = 3.56 \text{ mc}$$

Whether full advantage can be taken of this 400 line resolution depends on brightness of the star in question.

1.4.4.2.6 General Configuration

Vidicons are currently available which will operate completely transistorized, requiring only 12 watts of power at 12 volts dc. These additional circuits include the deflection and blanking amplifiers, the power supply, and the video amplifier. These components may be packaged, with the Vidicon and necessary controls, in less than 3/8 of a cubic foot. Weight of the complete unit packaged in this manner would be about 16 pounds.

1.4.4.3 Monitors

If a monitor size is to be chosen to allow the 400 line resolution to be easily viewed at the standard viewing distance of 18 inches, this size may be computed from known values. Acuity of the human eye varies between 0.8 and 1.5 minutes of arc, for minimum separable resolution. Thus a minimum height of 0.0075 inches is required per line, and with the 400 line resolution stated, a 3.0 inch high display would be acceptable. To allow a margin for fatigue, etc, a larger display should be proposed. However, it appears that a seven inch display is adequate for the following reasons.

It provides twice the 1.5 minutes of arc acuity.
It provides for 400 lines, while 100 may be adequate.
It provides for 1.5 minutes of arc, while 0.8 is true minimum.

A seven inch monitor may be packaged in a 10 x 10 x 8 inch area, weighing 25 pounds and requiring 38 watts of power.

Since two Vidicons are required, one for the 30 degree field and one for the 1 degree field, two monitors probably should be utilized also. This provides for system backup in case of failure of one monitor, since the remaining one may be manually switched between the two cameras as the operator's requirements change.

1.4.4.4 Block Diagram

Figure 1-19 is a preliminary block diagram of one channel of the video system; the second channel merely duplicates this and two way switching is indicated.

1.4.4.5 Optical System

The optical instrument must fit within the inner surface of an observing dome, diameter 21 inches, with sufficient clearance for free rotation. The geometrical center of the dome, point 0, is the common intersection for all optical axes of rotation of the telescope. The optic axis of the telescope must pass through this point.

Two concentric spherical shells make up the dome, and are part of the optical system. Optical characteristics will depend on the dome material, the shell thickness, and the shell separation.

A dichroic coating for infrared exclusion is necessary on the outer dome.

1.4.4.5.1 Finder Telescope

Visual observation of the celestial sphere is indirect. The finder telescope (field of view 30 degrees) will focus an image on the face of a vidicon tube. The image diameter needs to be 5/8inches to take advantage of the full tube face. No other use of the finder is made than to obtain an image on the vidicon.

The requirements for the system are that the optic axis of the finder in its mounting brackets must be carefully and accurately aligned parallel to the optic axis of the observing telescope.

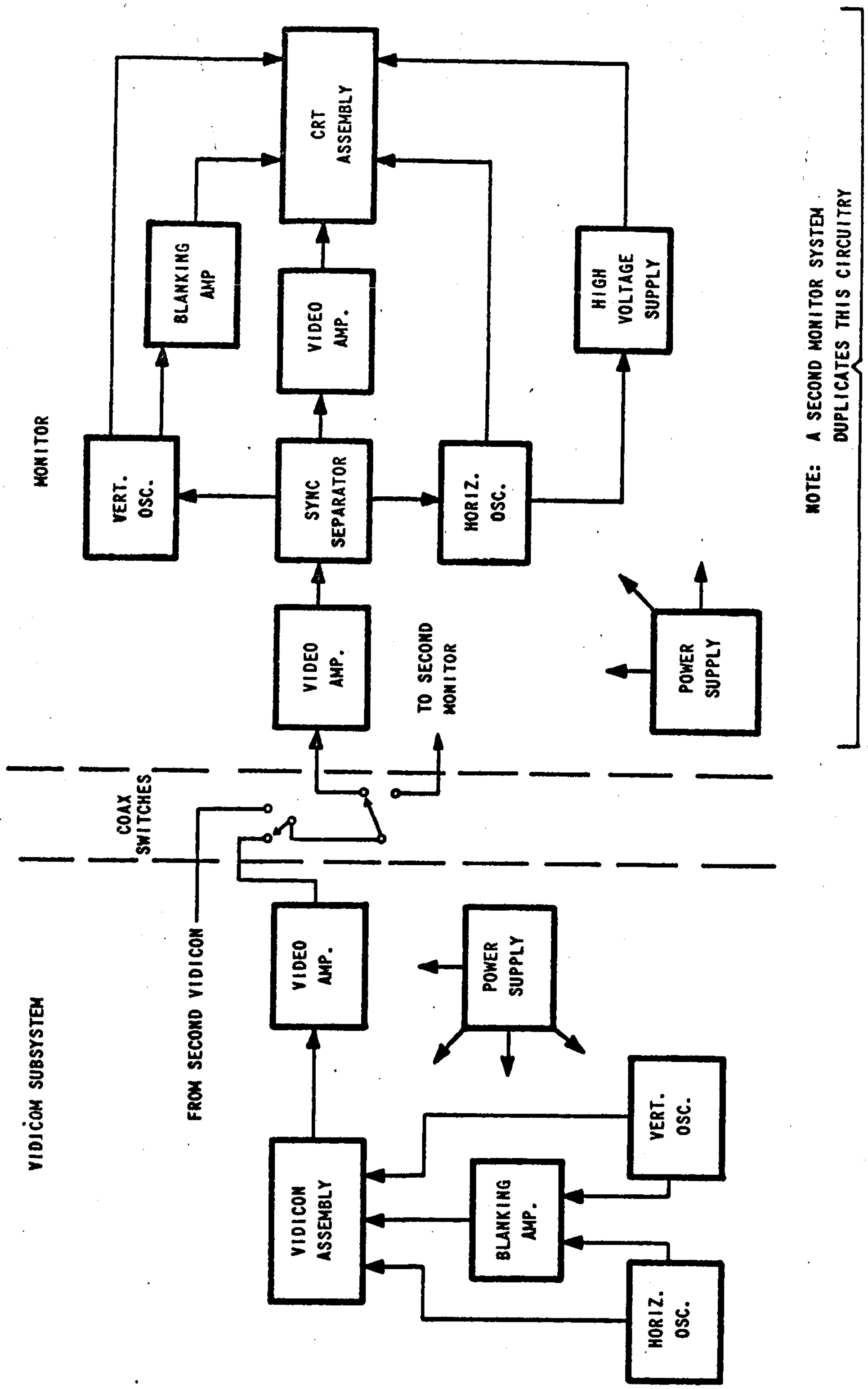


FIGURE 1-19
VIDICON SYSTEM BLOCK DIAGRAM

It is necessary that the center of the finder field, which is the point seen by the observing telescope, be marked by the intersection of crosshairs, or preferably by a ring or circle scaled to subtend 1 degree on the viewing screen. It may be possible to insert such an illuminated marker in the field of the finder, where it would be transmitted as part of this field by the vidicon. If this is not feasible, or if it is at all difficult, such a marker could be placed on the external face of the viewing screen, although the first alternative is preferable.

1.4.4.5.2 Observing Telescope

The observing instrument provides at least three images of a 1 degree star field. Figure 1-20 suggests appropriate dimensions.

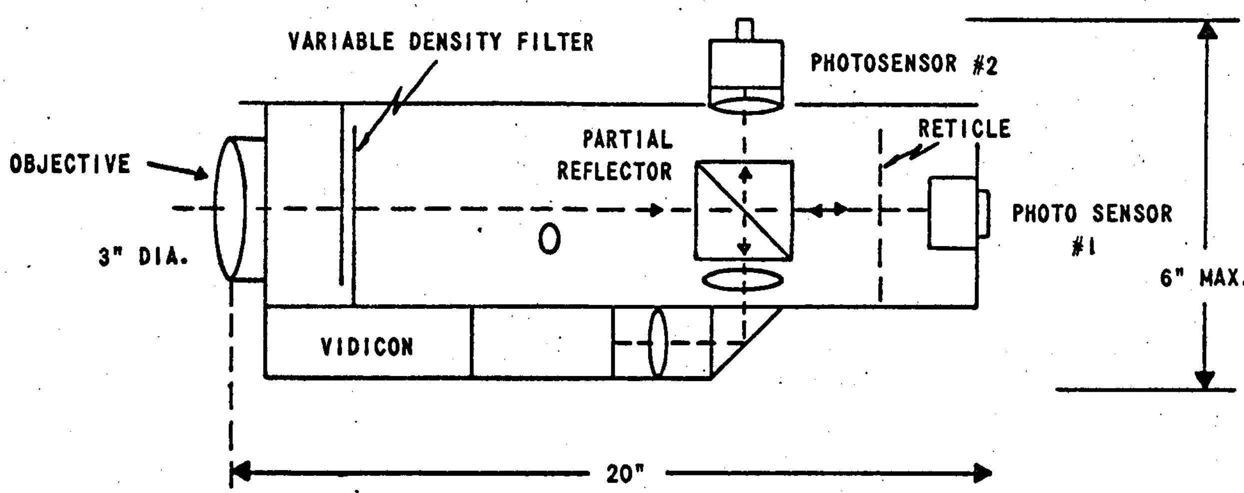


FIGURE 1-20

At the partial reflector the reflected light is focused on a vidicon so that the image (5/8 in. diameter) fills the tube face. The transmitted light is incident on the special reticle, partly transmitting and partly reflecting. The photosensors are preceded by whatever secondary lenses are demanded. Sensor No. 1 receives light transmitted by the reticle, and No. 2 receives light reflected back from the mirrored portions of the reticle.

The Vidicon transmits the image of the 1 degree field to a second viewing screen independent of that displaying the finder field.

The navigational stars which are to be observed are second magnitude and brighter, and no other star of greater than fifth magnitude lies within 1 degree of navigational stars. Nevertheless, it is possible that in space, stars of lower magnitude will appear in the field of the telescope along with the navigational star to create electronic confusion in the photosensors. For this reason the optical design of the telescope includes a variable density filter, in order to exclude fainter star images from the field.

The telescope field display on the viewing screen may provide a backup method for observing transits of stars and planets. It is essential that the image seen by the navigator shall include clearly defined 90 degree crosshairs, across which the star and planets will transit on the viewing screen. As in the case of the finder, these crosshairs can be placed on the screen itself or in the optical system of the telescope. There are stringent requirements on the orientation of these crosshairs with respect to the telescope motion, and with respect to the reticle markings. Since the rotation of the telescope from one 45 degree great circle to the other shifts the actual physical position of the reticle which establishes the physical orientation of the transit lines, it seems logical to suggest that illuminated crosshairs in the telescope be in some way physically oriented with respect to the reticle, and rotate with it.

The single set of two photosensors and one reticle can accurately determine star transit, and of course the rotation of the telescope about its optic axis provides information about either great circle transit.

On the other hand, it is much less likely that the accuracy of planet transit observation will be as satisfactory. The requirement that planet tangencies on two lines 90 degrees apart be read demands a fairly complex reticle, which must also give star transit data.

If it were possible to obtain data for a sequence of tangencies for each 45 degrees line, a computer smoothing process would considerably improve the resulting determination of the plant center location.

A brief consideration of this problem indicates that a method which gives such multiple readings is preferable, but a single reticle to give both star transit data and iterated planet tangency readings may be very difficult to design. (Two reticles each having about 100 reflective lines might be used, but a co-alignment problem would then be introduced).

If such a design proves impossible, but a single pair of planet tangency readings gives poor results, it might be necessary to employ a second set of photosensors and a second reticle. With this added capability, iterated tangency readings are possible at the cost of decreased light intensity at each photosensor, possibly requiring larger aperture optics.

The planet angular diameters may lie between 20 seconds and 1 minute. No method of detecting planet tangency is very useful if it cannot determine such tangencies within 0.1 arc seconds or less as read out on the angle scale of the table.

1.4.4.5.3 Optical System Components

Within the guide lines established above some choices for optical components have been made. All lenses are "in stock" items, as are the Vidicon and photomultiplier tubes. The only custom made element of the system is the reticle. The spatial arrangement and size of the optical package is indicated in Section 2.

Finder System

The finder telescope consists of a single 25mm cine lens in a "C" mount, with a 16mm format. Any of the following lenses would be satisfactory:

F/0.95	Angenieux
F/0.95	Carl Meyer
F/0.95	Cinor
F/0.99	Dallmeyer Speed

The primary image, formed one inch from the second principal point, will have a diameter of approximately 3/4 inches and cover a field of view of more than 35 degrees. The aperture of the lens is slightly over one inch, and reference to the Vidicon sensitivity analysis (Paragraph 1.4.4.1) shows that this will, with careful design and choice of the Vidicon tube and its electronics, provide enough light energy to give a usable image on the viewing screen.

The overall length of the lens mounting is approximately two inches including a light shield. The telescope tube is one inch long and the Vidicon tube and deflecting coils make a package six inches long and two inches in diameter. The overall size of the finder telescope assembly is two inches in diameter by nine inches in length.

Transit Telescope System

A well corrected telephoto lens 14 inches focal length, relative aperature F/4.5 in a 35mm format, will serve admirably as the objective.

The image is formed two inches or less from the mounting ring or flange on the lens mount, and is approximately 1.6 inches in diameter. The aperture stop is three inches in diameter and the field of view about 7 degrees. The central 1 degree field covers an area 0.25 inches in diameter. A Dallmeyer telephoto lens of these dimensions was made at one time and is sometimes available on the market. Any other lens of similar characteristics would be satisfactory.

With a three inch aperture the diffraction image is 0.00025 inches in diameter so the linear width of the 1 degree field is 1000 times the width of the diffraction limited image. Hence we may use this image with no further magnification, on the surface of the reticle.

In order to observe the transit field, a partial reflector placed in the beam in front of the image reflects some of the light through an auxiliary lens to the Vidicon tube face. The rest of the light strikes the reticle. If the star image falls on a transmitting area it strikes the sensitive area of a photomultiplier tube. If it strikes a reflective area on the reticle, it reflects to the sensitive area of a second photomultiplier. It is necessary to refocus this light with another auxiliary lens (due to the different path length).

A number of photomultipliers are available. For example, the DuMont 6365 is a highly sensitive six stage tube 3/4 inches diameter and 2-3/4 inches long. The choice is very large for these tubes. Photoconductive and photovoltaic sensors were considered, but tentatively rejected because of noise considerations.

RE-CAP OF STUDY PROGRESS BY MONTHS

FIFTH MONTH'S PROGRESS

1.4.5 Fifth Month's Work

Drawings of the preliminary design (feasibility) configuration were essentially completed during October, attention was turned to the overall performance accuracy analysis primary goal of this study. See Section 3 of this report.

Writing of this report began, with the goal of publication on or before 17 November 1963. This report and completion of the analysis occupied most of the 5th month.

A basic firm decision was made to use a transit detector reticle of reflective mirror lines deposited upon a transparent substrate by photo-resist etching. Apparently, the use of "grainless" Kodak Photo Resist ("KPR") to control the etching of a metallic coating represents the best state of the art in chemical formation of reticle patterns. According to Fischel the accuracy with which large master patterns may be photochemically reduced approaches molecular dimensions. At any rate, the dimensional control of this process exceeds that possible in lapping the knife edge of a wedge mirror for the more conventional bridge type null sensors.

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APPENDIX 1-A

TO SECTION 1

EVALUATION OF ALTERNATE METHODS FOR USING PSEUDO-DIURNAL TECHNIQUES

I. G. FOSTER

APPENDIX 1-A
EVALUATION OF ALTERNATE METHODS FOR USING PSEUDO-DIURNAL TECHNIQUES

The problem of planet center location (local vertical) is common to every method and identical in each. The best method of using the pseudo-diurnal techniques is to observe transits of individual stars, occuring consecutively, chosen to be reducible to isoazimuthal pairs. The criteria for choice include the number and distribution of the navigational stars, the physical operations involved in the observation, the time necessary to obtain the observations, and the complexity of the resulting mathematical reduction of data to a line-of-position.

Methods of Observation

The following methods of observation are involved in a pseudo-diurnal rotation:

1. Iso-Azimuthal

   a. The simultaneous transit of two stars across a reticle line which is the projection in the focal plane of a great circle through the zenith point of the observing telescope. Two such pair transits are required, each on a different great circle.

   b. The non-simultaneous transit of two stars across a reticle line, which is the projection in the focal plane of a great circle through the zenith point of the observing telescope. Two such groups of transits are required, each group on a different great circle.

   c. Other modes in which the planet is observed between star transits, or in which the planet is observed twice during the set of observations.

2. Iso-Almucantar

   a. The simultaneous transit of two stars across a reticle line, which is the approximate projection in the focal plane of a small circle (almucantar) lying in a plane normal to the optic axis of the telescope. Two such pair transits are required, on the same or different almucantars.

b. The transit of four stars. The first two, each timed individually as it transits the center of the almucantar line, are referred to one almucantar. The second pair, timed individually, is referred to a second almucantar.

c. The transit of each of three stars across the center of a single almucantar line in the field. Only one such set of three transits is needed.

d. Other modes in which the planet is observed between star transits, or the planet is observed twice during the set of observations.

3. Single Star (Versus Planet)

a. The transit of a single star through the center of the telescope field, i. e., through the zenith point of the observing telescope.

Evaluation of Alternate Methods

1. a The pseudo-diurnal path is the path of the optic axis point traced out as the telescope rotates about the pseudo-diurnal axis. This mode of observation depends upon the existence of two pairs of stars. The stars of a single pair lie on a great circle which intersects the pseudo-diurnal path on the celestial sphere. The great circle intersection with this path must be at such a point that the telescope will swing past it within a few minutes of the planet sighting. Of the approximately 1,500 pairs of stars available from the navigational group, only a few will be found in that part of the celestial sphere under observation. The choice of star pairs is then severely limited and in some cases there may be no pairs at all. This tends to extend observing time, increasing error due to rate instability.

It is not easy to determine with accuracy the instant at which the optic axis extended transits the great circle of the isoazimuthal pair. This point is illustrated in Figure 1-A-1 which shows two star images S_1 and S_2 moving in the field of the telescope. Whether S_1 and S_2 move in the same or opposite sense is unimportant. The discussion is the same. It is possible to shift S_1 and S_2 in a direction along VV' in the field of view and therefore one can put the images in such relative positions that the line S_1S_2 will always transit across the optic axis.

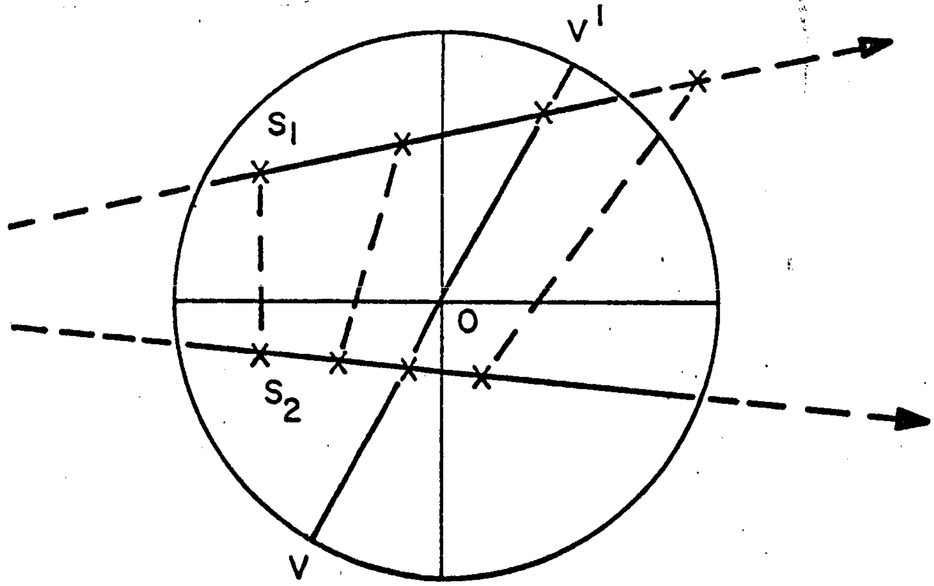


FIGURE 1-A-1

When one first sees the two stars S_1S_2 they are in the field, moving in the same sense, and their positions in the field of view might be changed simply by adjusting viewing mirrors. Lines would be scribed on the viewing mirrors in such a fashion that the images of these lines in the field of view represent the projection of a great circle through the zenith point on the field of view. The zenith point is the point at which the optic axis intersects the celestial sphere. The line VV' represents the projection of the great circle on which we judge S_1 and S_2 to lie. As the optic axis moves across the sphere, the great circle (shown by the line VV') will not change its direction, but the line joining S_1 and S_2 is changing direction.

If one has chosen with complete accuracy the direction VV' then the stars will transit VV' simultaneously. If not, then S_1 may reach VV' before or after S_2 and VV' has not been chosen properly -- it is not the great circle containing $S_1\overline{S_2}$. It is essential to provide a rotatable set of reticle lines for the observer's use. One should not used a fixed reference line like VV' as the transit indicator for manual operation, because if one errs in choosing VV', a correction can be made only if one has more data than a single instantaneous position.

Suppose now we describe a non-manual method of operation, using Figure 1-A-2. VV' represents the assumed great circle direction as represented by the lines scribed on the mirrors. A rotatable reticle is adjusted until its y axis is parallel to VV' as shown.

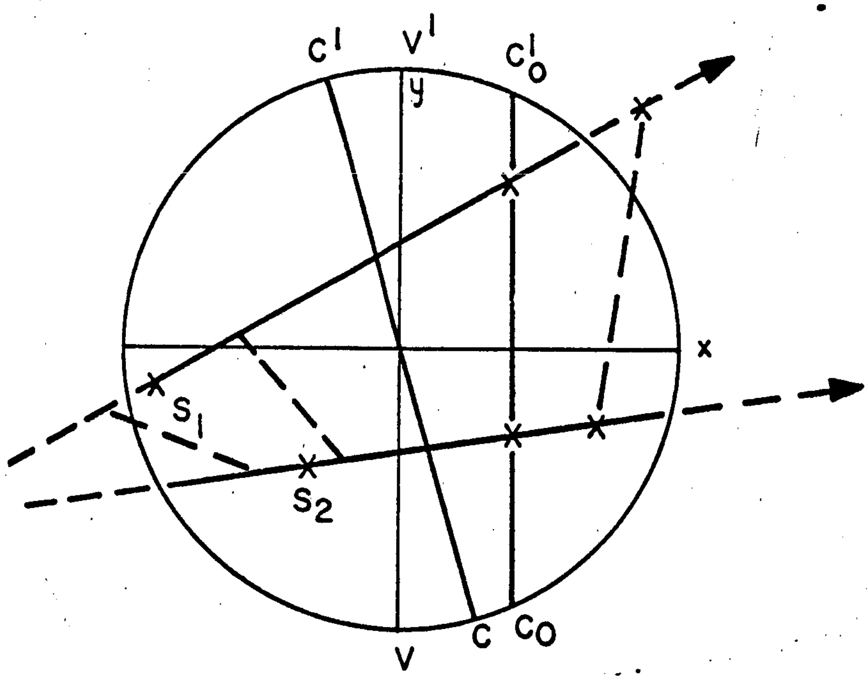


FIGURE 1-A-2

Now one can see that the great circle on which the stars truly lie is CC', but that in position C_0C_0' they do lie on a small circle which lies in a plane parallel to the plane containing VV'. If one has recorded a succession of x positions against time for each star, either manually or by some method involving photo-sensitive recording, one can compute the position of the y line along which the coincidence occurs. A lengthy correction will generate the position of the great circle VV' and two fictitious stars S_1'S_2' on it. Clearly if one attempted to determine C_0C_0' by personal judgment, it is more than likely that no reticle line would actually coincide with C_0C_0' by means of which coincidence could be judged.

The element of judgment which allows the navigator to rotate the reticle lines and hence truly find the great circle transit is not present in the non-manual method, which then demands that one make elaborate corrections. This must be balanced off against the possibly greater accuracy of transit time possible to compute from the many readings of the non-manual operation.

Finally in order to view both stars S_1S_2 at once, one needs two auxiliary components (mirrors) with stringent demands in regard to the stability of their axes of rotation.

1.b Much less stringent demands are made if it is possible to substitute single star observations for pair observations in the isoazimuthal method. First, if one can observe single star transits, then the number of pairs available is much increased. At any position on the pseudo-diurnal circle, one may observe nearly all the navigational stars. One need not search for a pair of stars lying on a single great circle but only for two stars which transit consecutively a given great circle carried across the celestial sphere by the pseudo-diurnal rotation. The allowable time between consecutive transits is limited by several factors, but could be in the order of a minute or two.

In addition, the problem of simultaneous transit is completely avoided. A star S_1 (Figure 1-A-3) appears in the field of view and moves toward the other edge. At time t_1, the star image transits VV', a line in

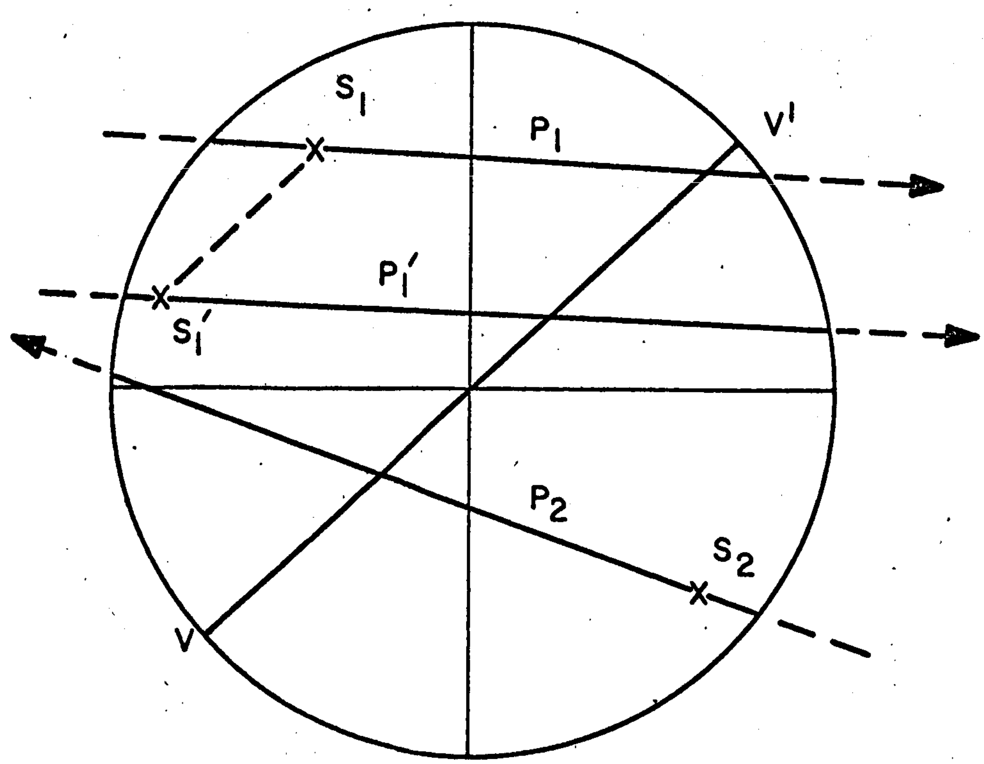


FIGURE 1-A-3

the field which is the image of a great circle through the zenith point. The viewing element (mirror) can be adjusted so that the image S moves along a line in the field parallel to p_1. Clearly the transit time is independent of the path followed, whether p or p'. Some time later a second star S_2 is brought into the field by the mirror and it too transits VV' at a later instant t_2. The direction of VV' is not changed between observations. A correction $\omega(t_2 - t_1)$ applied to the longitude of S_2 will give the coordinates of a fictitious star which would have transited VV' at the instant t_1, hence an iso-azimuthal pair is created.

Finally, one avoids the need for two mirrors since only one star need be observed at a time. No rotatable reticle in the field of view is required, since the only transit line necessary (VV') is associated with the viewing mirror.

On the other hand, the total observing time is quite a bit longer than for case (a). This can be a disadvantage for several reasons: stability of pseudo-diurnal axis must be maintained for a longer period, all windows or viewing ports have to be of greater aperture, and any angle measuring instrument has to have a capability for measuring a larger angle.

1\. c It is possible to make the isoazimuthal observations in the following order: planet, S_1, S_2, then repeat, taking planet, S_3, S_4. Corrections of star positions to planet position can in this case be accomplished only for the first pair separately and apart from the second pair, because it is necessary to interrupt the pseudo-diurnal rotation between planet sightings.

Mechanization is no different than case (b) above, and stars are available in the same numbers.

This system of observing suffers because one has not located a specific line of direction from vehicle to planet, but only two planes in which the planet is known to be at two different times. It is perhaps possible to use such data in a computer in a lengthy curve fitting or iteration process by which the planets' position relative to the vehicle could be established.

The only advantage of this mode would be the possible time saved in the total observing period.

2\. a In the "isoalmucantar" mode of observation (see Appendix 1-E) star pairs are chosen with reference to a different geometrical standard. There are the same number of pairs as described above, but the criterion for choosing them is simultaneous transit of the two stars of each pair across an almucantar which is a small circle on the celestial sphere lying in a plane normal to the zenith direction (optic axis). It appears that there are about the same number of stars either singly or in groups available for isoalmucantar observation as for isoazimuthal observation.

The image of the almucantar is not strictly a straight line (as is a great circle through the zenith), although its curvature in a small field is not great. At the point 0 (Figure 1-A-4) this curve is tangent to the straight reference line inscribed on the mirror. The chance of observing a simultaneous transit of two stars is practically nil, primarily because the curvature of the almucantar image depends upon its co-latitude from the zenith. At a co-latitude of 90 degrees, the

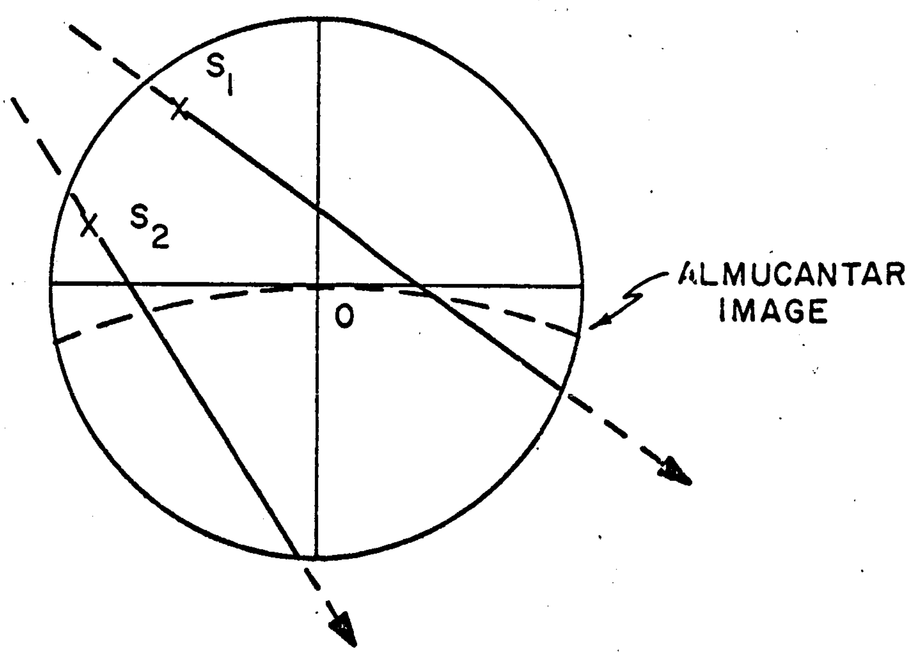


FIGURE 1-A-4

image will be a straight line. The location of the true almucantar in the field of view is known only approximately, only as well as the line approximates the curved image. Second is the practical difficulty of choosing a pair of stars which actually do lie on the same almucantar. This is analogous to the situation described under 1(a) where simultaneous transit did not occur on the great circle at all, but on the small circle.

For this reason alone-- the difficulty of obtaining a true simultaneous transit-- this method of observation is not considered further.

2. b On the other hand, one can avoid most of the difficulties of simultaneous transit by following a slightly different procedure, similar to method 1(b). In this case, one adjusts the viewing mirror so that it tracks along a particular almucantar, chosen because a group of at least two stars is so located that each star will transit the same almucantar, but at different times. The viewing mirror would be rotated to such a position that the star in question would transit the almucantar image at 0, thus removing the error due to almucantar image curvature. This requires considerable judgment on the part of the observer and surely would involve a difficult mechanization since transit has to occur at a particular point of a line, not at any point on the line.

Without changing the almucantar, one next observes the transit of the second star. Corrections carried out on each star provide information from which one can generate a fictitious great circle passing through the planet position. If one then repeats this sequence of observations for a second almucantar and two stars transiting consecutively, a second great circle is generated. The intersection of the two great circles establishes the planet position. The generation of the two great circles adds another operation to the mathematical process which is not present in the isoazimuthal method.

2. c A third mode of isoalmucantar observation involves finding an almucantar across which three stars will transit, not simultaneously, but within a relatively few minutes. From this data, two great circles can be generated and the planet located.

Fewer observations are needed, with a consequent saving of time, but groups of three stars are much less likely to occur than the pairs required in method 2(b). Instrumentation is similar to that required in 2(b).

2. d Other possible modes in which the planet is observed more than once, or is observed between star transits, lead to such mathematical difficulties in using the data as to be of little value.

3. a If we were able to discover a single star which lies exactly on the pseudo-diurnal path of the optic axis in the celestial sphere, then the planet position is known at once, simply by correcting the star position in the standard fashion. The possibility that one of the 57 navigational stars lies on this path is not zero, but is surely very small.

On the other hand, it is quite likely that one might find a star close to this path on the sphere. There exist possibilities of generating a fictitious star out of this which would lie on the pseudo-diurnal path and from which the planet coordinates are immediately available. However, this would entail the measurement of an angle in the field of view with a consequent loss in accuracy, defeating the object of the pseudo-diurnal technique.

Conclusion

In weighing the advantages and disadvantages of each method discussed, it appears that 1(b) is the best choice. Star availability is as good as for any method; instrumentation is no more complicated that for any case; and the operation necessary to determine star transits is less demanding when performed either manually or automatically. The one disadvantage is that more time is required to make the observations than in other methods. (It was decided to concentrate upon the single star-at-a-time method.)

Whatever method of observation is employed, the target planet center location must come first; this is the "local vertical" problem common to all methods of celestial navigation.

APPENDIX B TO SECTION 1

STAR COORDINATE TRANSFORMATION TABLES

HONEYWELL H-800

Joy Henefelt and I. G. Foster

AND

IRRADIANCE OF NAVIGATIONAL STARS

FOR CERTAIN PHOTOSENSITIVE MATERIALS

Honeywell Aero Report 2858-FR

TABLE 1-1

COORDINATES OF NAVIGATIONAL STARS

for β URSA MINORIS AS POLESTAR

CONSTELL NAME	ALPHA DEG	MIN	SEC	DELTA DEG	MIN	SEC	LAMBDA DEG	MIN	SEC	BETA DEG	MIN	SEC
A AND	1	37	3.00	28	53	10.00	233	5	27.28	16	39	3.50
A PHE	6	6	52.50	-42	30	25.00	221	23	.46	-54	9	20.13
A CAS	9	35	54.00	56	20	5.00	245	45	19.31	42	30	18.24
B CET	10	25	58.50	-18	11	22.00	233	38	29.65	-31	11	8.89
A ERI	24	5	4.50	-57	25	23.00	236	52	1.31	-71	40	28.36
A U MI	29	28	27.00	89	5	33.00	269	16	20.66	73	25	19.07
A HYI	29	24	3.00	-61	44	58.00	242	3	5.29	-76	34	5.20
T ERI	44	12	51.00	-40	27	8.00	272	4	13.28	-56	8	17.33
A CET	45	5	7.50	3	56	44.00	272	25	56.21	-11	44	2.67
A PER	50	25	3.00	49	43	52.00	276	1	18.27	34	8	50.61
A TAU	68	26	55.50	16	26	11.00	294	38	45.59	2	13	10.95
B ORI	78	11	22.50	-8	14	35.00	307	56	30.06	-20	51	13.93
A AUR	78	29	16.50	45	57	45.00	298	52	14.42	32	39	15.13
G ORI	81	47	10.50	6	19	4.00	308	3	59.19	-6	3	54.37
B TAU	80	59	16.50	28	34	41.00	304	27	31.16	15	54	27.26
E ORI	83	35	1.50	-1	13	27.00	312	11	32.74	-13	0	13.69
A ORI	88	17	31.50	7	24	6.00	315	13	31.74	-3	39	2.29
A CAR	95	46	57.00	-52	40	30.00	344	22	53.96	-59	46	38.25
A C MA	100	52	46.50	-16	39	57.00	333	20	25.17	-24	22	25.90
E C MA	104	17	34.50	-28	55	14.00	340	40	16.11	-35	19	25.57
A C MI	114	20	30.00	5	19	16.00	340	54	53.50	0	15	20.43
B GEM	115	45	49.50	28	7	2.00	336	2	54.45	22	35	43.10
E CAR	125	26	21.00	-59	23	24.00	18	41	17.66	-57	46	35.23
L VEL	136	39	30.00	-43	16	57.00	17	50	53.67	-40	16	25.57
B CAR	138	12	3.00	-69	33	53.00	39	25	19.61	-63	15	46.55
A LEO	151	36	3.00	12	8	55.00	15	0	31.68	16	45	15.43
A U MA	165	21	54.00	61	57	4.00	0	48	50.56	66	40	43.32

TABLE 1-1 (Cont' d)

B LEO	176	47	36.00	14	46	44.00	39	48	30.63	25	18	49.14
G CRV	183	28	30.00	-17	20	12.00	52	42	21.04	-4	58	57.42
A CRU	186	7	52.50	-62	53	40.00	65	24	24.87	-49	17	18.21
G CRU	187	16	31.50	-56	54	23.00	64	12	38.67	-43	19	47.83
E U MA	193	6	6.00	56	9	37.00	41	17	6.03	68	31	48.70
A VIR	200	48	36.00	-10	58	9.00	68	29	11.13	3	37	28.97
ET U MA	206	31	15.00	49	29	50.00	65	22	41.24	64	15	25.54
B CEN	210	17	55.50	-60	11	44.00	81	21	21.47	-44	45	16.92
T CEN	211	7	27.00	-36	11	20.00	80	1	37.99	-20	46	2.37
A BOO	213	29	36.00	19	22	27.00	79	24	18.78	34	50	16.05
A CEN	219	16	4.50	-60	41	2.00	87	37	29.94	-45	0	35.22
A LIB	222	12	24.00	-15	53	21.00	89	31	37.62	0	11	46.98
B U MI	222	41	54.00	74	18	24.00	0	0	.00	90	0	.00
A CR B	233	16	48.00	26	50	17.00	102	46	53.29	42	12	47.06
A SCO	246	47	1.50	-26	21	6.00	111	56	40.76	-11	53	28.37
B TR A	251	10	55.50	-68	57	45.00	107	6	28.44	-54	24	37.86
ET OPH	257	3	48.00	-15	40	51.00	122	57	25.75	-2	35	33.82
L SCO	262	46	24.00	-37	4	43.00	124	22	32.42	-24	32	19.01
A OPH	263	18	13.50	12	35	8.00	134	8	49.25	24	13	2.13
G DRA	268	56	10.50	51	29	33.00	155	42	40.54	60	26	24.38
E SGR	275	25	43.50	-34	24	15.00	136	0	35.11	-24	7	50.38
A LYR	278	59	16.50	38	44	53.00	159	3	16.15	46	2	31.27
S SGR	283	14	30.50	-26	20	40.00	145	6	12.00	-17	56	25.30
A AQL	297	14	39.00	8	46	9.00	167	26	9.45	12	35	30.75
A PAV	305	40	50.50	-56	51	20.00	151	49	.01	-52	0	3.25
A CYG	310	2	31.50	45	8	50.00	192	49	1.96	43	44	9.49
E PEG	324	35	30.00	9	42	16.00	194	59	36.80	5	53	59.48
A GRU	331	28	37.50	-47	8	29.00	181	6	51.48	-49	54	3.88
A PS A	343	54	9.00	-29	49	8.00	201	53	41.07	-36	53	30.79
A PEG	345	43	13.50	15	0	21.00	215	27	59.33	6	8	2.50

TABLE 1-2

COORDINATES OF NAVIGATIONAL STARS

for α CEPHEI AS POLESTAR

CONSTELL NAME	ALPHA DEG	MIN	SEC	DELTA DEG	MIN	SEC	LAMBDA DEG	MIN	SEC	BETA DEG	MIN	SEC
A AND	1	37	3.00	28	53	10.00	149	8	12.81	46	45	28.75
A PHE	6	6	52.50	-42	30	25.00	125	10	49.13	-21	24	2.98
A CAS	9	35	54.00	56	20	5.00	189	24	19.89	64	26	6.70
B CET	10	25	58.50	-18	11	22.00	137	35	43.97	0	0	6.34
A ERI	24	5	4.50	-57	25	28.00	129	18	36.93	-39	48	57.64
A U MI	29	28	27.00	89	5	33.00	268	8	17.65	62	43	32.43
A HYI	29	24	3.00	-61	44	58.00	128	53	15.34	-44	53	48.28
T ERI	44	12	51.00	-40	27	8.00	154	29	36.44	-32	53	53.18
A CET	45	5	7.50	3	56	44.00	177	58	54.62	5	30	16.99
A PER	50	25	3.00	49	43	52.00	209	19	43.84	42	9	29.16
A TAU	68	26	55.50	16	26	11.00	204	13	54.27	6	5	26.58
B ORI	78	11	22.50	-8	14	35.00	202	18	.11	-20	20	10.20
A AUR	78	29	16.50	45	57	45.00	226	7	51.81	28	44	49.60
G ORI	80	47	10.50	6	19	4.00	210	58	33.43	-8	9	24.65
B TAU	80	59	16.50	28	34	41.00	220	2	45.66	12	11	47.71
E ORI	83	35	1.50	-1	13	27.00	210	31	17.67	-16	11	13.17
A ORI	88	17	31.50	7	24	6.00	218	22	3.19	-10	0	40.47
A CAR	95	46	57.00	-52	40	30.00	182	51	26.63	-65	13	47.05
A C MA	100	52	46.50	-16	39	50.00	221	40	40.81	-36	56	29.38
E C MA	104	17	34.50	-28	55	14.00	219	11	38.87	-49	27	54.23
A C MI	114	20	30.00	5	19	16.00	243	23	1.07	-19	35	4.36
B GEM	115	45	49.50	28	7	2.00	249	14	59.22	2	30	45.07
E CAR	125	26	21.00	-59	23	24.00	197	50	56.95	-82	34	19.70
L VEL	136	39	30.00	-43	16	57.00	263	52	21.40	-70	47	8.57
B CAR	138	12	3.00	-69	33	53.00	93	25	54.54	-82	50	47.91
A LEO	151	36	3.00	12	9	55.00	282	18	56.38	-14	49	7.86
A U MA	165	21	54.00	61	57	4.00	284	42	48.37	35	54	57.90

TABLE 1-2 (Cont' d)

B LEO	176	47	36.00	14	46	44.00	306	17	20.16	-7	26	39.64
G CRV	183	28	30.00	-17	20	12.00	324	40	58.11	-35	34	12.21
A CRU	186	7	52.50	-62	53	40.00	22	9	34.68	-69	1	2.75
G CRU	187	16	31.50	-56	54	23.00	8	50	35.52	-65	48	54.15
E U MA	193	6	6.00	56	9	37.00	303	32	23.68	35	42	13.74
A VIR	200	48	36.00	-10	38	8.00	339	7	24.71	-22	43	24.47
ET U MA	206	31	15.00	49	29	50.00	316	5	12.52	33	51	1.47
B CEN	210	17	55.50	-60	11	44.00	28	42	59.92	-57	37	28.36
T CEN	211	7	27.00	-36	11	20.00	3	38	.51	-39	50	35.48
A BOO	213	29	36.00	19	22	27.00	337	6	22.68	10	1	58.09
A CEN	219	16	4.50	-60	41	2.00	34	9	36.85	-54	22	38.48
A LIB	222	12	24.00	-15	53	21.00	1	10	37.86	-17	22	27.99
B U MI	222	41	54.00	74	18	24.00	299	33	27.24	57	0	21.09
A CR B	233	16	48.00	26	50	17.00	350	4	29.69	25	20	19.85
A SCO	246	47	1.50	-26	21	6.00	27	21	13.07	-15	38	50.97
B TR A	251	10	55.50	-68	57	45.00	58	46	27.94	-49	58	27.56
ET OPH	257	3	48.00	-15	40	51.00	31	25	8.93	-1	52	57.94
L SCO	262	46	24.00	-37	4	43.00	45	3	36.62	-19	21	21.37
A OPH	263	18	13.50	12	35	3.00	25	12	27.49	26	25	17.37
G DRA	268	56	10.50	51	29	33.00	358	40	54.97	61	17	1.97
E SGR	275	25	43.50	-34	24	15.00	53	57	39.75	-13	4	9.10
A LYR	278	55	16.50	38	44	53.00	24	58	42.22	56	1	36.47
S SGR	283	14	34.50	-26	20	40.00	57	59	54.96	-3	21	28.40
A AQL	297	14	39.00	8	46	9.00	63	15	50.72	33	58	6.69
A PAV	305	40	58.50	-56	51	20.00	81	23	45.19	-29	45	38.95
A CYG	310	2	31.50	45	8	57.00	68	17	5.94	71	53	44.21
E PEG	325	35	30.00	9	42	16.00	97	37	42.60	37	5	10.36
A GRU	331	28	37.50	-47	8	28.00	98	41	35.37	-19	59	31.34
A PS A	343	54	9.00	-29	40	3.00	111	7	54.27	-4	19	7.66
A PEG	345	43	43.50	15	8	21.00	123	27	30.63	39	4	18.21

TABLE 1-3

COORDINAT F NAVIGATIONAL STARS

for β CARINAE AS POLESTAR

CONSTELL NAME	ALPHA DEG	MIN	SEC	DELTA DEG	MIN	SEC	LAMBDA DEG	MIN	SEC	BETA DEG	MIN	SEC
A AND	1	37	3.00	28	53	10.00	35	22	39.00	-42	25	58.96
A PHE	6	6	52.50	-42	30	25.00	51	56	49.13	27	25	42.55
A CAS	9	35	54.00	56	20	5.00	4	25	19.12	-64	14	45.98
B CET	10	25	58.50	-18	11	22.00	41	3	38.87	5	7	36.26
A ERI	24	5	4.50	-57	25	28.00	45	31	.98	45	27	59.49
A U MI	29	28	27.00	89	5	33.00	272	29	39.38	-69	50	18.73
A HYI	29	24	3.00	-61	44	38.00	45	9	16.10	50	33	3.84
T ERI	44	12	51.00	-40	27	8.00	19	59	15.77	36	7	20.22
A CET	45	5	7.50	3	56	44.00	1	32	24.19	-4	47	4.27
A PER	50	25	3.00	49	43	32.00	335	49	57.02	-44	55	56.48
A TAU	68	26	55.50	16	26	11.00	335	30	47.34	-8	34	59.89
B ORI	78	11	22.50	-8	14	35.00	334	14	55.20	17	52	56.15
A AUR	78	29	16.50	45	57	45.00	316	0	25.48	-33	27	7.67
G ORI	80	47	10.50	6	19	4.00	327	11	6.79	4	48	20.54
B TAU	80	59	16.50	28	34	41.00	320	18	47.21	-16	23	32.97
E ORI	83	35	1.50	-1	13	27.00	326	43	13.48	12	50	5.99
A ORI	88	17	31.50	7	24	6.00	319	41	52.07	5	52	10.32
A CAR	95	46	57.00	-52	40	30.00	340	52	40.54	64	20	57.87
A C MA	100	52	46.50	-16	39	50.00	313	25	9.92	32	19	27.27
E C MA	104	17	34.50	-28	55	14.00	313	38	57.68	44	50	29.49
A C MI	114	20	30.00	5	19	15.00	294	27	10.62	13	21	28.72
B GEM	115	45	40.50	28	7	2.00	289	55	43.74	-9	1	55.09
E CAR	125	26	21.00	-59	23	24.00	304	18	13.66	78	29	15.75
L VEL	136	39	30.00	-43	16	57.00	272	32	7.55	63	42	21.10
B CAR	138	12	3.00	-69	33	53.00	0	0	.00	90	0	.00
A LEO	151	36	3.00	12	[illegible]	55.00	256	46	57.50	7	44	56.38
A U MA	165	21	54.00	61	57	4.00	252	57	14.04	-42	55	.46

TABLE 1-3 (Cont'd)

B LEO	176	47	36.00	14	46	44.00	232	53	29.17	1	23	22.41
G CRV	183	28	30.00	-17	20	12.00	217	45	58.29	30	54	55.49
A CRU	186	7	52.50	-62	53	40.00	175	49	7.71	70	10	33.58
G CRU	187	16	31.50	-56	54	23.00	185	54	12.50	65	29	48.63
E U MA	193	6	6.00	56	9	37.00	232	19	34.77	-41	47	59.33
A VIR	200	48	36.00	-10	59	3.00	202	15	37.62	19	37	59.11
ET U MA	206	31	15.00	49	29	30.00	219	5	25.79	-38	57	28.43
B CEN	210	17	55.50	-60	11	44.00	161	19	56.78	60	3	.80
T CEN	211	7	27.00	-36	11	20.00	181	10	58.04	39	29	49.33
A BOO	213	29	36.00	19	22	27.00	200	27	2.31	-13	8	3.59
A CEN	219	16	4.50	-60	41	2.00	154	15	39.23	57	31	21.53
A LIB	222	12	24.00	-15	53	21.00	179	54	42.46	16	57	16.89
B U MI	222	41	54.00	74	18	24.00	233	14	27.02	-63	15	46.55
A CR B	233	16	48.00	26	50	17.00	185	22	25.97	-26	47	4.97
A SCO	246	47	1.50	-26	21	6.00	153	32	56.74	18	26	8.73
B TR A	251	10	55.50	-68	57	45.00	125	51	38.16	55	39	24.14
ET OPH	257	3	48.00	-15	40	51.00	147	51	16.10	5	13	16.95
L SCO	262	46	24.00	-37	4	43.00	135	59	3.58	24	0	37.36
A OPH	263	18	13.50	12	35	8.00	150	36	15.55	-23	35	12.30
G DRA	268	56	19.50	51	29	33.00	167	9	44.35	-61	3	41.19
E SGR	275	25	43.50	-34	24	15.00	126	13	33.43	18	32	32.34
A LYR	278	55	16.50	38	44	53.00	148	53	17.32	-52	52	18.94
S SGR	283	14	34.50	-26	20	40.00	121	20	24.48	9	10	20.50
A AQL	297	14	39.00	8	46	9.00	113	32	3.34	-27	43	.46
A PAV	305	40	58.50	-56	51	20.00	98	30	16.56	36	44	38.66
A CYG	310	2	31.50	45	9	50.00	103	49	34.86	-65	14	21.52
E PEG	325	35	39.00	9	42	16.00	81	35	6.70	-29	57	1.68
A GRU	331	28	37.50	-47	8	28.00	79	53	34.19	27	6	49.16
A PS A	343	54	9.00	-29	49	8.00	67	27	2.82	11	7	43.33
A PEG	345	43	43.50	15	0	21.00	57	55	16.36	-32	47	54.35

TABLE 1-4

COORDINATES OF NAVIGATIONAL STARS

for α CARINAE AS POLESTAR

CONSTELL NAME	ALPHA DEG	MIN	SEC	DELTA DEG	MIN	SEC	LAMBDA DEG	MIN	SEC	BETA DEG	MIN	SEC
A AND	1	37	3.00	28	53	10.00	344	29	26.19	-25	0	18.64
A PHE	6	6	52.50	-42	30	25.00	28	51	39.23	32	40	35.90
A CAS	9	35	54.00	56	20	5.00	316	0	28.97	-39	45	10.00
B CET	10	25	58.50	-18	11	22.00	7	41	59.31	17	9	13.12
A ERI	24	5	4.50	-57	25	28.00	36	22	22.37	50	35	24.73
A U MI	29	28	27.00	89	5	33.00	271	21	32.65	-52	18	9.10
A HYI	29	24	3.00	-61	44	58.00	41	28	20.03	54	39	2.32
T ERI	44	12	51.00	-40	27	8.00	1	39	35.21	53	23	30.87
A CET	45	5	7.50	3	56	44.00	324	49	.50	19	10	26.32
A PER	50	25	3.00	49	43	52.00	299	10	36.93	-19	21	13.88
A TAU	68	26	55.50	16	26	11.00	297	24	51.41	16	57	17.81
B ORI	78	11	22.50	-8	14	35.00	294	16	32.02	43	18	56.02
A AUR	78	29	16.50	45	57	45.00	282	6	12.05	-9	44	36.68
G ORI	80	47	10.50	6	19	4.00	287	12	45.21	29	38	41.24
B TAU	80	59	16.50	28	34	41.00	283	4	45.03	7	43	29.92
E ORI	83	35	1.50	-1	13	27.00	285	27	15.87	37	33	11.66
A ORI	88	17	31.50	7	24	6.00	278	32	56.31	29	35	4.95
A CAR	95	46	57.00	-52	40	30.00	0	0	.00	90	0	.00
A C MA	100	52	46.50	-16	39	50.00	261	43	15.81	53	45	56.31
E C MA	104	17	34.50	-28	55	14.00	251	51	2.00	65	25	39.33
A C MI	114	20	30.00	5	19	16.00	248	33	21.23	29	54	22.08
B GEM	115	45	49.50	28	7	2.00	252	18	33.52	7	20	37.90
E CAR	125	26	21.00	-59	23	24.00	145	59	42.90	72	18	24.00
L VEL	136	39	30.00	-43	16	57.00	182	38	47.28	61	30	56.93
B CAR	138	12	3.00	-69	33	53.00	122	57	38.09	64	20	57.87
A LEO	151	36	3.30	12	8	55.00	214	54	30.80	9	32	10.49
A U MA	165	21	54.00	61	57	4.00	236	29	26.81	-37	2	16.83

TABLE 1-4 (Cont' d)

B LEO	176	47	36.00	14	46	44.00	196	3	10.56	-6	23	13.55
G CRV	183	28	30.00	-17	20	12.00	171	3	20.40	15	5	8.74
A CRU	186	7	52.50	-62	53	40.00	130	3	15.49	44	55	32.32
G CRU	187	16	31.50	-56	54	23.00	136	25	37.71	41	6	57.29
E U MA	193	6	6.00	56	9	37.00	210	59	45.37	-44	42	34.91
A VIR	200	48	36.00	-10	58	8.00	161	28	14.53	0	10	25.18
ET U MA	206	31	15.00	49	29	50.00	204	36	18.70	-48	4	55.75
B CEN	210	17	55.50	-60	11	44.00	123	14	8.10	34	24	.45
T CEN	211	7	27.00	-36	11	20.00	139	3	39.26	15	4	28.84
A BOO	213	29	36.00	19	22	27.00	169	58	16.24	-31	59	27.28
A CEN	219	16	4.50	-60	41	2.00	118	46	38.91	31	55	35.70
A LIB	222	12	24.00	-15	53	21.00	141	17	41.85	-7	23	9.20
B U MI	222	41	54.00	74	18	24.00	244	33	20.88	-59	46	38.25
A CR B	233	16	48.00	26	50	17.00	157	31	37.36	-49	16	40.37
A SCO	246	47	1.50	-26	21	6.00	115	57	25.20	-7	1	16.01
B TR A	251	10	55.50	-68	57	45.00	100	15	40.78	32	58	36.56
ET OPH	257	3	48.00	-15	40	51.00	109	9	56.88	-19	45	9.82
L SCO	262	46	24.00	-37	4	43.00	100	20	46.90	0	27	53.93
A OPH	263	18	13.50	12	35	8.00	108	37	38.20	-48	40	56.31
G DRA	268	56	10.50	51	29	33.00	167	1	14.33	-85	37	54.55
E SGR	275	25	43.50	-34	24	15.00	90	17	32.09	-2	55	13.03
A LYR	278	55	16.50	38	44	53.00	79	54	3.05	-75	54	18.63
S SGR	283	14	34.50	-26	20	30.00	83	11	56.69	-10	42	43.99
A AQL	297	14	39.00	8	46	9.00	60	29	39.67	-42	45	36.37
A PAV	305	40	58.50	-56	51	20.00	72	52	31.24	22	14	12.71
A CYG	310	2	31.50	45	8	50.00	4	51	3.96	-66	31	4.08
E PEG	325	35	30.00	9	42	16.00	28	11	36.12	-31	18	52.27
A GRU	331	28	37.50	-47	8	28.00	53	8	8.50	20	30	57.34
A PS A	343	54	9.00	-29	49	8.00	34	45	16.44	11	30	3.17
A PEG	345	43	43.50	15	0	21.00	6	41	15.46	-23	59	55.53

TABLE 1-5

COORDINATES OF PLANETS

for β URSA MINORIS AS POLESTAR

VENUS	106	52	30.00	24	12	22.00	328	59	43.59	16	41	40.82
EARTHMON	308	19	29.00	-18	47	9.00	170	34	48.04	-16	53	16.15
MARS	211	17	41.00	-12	9	36.00	78	50	15.23	3	14	1.42
JUPITER	7	40	21.00	1	53	1.00	234	15	.29	-10	56	14.87
SATURN	321	54	17.00	-16	7	34.00	184	23	32.30	-17	59	50.19

for β CARINAE AS POLESTAR

VENUS	106	52	30.00	24	12	22.00	298	30	10.87	-6	26	29.31
EARTHMON	308	19	29.00	-18	47	9.00	99	20	52.22	-1	22	7.27
MARS	211	17	41.00	-12	9	36.00	191	38	60.00	17	15	21.69
JUPITER	7	40	21.00	1	53	1.00	38	10	5.75	-14	55	31.96
SATURN	321	54	17.00	-16	7	34.00	86	25	55.67	-4	16	8.09

for α CEPHEI AS POLESTAR

VENUS	106	52	30.00	24	12	22.00	240	36	40.57	0	26	8.17
EARTHMON	308	19	29.00	-18	47	9.00	79	23	9.62	8	18	39.30
MARS	211	17	41.00	-12	9	35.00	349	30	9.78	-19	7	4.61
JUPITER	7	40	21.00	1	53	1.00	142	22	28.08	19	42	20.41
SATURN	321	54	17.00	-16	7	34.00	92	25	51.83	11	25	16.32

for α CARINAE AS POLESTAR

VENUS	106	52	30.00	24	12	22.00	259	38	42.06	12	30	42.49
EARTHMON	308	19	29.00	-18	47	9.00	58	27	53.87	-13	10	8.93
MARS	211	17	41.00	-12	9	36.00	152	19	59.88	-5	2	10.66
JUPITER	7	40	21.00	1	53	1.00	357	21	18.92	0	21	8.82
SATURN	321	54	17.00	-16	7	34.00	45	13	27.52	-10	32	10.39

TABLE 1-6
NAVIGATIONAL STAR COORDINATES

Constellation	Star Name	Magnitude	Right Ascension			δ Declination			α R. A. Angle		
			H	M	S	°	'	"	°	'	"
αAnd	Alpheratz	2.1	0	06	28.2	+28	53	10	1	37	3
αPhe	Ankaa	2.4	0	24	27.5	-42	30	25	6	6	52.5
αCas	Schedar	2.3	0	28	23.6	+56	20	05	9	35	54
βCet	Diphda	2.2	0	41	43.9	-18	11	22	10	25	58.5
αEri	Achernar	0.6	1	36	20.3	-57	25	28	24	5	4.5
αU Mi	Polaris	2.1	1	57	53.8	+89	05	33	29	28	27
αHyi	Alphard	3.0	1	57	36.2	-61	44	58	29	24	3
θEri	Acamar	3.4	2	56	51.4	-40	27	08	44	12	51
αCet	Menkar	2.8	3	00	20.5	+ 3	56	44	45	5	7.5
αPer	Mirfak	1.9	3	21	40.2	+49	43	52	50	25	3
αTau	Aldebaran	1.1	4	33	47.7	+16	26	11	68	26	55.5
βOri	Rigel	0.3	5	12	45.5	- 8	14	35	78	11	22.5
αAur	Capella	0.2	5	13	57.1	+45	57	45	78	29	16.5
γOri	Bellatrix	1.7	5	23	08.7	+ 6	19	04	80	47	10.5
βTau	Elnath	1.8	5	23	57.1	+28	34	41	80	59	16.5
εOri	Alnilam	1.7	5	34	20.1	- 1	13	27	83	35	1.5
αOri	Betelgeuse	0-1	5	53	10.1	+ 7	24	06	88	17	31.5
αCar	Canopus	-0.9	6	23	07.8	-52	40	30	95	46	57
αC Ma	Sirius	-1.6	6	43	31.1	-16	39	50	100	52	46.5
εC Ma	Adhara	1.6	6	57	10.3	-28	55	14	104	17	34.5
αC Mi	Procyon	0.5	7	37	22.0	+ 5	19	16	114	20	30
βGem	Pollux	1.2	7	43	03.3	+28	07	02	115	45	49.5
εCar	Avior	1.7	8	21	45.4	-59	23	24	125	26	21
λVel	Suhail	2.2	9	06	38.0	-43	16	57	136	39	30
βCar	Miaplacidus	1.8	9	12	48.2	-69	33	53	138	12	3
αLeo	Regulus	1.3	10	06	24.2	+12	08	55	151	36	3
αU Ma	Dubhe	1.9	11	01	27.6	+61	57	04	165	21	54
βLeo	Denebola	2.2	11	47	10.4	+14	46	44	176	47	36
γCrv	Gienah	2.8	12	13	54.0	-17	20	12	183	28	30

TABLE 1-6 (Cont'd)

Constellation	Star Name	Magnitude	Right Ascension H M S	δ Declination ° ' "	α R. A. Angle ° ' "
αCru	Acrus	1.0	12 24 31.5	-62 53 40	186 7 52.5
γCru	Gacrus	1.6	12 29 06.1	-56 54 23	187 16 31.5
εU Ma	Alioth	1.7	12 52 24.4	+56 09 37	193 6 6
αVir	Spica	1.2	13 23 14.4	-10 58 08	200 48 36
ηU Ma	Alkaid	1.9	13 46 05.0	+49 29 50	206 31 15
βCen	Hadar	0.9	14 01 11.7	-60 11 44	210 17 55.5
θCen	Menkent	2.3	14 04 29.8	-36 11 20	211 7 27
αBoo	Arcturus	0.2	14 13 58.4	+19 22 27	213 29 36
αCen	Rigil Kentaurus	0.1	14 37 04.3	-60 41 02	219 16 4.5
αLib	Zubenelgenubi	2.9	14 48 49.6	-15 53 21	222 12 24
βU Mi	Kochab	2.2	14 50 47.6	+74 18 24	222 41 54
αCr B	Alphecca	2.3	15 33 07.2	+26 50 17	233 16 48
αSco	Antares	1.2	16 27 08.1	-26 21 06	246 47 1.5
αTR A	Atria	1.9	16 44 43.7	-68 57 45	251 10 55.5
ηOph	Sabik	2.6	17 08 15.2	-15 40 51	257 3 48
λSco	Shaula	1.7	17 31 05.6	-37 04 43	262 46 24
αOph	Rasalhague	2.1	17 33 12.9	+12 35 08	263 18 13.5
γDra	Eltanin	2.4	17 55 44.7	+51 29 33	268 56 10.5
εSgr	Kaus Australis	1.9	18 21 42.9	-34 24 15	275 25 43.5
αLyr	Vega	0.1	18 35 41.1	+38 44 53	278 55 16.5
σSgr	Nunki	2.1	18 52 58.3	-26 20 40	283 14 34.5
αAql	Altair	0.9	19 48 58.6	+ 8 46 09	297 14 39
αPav	Peacock	2.1	20 22 43.9	-56 51 20	305 40 58.5
αCyg	Deneb	1.3	20 40 10.1	+45 08 50	310 2 31.5
εPeg	Enif	2.5	21 42 22.0	+ 9 42 16	325 35 30
αGru	Al Nair	2.2	22 05 54.5	-47 08 28	331 28 37.5
αPs A	Fomalhaut	1.3	22 55 36.6	-29 49 08	343 54 9
αPeg	Markab	2.6	23 02 54.9	+15 00 21	345 43 43.5

Star Irradiance for Certain Photosensitive Materials

The pseudo-polestar and rate reference star must be tracked by a star tracker, and the navigational star and planet transits must be observed by photosensors of some type. The telescope design will be influenced by the requirement that enough radiation from the star must be gathered by the instrument to produce a good response from the photosensor. This problem is not considered here, but data on irradiance is given.

The values given in Table 1-7 are irradiances to be expected outside the atmosphere of the earth, values of irradiance at the observing instrument, before passage through the optical system. They are computed on the basis of an equation described in Reference 1-21.

This data will influence the detailed design of the optical instruments used in the P. D. technique.

TABLE 1-7
EFFECTIVE IRRADIANCE OF THE NAVIGATIONAL STARS
FOR FOUR PHOTOSENSITIVE DETECTORS

Units are watts/cm^2 x 10^{13}

	Detector			
Star	Silicon	S-4	S-11	S-20
α And	1.31	2.30	2.15	2.46
α Phe	1.174	0.497	0.542	0.702
α Cas	1.135	0.518	0.559	0.712
β Cet	1.50	0.696	0.764	0.945
α Eri	6.07	13.00	11.95	13.62
α U Mi	1.295	1.02	0.932	1.14
α Hyi	-	-	-	-
θ Eri	-	-	-	-
α Cet	-	-	-	-
α Per	1.58	1.23	1.26	1.51
α Tau	6.64	1.92	2.20	2.98
β Ori	8.22	14.45	13.55	15.50
α Aur	8.35	5.18	5.44	6.65
γ Ori	2.46	6.02	5.71	6.58
β Tau	1.27	2.24	2.10	2.40
ϵ Ori	2.49	7.20	7.20	8.31
α Ori	12.50	1.87	2.24	3.53
α Car	15.42	15.05	15.05	17.65
α C Ma	29.20	45.70	43.50	49.80
ϵ C Ma	2.868	7.03	6.66	7.68
α C Mi	6.08	4.74	4.84	5.80
β Gem	3.70	1.69	1.82	2.32
ϵ Car	2.01	0.854	0.93	1.20
λ Vel	1.90	0.563	0.645	0.874
β Car	1.44	2.32	2.19	2.51
α Leo	2.76	4.86	4.56	5.21
α U Ma	1.88	0.938	1.01	1.268
β Leo	1.05	1.46	1.40	1.62
γ Crv	0.865	1.52	1.43	1.63

TABLE 1-7 (Cont'd)

Star	Detector			
	Silicon	S-4	S-11	S-20
α Cru	6.20	15.90	14.80	17.00
γ Cru	6.60	0.60	0.815	1.49
ε U Ma	1.13	1.76	1.68	1.92
α Vir	4.98	12.72	11.86	13.60
η U Ma	2.313	5.251	4.94	5.70
β Cen	6.90	17.80	16.50	19.00
θ Cen	1.38	0.586	0.638	0.825
α Boo	10.20	4.34	4.74	6.13
α Cen	12.40	7.54	7.90	9.72
α Lib	-	-	-	-
β U Mi	0.92	0.292	0.331	0.439
α Cr B	1.04	1.67	1.59	1.82
α Sco	6.18	1.02	1.21	1.85
α Tr A	2.265	0.833	0.935	1.205
η Oph	0.827	1.22	1.17	1.34
λ Sco	2.64	6.49	6.15	7.09
α Oph	1.16	1.39	1.37	1.565
γ Dra	1.85	0.531	0.609	0.825
ε Sgr	1.785	3.07	2.91	3.30
α Lyr	7.72	12.45	11.80	13.50
σ Sgr	1.86	4.23	3.97	4.59
α Aql	3.56	3.98	3.91	4.56
α Pav	2.07	4.70	4.42	5.10
α Cyg	2.50	3.69	3.54	4.06
ε Peg	1.425	0.489	0.503	0.719
α Gru	1.87	4.01	3.69	4.29
α Ps A	2.55	3.54	3.41	3.92
α Peg	0.895	1.75	1.458	1.65
α Cep	0.765	0.854	0.842	0.980

Documentation of NAVSTRJH (Program for Navigational Star Coordinate Transformations) - J. R. Henefelt

IDENTIFICATION:

Name: NAVSTRJH

Program Number: BF7

Computer: Honeywell 800

Language: MH Fortran

Programmer: J. R. Henefelt

Associate: I. Foster

Date: 19 July 1963

Timing: Input Dependent

PURPOSE:

To find the coordinates of a given number of stars with respect to another star (the pseudo-pole star).

METHOD:

Coordinate transformation is made. The following equations are used to get the new coordinates of the stars:

$$\lambda = \tan^{-1}\left(\frac{\cos(\alpha' - \alpha)\cos\delta\sin\delta'}{\sin(\alpha' - \alpha)\cos\delta}\right) \qquad (1)$$

$$\beta = \sin^{-1}(\cos(\alpha' - \alpha)\cos\delta\cos\delta' + \sin\delta\sin\delta') \qquad (2)$$

USAGE:

a. Input

Card No. 1

Col.	1 - 6	degree portion of α'	-	F6.0
	7 - 10	minute portion of α'	-	F4.0

11 - 15	second portion of α'	-	F5.1
16 - 25	degree portion of δ'	-	F10.0
26 - 29	minute portion of δ'	-	F4.0
30 - 34	second portion of δ'	-	F5.1

Card No. 2, etc.

Same format as Card No. 1

b. Output

A sample page of output is attached.

c. Special subroutine used

1. AAAJH - subroutine used to print the output data.

FLOW CHARTS:

Attached is a flow chart for NAVSTRJH and Subroutine AAAJH.

NAVSTRJH

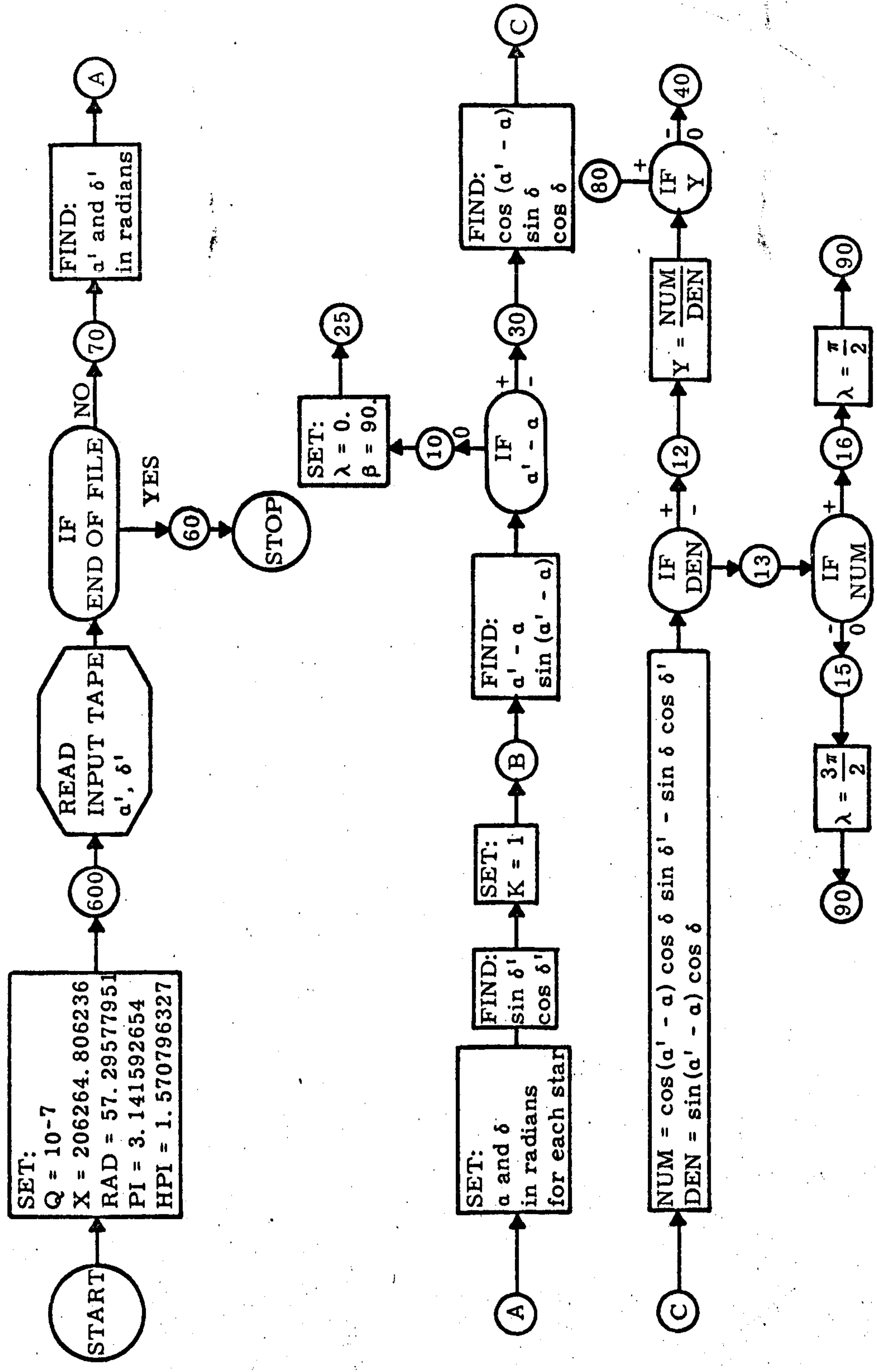


FIGURE 1-B-1 (sheet 1 of 2)
NAVSTRJH

NAVSTRJH (Continued)

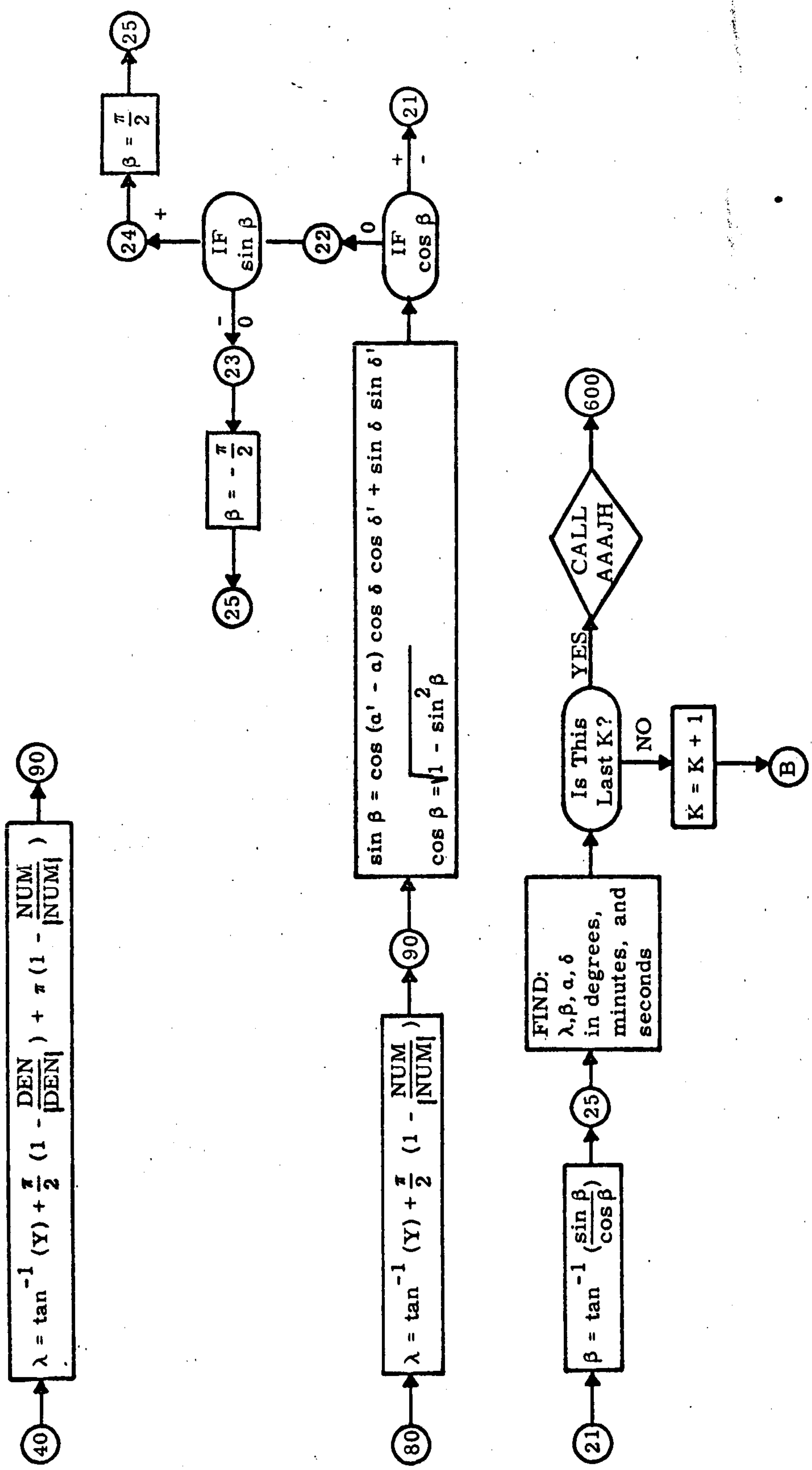


FIGURE 1-B-1 (sheet 2 of 2)
NAVSTRJH

SUBROUTINE AAAJH

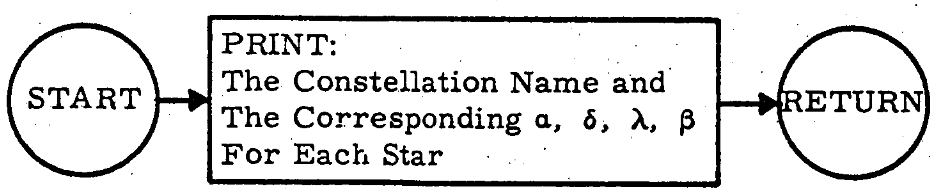


FIGURE 1-B-2
SUBROUTINE AAAJH

APPENDIX C TO SECTION 1

MIDARM SPECIFICATIONS
AND EVALUATION

"Midarm" (Micro Dynamic Angle and Rate Monitor) is the trade-mark name for a general purpose, wide angle, electro-optical angle and angular rate measuring and rate control system developed by Razdow Laboratories, Inc., 377 Fifth Street, Newark 7, New Jersey.

The published specifications of Midarm vary with the model, but some claims are:

- 0.0002 degree/hour rate accuracy in less than one minute of time
- 0.02 arc second angular repeatability
- 0.2 to 0.05 arc second angular accuracy
- Digital pulse resolution of 12.8 arc seconds per pulse
- Sinusoidal analog voltage outputs of up to 20 volts per arc second 12.9 arc sec/cycle
- Analog accuracy of 34 sec/cycle model: 0.2 arc sec/cycle
- Angular range 2.5 degrees (claimed to be extendable to 360 degrees in "Mark IV" through use of a 12-inch diameter polygon wheel with 72 optically-flat, edge-facets
- Weight of optical unit 13 lbs.
- Weight of power supply 23 lbs.
- Weight of control unit with modules 18 lbs.

Application of Midarm to measurement of dynamic accuracy of gears to 0.05 arc second is discussed in Reference 1-22. Reference 1-23 notes the use of "Midarm" and "Ultradex" to testing of M.I.T.'s Apollo space sextants to an accuracy of ±0.02 arc second composite error.

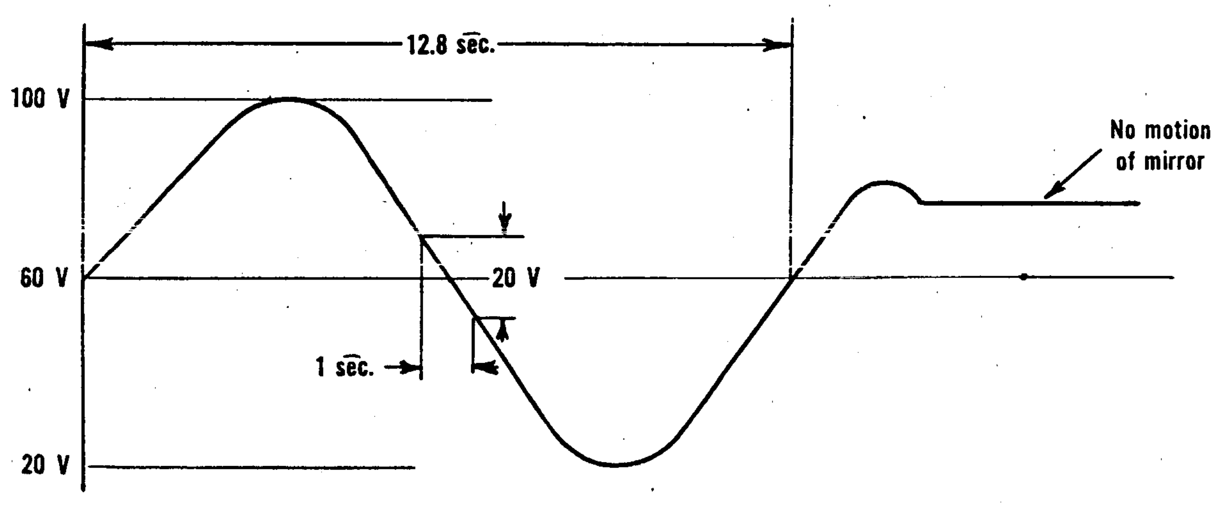
MIDARM OUTPUTS

MIDARM measures angles of rotation. It has a continuous analog output and a digital pulse output over the wide range of 2½ degrees.

The analog output is in the form of a sinewave with a period of 12.8 arc-seconds. The digital pulse output occurs every 12.8 arc-seconds of mirror rotation.

ANALOG OUTPUT

The analog output can be used for the indication of ultra high resolution angular changes. It can be fed to a recorder input. The sinewave minima and maxima are respectively +20 and +100 volts DC with a center reference voltage of 60 volts DC.

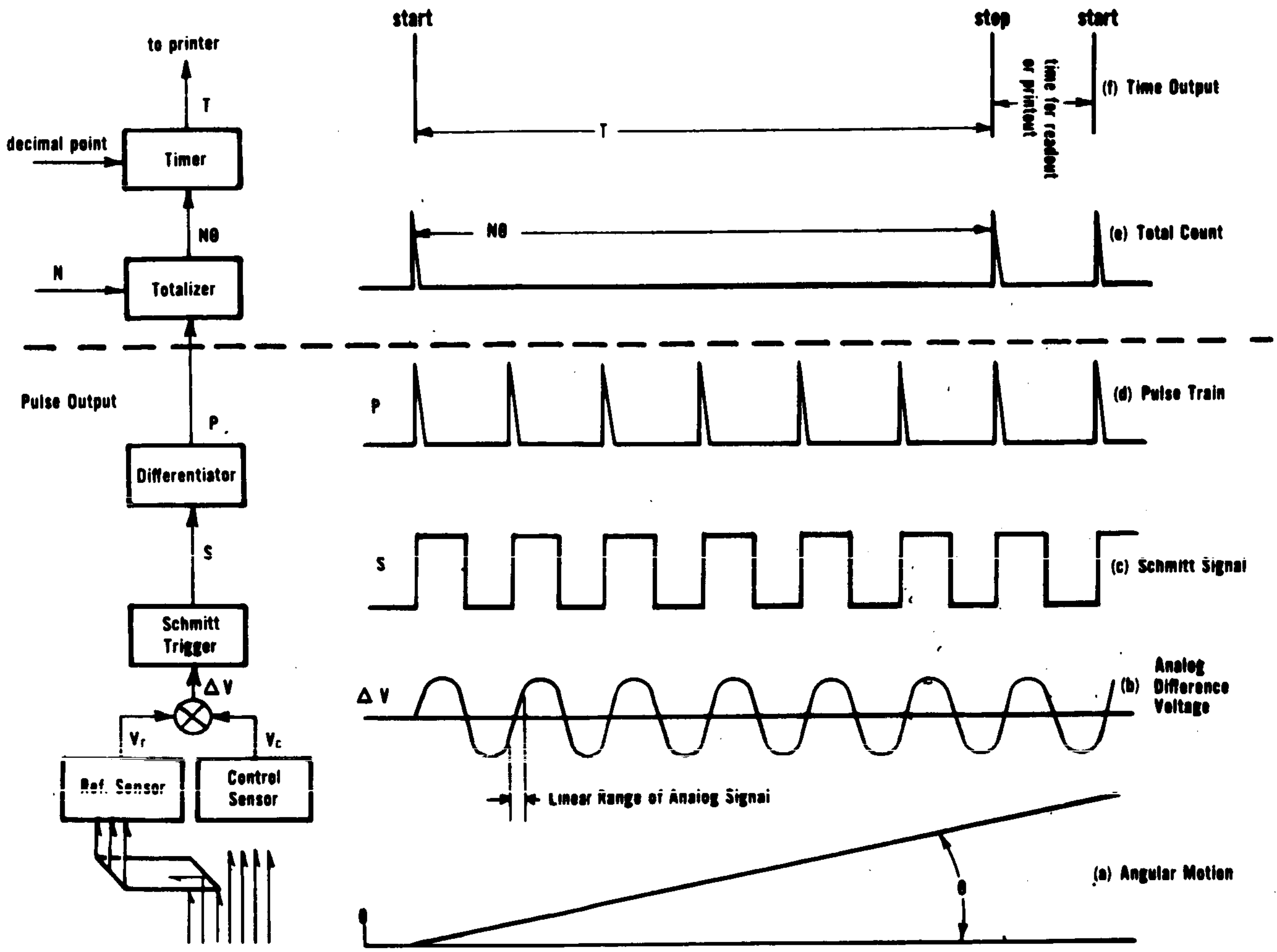


The maximum voltage gradient is at least 20 volts per arc-second meaning that a one volt change is equivalent to 0.05 arc-second of mirror rotation.

FIGURE 1-C-1

DIGITAL OUTPUT

The development of the pulse output is illustrated below:



To determine rate, an electronic counter is used in conjunction with the MIDARM digital pulse output. The counter will indicate the time between pulses or "N" pulses selectable on the counter.

$$\text{RATE} = \frac{\text{Angular Displacement (indicated by number of pulses)}}{\text{Time (indicated on counter)}}$$

A digital printer can be used to record the time between pulses.

FIGURE 1-C-2

In later versions of Midarm a two phase output waveform is used which allows linear interpolation over a whole bit length of 34 arc seconds.

MIDARM Evaluation by Honeywell DATE: 7 June 1962

A Micro Dynamic Angle and Rate Monitoring System (abbreviated MIDARM-2) was loaned to Evaluation by Razdow Laboratories, Inc., New Jersey. An evaluation of the unit was performed during the month of April 1962. The device was tested in our laboratory from two viewpoints; (1) as a general purpose angle readout or rate measuring tool, and (2) for the proposed use in a 360-degree readout scheme for a two-axis CD servo table.

The accuracy of the MIDARM device was checked over its full 2-1/2 degree operating range. Both a Leitz Dividing Head (calibrated to 1 arc second accuracy) and a Wild T-3 Theodolite were used to determine the MIDARM accuracy. The data from the dividing head and the theodolite compared to within 1 arc second. Readings were taken every 100 pulses or 1,290 arc seconds. The test was made with a 1/20 mirror; and as shown by the CW and CWW measurement, the test reflected good repeatability. Overnight gyro drift runs were made on the Fecker Table with the gyro drift rate trimmed to a 0.03 deg/hr rate. Resolutions of 3 arc seconds and accuracies of 1 arc second were possible using the MIDARM readout as well as the accumulation of data in an analog fashion without constant manpower surveillance. This test is similar to the SINS open-loop test used for testing the GG159 gyro, except that with the use of MIDARM, the data can be automatically logged without constant surveillance.

At present, a MIDARM system can fill a need for measurement of small incremental angles over a range of 2-1/2 degrees. The resolution and accuracy of the analog readout for the MIDARM appears to be in the 0.1 arc second region. This is based on comparative type of measurements that frankly cannot be tied down to this level due to limitations of optical standards in the laboratory. The accuracy of the pulses every 12.91 arc seconds appears to be better than 0.1 arc second, but this cannot be determined with our present optical equipment. The basic uses of the device are to: 1) increase the range of present automatic autocollimators and 2) provide an extremely accurate rate measuring tool that was not previously available to the Evaluation laboratory.

The basic MIDARM has shown itself to be as accurate as we are able to measure, to be adaptable to various type tests of a developmental nature, and to be simple and easy to use. In the process of evaluating the MIDARM, it was apparent that it would increase the incremental resolution of our present servo table readout by a factor of 100. Incremental readings of 3 arc seconds, as compared with 6 arc minutes, could easily be distinguished and recorded in an analog fashion. Data from drift measurements in the 0.001 deg/hr range was available in minutes instead of hours.

Honeywell Aero Report 2858-FR

APPENDIX D TO SECTION 1

DISCUSSION OF GRAPHICAL METHODS

V. S. Kardashian

GEOMETRICAL DETERMINATION OF LINE OF SIGHT COORDINATES BASED UPON THE METHOD OF ISO-AZIMUTHAL TRANSITS
(Abstracted from MR 7984 of 8 August 1962)

INTRODUCTION

The system equations for the determination of the celestial coordinates of a planetary target were derived in Reference 1-2.* The general solution was expressed in equations (36) and (29) on pages 28 and 26, respectively, of Reference 1-2. For manual calculations, the solution of these equations is rather tedious.

Pre-selection of pairs of stars for observation could reduce the calculations by almost a factor of two. This method requires a much larger data storage system. The form of the equations is given in equations (11) and (12)** of Appendix 1-E***.

This paper details a geometrical solution to the problem of the determination of the bearing of a line of sight in space. The required observations are the timing of events which were referred to as the "Iso-azimuthal transits of Pairs of Stars" in R-RD 6206. The geometrical solution is based upon the properties of the gnomonic projection which is briefly discussed in Note 1 to this appendix.

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* Vahram S. Kardashian, "A Self-Contained Celestial Navigator for Interplanetary Travel", Minneapolis-Honeywell Regulator Co., MPG Research, July 27, 1961.

** In these equations, although the form would remain unaltered, the significance of the symbols would be slightly modified. The symbols λ and β represent longitude and latitude coordinates referenced to the pseudo-pole of the celestial sphere. The superscript z identifies the line of sight direction as in Appendix 1-E (MR 7893). The superscripts E and N, however, identify the intersection points of the great circle through the pair of stars with the equator and the parallel of latitude 45 degrees (or -45 degrees), respectively.

*** Vahram S. Kardashian, "Method of Precise Optical Navigation in Space Based upon An Apparent Pseudo-Diurnal Motion of The Celestial Sphere", Minneapolis-Honeywell Memorandum MR 7893, April 11, 1962 portion included as Appendix 1-E).

THE GENERAL GRAPHICAL SOLUTION

In this section we consider a gnomic projection of a region of the celestial sphere containing the apparent celestial coordinates of the target planet. The coordinates are referenced to a pre-established pseudo-pole in the celestial sphere. Let the scale of representation on the gnomonic map be sufficiently large so that we could plot on it the celestial coordinates of the two pairs of stars whose iso-azimuthal transit were observed. The straight line joining the plotted positions of two stars represents the projection of a great circle arc between the two stars. If one were to assume that the position of a planetary target in the celestial sphere is represented by the intersection point of two great circles*, it follows that a plot of the position of the planetary target may be described by the intersection point of two straight lines, each line defining a great circle plane. Each line is constructed by running a straight line through the plotted positions of the pair of stars whose iso-azimuthal transit was observed.

In practice, the collimating axis of the telescope passes successively through two great circle planes defined by pairs of known stars. The instants of transit are recorded, and these events are referenced to the time at which the telescope line of sight is oriented to the planetary target. Allowance for the time lapse between the observations of the planetary target, and the successive moments of iso-azimuthal transit of pairs of stars is made by applying a correction to the meridional components of each pair of stars. These corrections represent the angular rotation of the telescope in the time interval from the observation of the reference target to the observation of the reference planes. It is mathematically represented by $\omega \Delta t$ where ω is the pseudo-diurnal rate and Δt is the time lapse.**

The graphical solution consists first in replotting the positions of the stars corrected for the meridional displacement of the great circle planes. This is followed by the construction of two straight lines which describe the corrected great circle arcs, and the determination of their intersection point. Unless the scale of the gnomonic chart is very small, the accuracy of the method is very poor. To accommodate this small scale and achieve theoretical accuracies of the order of 10 seconds of arc, 36 meters square charts would be required. Hence, an alternate practical method is described in the next section.

---

* See document R-RD 6206 (Reference 1-2).

** (In applying this to single star transits, a compass, scale and parallel rule can be applied to run each star back through the observed angle $\theta = \omega t$) (CWB).

THE USE OF SPECIAL CHARTS

For a particular space mission for which a trajectory is selected, the geometrical solution described above can be read to an accuracy of 5 to 10 seconds of arc if there is made available an equatorial gnomonic projection of the strip of the celestial sphere where the planet target is precomputed to be during the course of the mission.

Several of these charts covering a field of view 0 degree 0.5 to 1 degree 0.0 wide and some 30 degrees to 60 degrees long would be sufficient to chart the positions of the planetary targets throughout the entire phase of the mission. These long charts can be rolled over spools for convenient storage much like photographic films for cameras. They may be unrolled mechanically for viewing of selected or desired regions of the celestial sphere. If these charts are 36 cm wide and cover a field of view one degree wide, the scale representation would be as small as 10 seconds of arc per millimeter at the point at which the plane is tangent to the sphere. At points other than the tangent point, the distortion inherent in a gnomonic projection reduces the scale and permits a higher accuracy in plotting and reading. Should a higher reading accuracy be required, it can be achieved by restricting the width of the field of view to a fraction of a degree or widening the chart.

Obviously, it is not possible to plot the positions of most of the navigational stars on the gnomonic projection of a narrow belt of the celestial sphere. However, the great circle planes defined by known pairs of stars can be described on this special chart. It requires a table giving two sets of precomputed coordinates for all pairs of navigational stars. The two sets of coordinates would represent, for instance, the intersection of the great circle plane defined by the two stars with two parallels of latitude on the edges of the belt. The meridional correction for the time lapse between observations can now be made by direct translation of the straight line parallel to itself. The line is translated by an amount $\omega \Delta t$. It is to be noted that unless the gnomonic projection is equatorial, the slopes of the straight lines will alter in translation.

It will also be noted that if the number of navigational stars which could be used for observation are limited to "n", the number of combinations of pairs of stars is given by $_nC_2 = n!/(n-2)!\ 2!$. When n is greater than 3, $_nC_2$ is greater than n. This requires, therefore, a greater storage of data-- on tape or in tables-- than would have been required if the coordinates of each of the n navigational stars were stored separately. Furthermore, allowance for corrections of the coordinates of stars for aberration, parallax, or proper motion adds a greater penalty to the storage problem. Besides, the corrections become obsolete after each mission.

Instead of using precomputed coordinates for a particular pair of stars, one could equally well have the gnomonic chart strips incorporate predrawn straight lines representing great circle planes through the brighter stars. In order to obtain the bearing of a target planet for which two iso-azimuthal transits have been observed, each of the straight lines representing the observed great circle planes are translated in longitude and their intersection point read-off from the charts. The actual operation could possibly be made more expedient if a duplicate transparent chart were made to slide over the lower chart. The amount of slide is determined by the product of the pseudo-diurnal rate by the time elapsed between the observations of iso-azimuthal transit. The intersection point is then read-off from the chart. The coordinates of the target planet are now defined to be the coordinates of the point of intersection corrected in longitude for the time lapse between target sighting and the instant of first iso-azimuthal transit.

Note 1 to Appendix 1-D: THE GNOMONIC PROJECTION

Of all the chartographer's projections*, the gnomonic is probably the oldest, believed to have been developed by Thales about 600 B.C. The gnomonic projection is defined as the geometrical projection of points on a sphere onto a plane such that the origin of the projecting rays is the center of the sphere. This perspective projection can be visualized by placing a light at the center of a sphere and observing the projection of latitude and longitude graticule lines onto a plane surface.

If the tangent plane is tangent to a point on the equator, the projection is called an equatorial gnomonic. All meridian circles appear as straight parallel lines on this projection. Their distance from the meridian line through the point of tangency is given by the relation $r \tan \lambda$ where r is the radius of the sphere, and λ is the longitude difference between the point of contact and the meridian in question. If the plane is tangent at a point on the sphere not on the equator, then the projection is called oblique gnomonic. For this case, the meridians appear as straight lines converging toward the nearer pole. In both the equatorial and oblique gnomonic projections, the small-circles of latitude appear as hyperbolic, parabolic, and elliptic. If the plane surface is tangent at the pole, the projection is called polar gnomonic. The meridian lines project as straight radial lines, and the parallels of latitude project as circles.

---

* A projection is defined as the method of representing all or part of the surface of a sphere (or spheroid) upon a plane surface.

The gnomonic projection has a variable distance scale, and the projection is neither orthomorphic** nor equal area. The only distinguishing feature of this projection is the characteristic that the projections of all great circles onto the tangent plane are straight lines. This useful property permits the geometrical solution described in this memorandum.

** An orthomorphic projection has, by definition, correct angular relationship between points on the sphere.

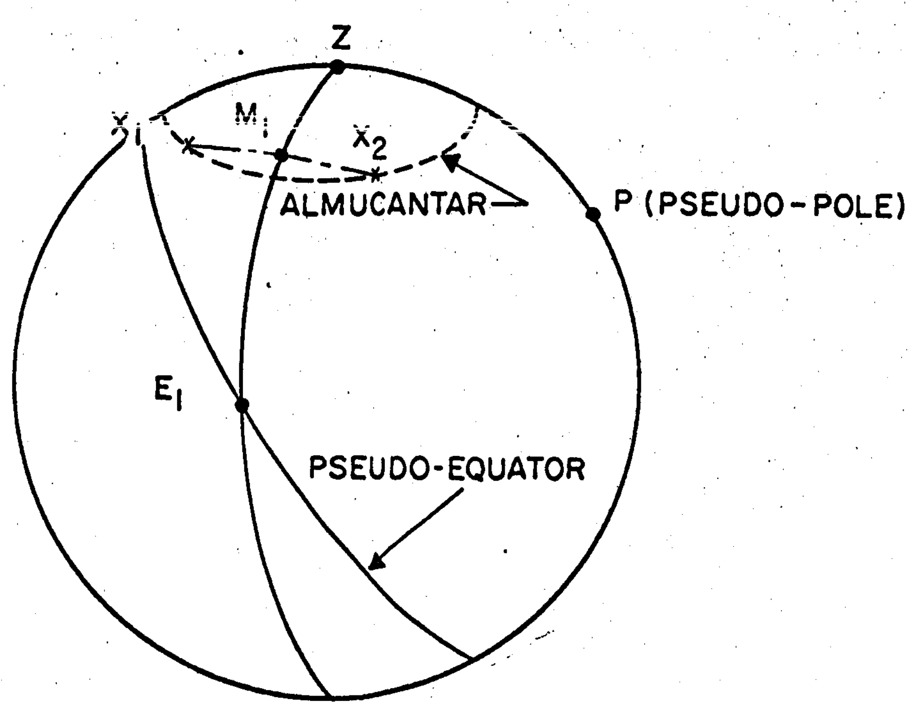
APPENDIX E TO SECTION 1

THE ISO-ALUMCANTAR METHOD

V. S. Kardashian

APPENDIX 1-E
THE ISO-ALMUCANTAR METHOD

In space, the apparent position of a target planet establishes a local vertical, and hence the direction of the line of sight to the target planet may be referred to as the zenith of the observer. Observation of two stars on the same almucantar* defines the zenith Z to lie on the perpendicular bisector of the great circle arc joining the pair of stars X_1 and X_2. The great circle arc representing the perpendicular bisector of X_1X_2 is denoted in the figure below by M_1E_1, where M_1 is the midpoint of X_1X_2, and E_1 represents the intersection of this great circle with the equator. The observer awaits for this event to occur. Visual inspection of the sky is sufficient to predict which pair of stars will provide imminent observation of the event of iso-almucantar passage. If, simultaneously, an observation of iso-almucantar passage of any other pair of stars is observed-- not necessarily on the same almuncantar-- then the two great circles M_1E_1 and M_2E_2 intersect at the zenith.



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* An almucantar is defined as a small circle drawn on the celestial sphere about the zenith. All points on the almucantar are at equal elevation.

However, since simultaneous observations are not practically feasible even if the two events were to occur simultaneously, each event must be accurately timed and the celestial coordinates of the stars corrected for the interval of time elapsed between the observations in a manner analogous to Equation (13) page 22 of Document R-RD 6206 (Reference 1-2). Thus, if β_1 and λ_1 represent the celestial coordinates of a star X_1 relative to a coordinate system whose z-axis is oriented toward the pseudo-pole of the pseudo-diurnal rotation, the adjusted coordinates become β_1 and $\lambda_1 - \omega(t_1 - t)$ respectively, where $(t_1 - t)$ represents the elapsed interval of time since planetary sighting.

Alternately, the correction for elapsed time between observations may be applied to the points M_1 (β_1^M, λ_1^M) and E_1 (O, λ_1^E). The point M_1 is defined by the following equations where β_1^M and λ_1^M are expressed solely in terms of the celestial coordinates of X_1 and X_2:

$$\lambda_1^M = \lambda_1 - \cot^{-1} \frac{\cos\beta_1 \cot X_1M - \sin\beta_1 \cos P\hat{X}_1X_2}{\sin P\hat{X}_1X_2} \qquad (1\text{-E-}1)^*$$

and

$$\beta_1^M = \frac{\sin\beta_1 \cos(\lambda_1 - \lambda_1^M) + \sin(\lambda_1 - \lambda_1^M) \cot P\hat{X}_1X_2}{\cos\beta_1} \qquad (1\text{-E-}2)$$

where

$$\cot P\hat{X}_1X_2 = \frac{\cos\beta_1 \tan\beta_2 - \sin\beta_1 \cos(\lambda_2 - \lambda_1)}{\sin(\lambda_2 - \lambda_1)} \qquad (1\text{-E-}3)$$

and

$$\cos 2X_1M = \sin\beta_1 \sin\beta_2 + \cos\beta_1 \cos\beta_2 \cos(\lambda_2 - \lambda_1) \qquad (1\text{-E-}4)$$

The intersection E_1 of the perpendicular bisector of X_1X_2 with the pseudo-equator has the following coordinates:

$$\lambda_1^E = \tan^{-1} \sin\left[\beta_1^M \left(\frac{\cos\beta^M \tan\beta_1 - \sin\beta^M \cos(\lambda_1 - \lambda_1^M)}{\sin(\lambda_1 - \lambda_1^M)}\right)\right] + \lambda_1^M \qquad (1\text{-E-}5)^{**}$$

$$\beta_1^E = 0$$

---

* See Note 1 for derivation (page 1-E-7).

** See Note 2 for derivation (page 1-E-8).

The derivation of these equations is given on pages 1-E-7, 8. In practice, the coordinates of both M and E can be precomputed and tabulated for any given pair of stars.

Thus, in the following table, the positions of points M and E adjusted for the lapse of time since the initial sighting of the planet are expressed.

$$\left.\begin{array}{lll} & \underline{\text{Corrected Latitude}} & \underline{\text{Corrected Longitude}} \\ M_1^{\dagger} & \beta_1^{M\dagger} = \beta_1^{M} & \lambda_1^{M\dagger} = \lambda_1^{M} - \omega(t_1 - t) \\ E_1^{\dagger} & \beta_1^{E\dagger} = 0 & \lambda_1^{E\dagger} = \lambda_1^{E} - \omega(t_1 - t) \\ M_2^{\dagger} & \beta_2^{M\dagger} = \beta_2^{M} & \lambda_2^{M\dagger} = \lambda_2^{M} - \omega(t_2 - t) \\ E_2^{\dagger} & \beta_2^{E\dagger} = 0 & \lambda_2^{E\dagger} = \lambda_2^{E} - \omega(t_2 - t) \end{array}\right\} \quad (1\text{-E-}6)$$

In the above equations, the dagger represents the corrected positions, ω is the pseudo-diurnal rate, and t, t_1, and t_2 represent the instants at which the target planet is sighted, and the times of observation of the first and second pairs of stars respectively. The coordinate system chosen has the z, x, and y axes directed to the pseudo-pole, the intersection of the pseudo-equator with the celestial equator, and a direction 90 degrees from the x-axis in the plane of the pseudo-equator. The system forms a right hand coordinate system and is illustrated in Figure 5, page 19 of Document R-RD 6206 (Reference 1-2).

The planes of the great circle arcs $M_1^{\dagger}\,E_1^{\dagger}$ and $M_2^{\dagger}\,E_2^{\dagger}$ each contain the zenith direction at time t. The zenith direction is then obtained by solving for the intersection point of these two planes. The solution is expressed by equation (36) page 28 and equation (28) page 26 of Document R-RD 6206. (Reference 1-2). These are reproduced below with a modified notation to conform to the present form. The daggers have been discarded for convenience.

$$\tan\lambda^{z} = \frac{\dfrac{\sin\lambda_1^{M}\tan\beta_1^{E} - \sin\lambda_1^{E}\tan\beta_1^{M}}{\sin(\lambda_1^{E} - \lambda_1^{M})} - \dfrac{\sin\lambda_2^{M}\tan\beta_2^{E} - \sin\lambda_2^{E}\tan\beta_2^{M}}{\sin(\lambda_2^{E} - \lambda_2^{M})}}{\dfrac{\cos\lambda_1^{M}\tan\beta_1^{E} - \cos\lambda_1^{E}\tan\beta_1^{M}}{\sin(\lambda_1^{E} - \lambda_1^{M})} - \dfrac{\cos\lambda_2^{M}\tan\beta_2^{E} - \cos\lambda_2^{E}\tan\beta_2^{M}}{\sin(\lambda_2^{E} - \lambda_2^{M})}} \quad (1\text{-E-}7)$$

$$\tan\beta^{z} = \frac{\dfrac{\tan\beta_1^M}{\sin(\lambda_1^M - \lambda^z)} - \dfrac{\tan\beta_1^E}{\sin(\lambda_1^E - \lambda^z)}}{\cot(\lambda_1^M - \lambda^z) - \cot(\lambda_1^E - \lambda^z)} \qquad (1\text{-E-}8)$$

However, it will be noted that

$$\beta_1^E = \beta_2^E = 0$$

These equations are therefore simplified and become:

$$\tan\lambda^{z} = \frac{\sin(\lambda_1^E - \lambda_1^M)\sin\lambda_2^E \tan\beta_2^M - \sin(\lambda_2^E - \lambda_2^M)\sin\lambda_1^E \tan\beta_1^M}{\sin(\lambda_1^E - \lambda_1^M)\cos\lambda_1^E \tan\beta_2^M - \sin(\lambda_2^E - \lambda_2^M)\cos\lambda_1^E \tan\beta_1^M} \qquad (1\text{-E-}9)$$

and

$$\tan\beta^{z} = \frac{\tan\beta_1^M}{\cos(\lambda_1^M - \lambda^z) - \sin(\lambda_1^M - \lambda^z)\cot(\lambda_1^E - \lambda^z)} \qquad (1\text{-E-}10)$$

We recall that in the above equations λ_1^M, λ_1^E, λ_2^M, and λ_2^E are variants where each of these quantities are corrected for the apparent rotation of the celestial sphere in the time interval between observations. On the other hand, β_1^E and β_2^E were arbitrarily chosen to have zero pseudo-latitude. Since for any given pair of stars the midpoint M was precomputed we could, if we so chose, precompute and tabulate another point $N(\lambda^N, \beta^N)$ such that N lies on the great circle joining E to M, and such that the latitude of N is arbitrarily chosen to be +45 degrees. This would further simplify the above equations giving:

$$\tan\lambda^{z} = \frac{\sin(\lambda_1^E - \lambda_1^N)\sin\lambda_2^E - \sin(\lambda_2^E - \lambda_2^N)\sin\lambda_1^E}{\sin(\lambda_1^E - \lambda_1^N)\cos\lambda_2^E - \sin(\lambda_2^E - \lambda_2^N)\cos\lambda_1^E} \qquad (1\text{-E-}11)$$

and

$$\cot\beta^{z} = \cos(\lambda_1^N - \lambda^z) - \sin(\lambda_1^N - \lambda^z)\cot(\lambda_1^E - \lambda^z) \qquad (1\text{-E-}12)$$

The observational scheme so far described consists in successively measuring the instant of iso-almucantar passage of one pair of stars followed by a similar event of a second pair. If we assume that the pseudo-polar axis is chosen to lie in a direction at right angles approximately to the direction of the zenith, and we assume we are restricted to observations of the 55 navigational stars only, then there exists 1,485 $\left(\frac{55!}{2!\,53!}\right)$ combinations of pairs. Hence, 1,485 events may be observed per one apparent rotation of the celestial sphere. For a diurnal rate of 10 revolutions per day, an average rate of observation would be one event per 5.8 seconds of time. Establishing planetary positions therefore, is very rapid.

With the pseudo-diurnal rotation an alternate mode of observation of iso-almucantar passage is also feasible. This mode would restrict the search of stars to a predetermined almucantar. The observation consists in noting the time of passage of a single star across the preselected almucantar. Two other similar observations of individual stars across the same almucantar are sufficient to determine the zenith direction. Denote by t, t_1, t_2, and t_3 the time of target planet sighting, the times of passage of stars X_1, X_2, and X_3 across the preselected almucantar respectively. The positions of these stars are corrected for time effect as shown in the table below:

	Corrected Latitude	Corrected Longitude
$X_1^\dagger$	$\beta_1^\dagger = \beta_1$	$\lambda_1^\dagger = \lambda_1 - \omega(t_1 - t)$
$X_2^\dagger$	$\beta_2^\dagger = \beta_2$	$\lambda_2^\dagger = \lambda_2 - \omega(t_2 - t)$
$X_3^\dagger$	$\beta_3^\dagger = \beta_3$	$\lambda_3^\dagger = \lambda_3 - \omega(t_3 - t)$

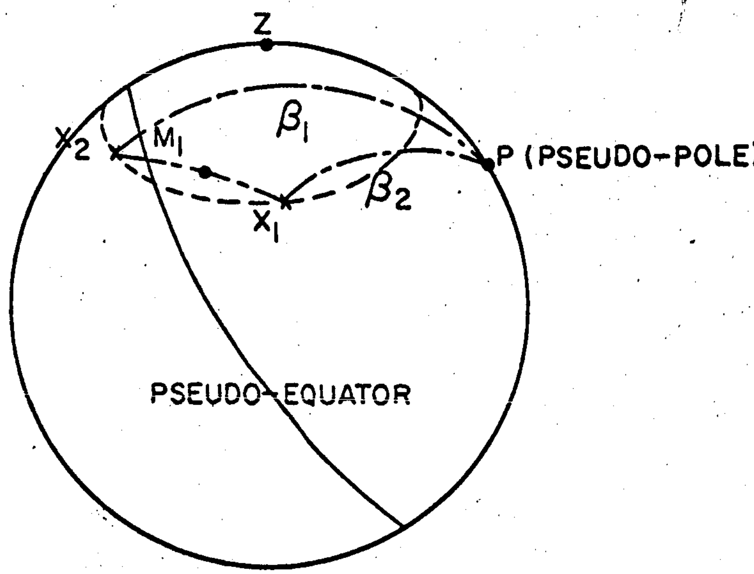
(1-E-13)

For any pair of stars, the formulae in the appendix give the coordinates of M and E. Again, the planes of the great circle arcs $E_1^\dagger M_1^\dagger$ and $E_2^\dagger M_2^\dagger$ each contain the zenith direction at time t. The zenith direction is then solved for by equations (1-E-9 and 1-E-10).

Greater precision may be achieved by observing more than 3 stars with azimuthal separation of 360°/n where n represents the number of stars observed. The zenith coordinates may then be chosen such that the root mean square deviations of the calculated positions is a minimum. This method is similar in principle to the impersonal astrolabe which has been in use in France since 1956. It has proven itself so highly accurate that the instrument has not been restricted to the determination of latitude and longitude, but also is most useful in improving star catalogues. It records star transit under favorable conditions to an accuracy of ±0".09 (standard deviation). The average value of the standard deviation for one transit is ±0".17. In a few years, it is hoped, the precision of the determination of the position lines will be accurate to 0".01. This example which illustrates the unprecedented precision capabilities of modern positional astronomy warrants the devotion of our interests and efforts to navigational systems in space based upon artificially generated pseudo-diurnal rotation of the celestial sphere.

NOTE 1. THE COORDINATES OF POINT M

Consider the diagram below. The cosine and four-parts formulae of spherical trigonometry give respectively:



$$M_1X_1 = M_1X_2 = 1/2\cos^{-1}(\sin\beta_1\sin\beta_2 + \cos\beta_1\cos\beta_2\cos(\lambda_2-\lambda_1))$$

and

$$\cot P\hat{X}_1X_2 = \frac{\cos\beta_1\tan\beta_2 - \sin\beta_1\cos(\lambda_2-\lambda_1)}{\sin(\lambda_2-\lambda_1)}$$

Also, from the four-parts formula

$$\cot(\lambda_1 - \lambda_1^M) = \frac{\cos\beta_1\cot M_1X_1 - \sin\beta_1\cos P\hat{X}_1X_2}{\sin P\hat{X}_1X_2}$$

or

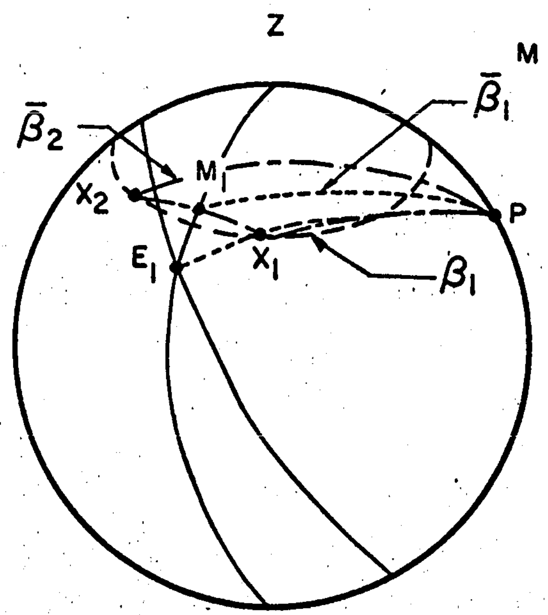
$$\lambda_1^M = \lambda_1 - \cot^{-1}\left(\frac{\cos\beta_1\cot M_1X_1 - \sin\beta_1\cos P\hat{X}_1X_2}{\sin P\hat{X}_1X_2}\right)$$

and

$$\beta_1^M = \tan^{-1}\left(\frac{\sin\beta_1\cos(\lambda_1-\lambda_1^M) + \sin(\lambda_1-\lambda_1^M)\cot P\hat{X}_1X_2}{\cos\beta_1}\right)$$

NOTE 2. THE COORDINATES OF POINT E

Consider the diagram below. The four parts formula of spherical trigonometry gives:



$$\cot X_1 \hat{M}_1 P = \frac{\cos \beta^M \tan \beta_1 - \sin \beta_1^M \cos(\lambda_1 - \lambda_1^M)}{\sin(\lambda_1 - \lambda_1^M)}$$

and

$$\sin \beta_1^M \cos(\lambda_1^E - \lambda^M) = \sin(\lambda^E - \lambda^M) \tan P\hat{M}_1 X_1$$

$$\tan(\lambda_1^E - \lambda_1^M) = \frac{\sin \beta_1^M}{\tan P\hat{M}}$$

it follows

$$\lambda_1^E = \lambda_1^M + \tan^{-1} \left\{ \sin\beta_1^M \left[\frac{\cos \beta_1^M \tan\beta_1 \sin\beta^M \cos(\lambda_1 - \lambda^M)}{\sin(\lambda_1 - \lambda_1^M)} \right] \right\}$$

APPENDIX F TO SECTION 1

SIMPLIFIED EXPLANATION OF THE ORIGINAL PSEUDO-DIURNAL TRANSIT CONCEPTS

V. S. Kardashian

APPENDIX 1-F
SIMPLIFIED DESCRIPTION OF THE ORIGINAL PDT CONCEPT

By Vahram S. Kardashian

INTRODUCTION

The motivation for development of a new technique for self-contained celestial navigation of spacecraft is the apparent need for greater accuracy than contemporary techniques can provide. Studies of the guidance and navigation requirements for lunar missions*, for example, suggest that self-contained mid-course navigational systems, to be useful, must determine lines of position to an accuracy of about two seconds of arc. Basic sighting instruments, on the other hand, can measure angles to an accuracy in the neighborhood of 10-20 sec. of arc. The random components of these angular measurements (but not the bias errors) can be reduced with the aid of statistical techniques such as maximum likelihood estimators or Kalman optimal filters to improve the accuracy to about 5-10 sec. of arc. Navigational techniques based on angular measurements therefore appear to be marginal.

The navigational technique described in this document entirely avoids direct angular measurements. Instead, it is based on the precise timing of particular celestial events. These time measurements together with stored data pertaining to the positions of the observational stars and near bodies (such as the moon, earth or other planets) are sufficient to obtain the desired position fix in space. **

The concept is first developed in Section I of this appendix by considering position determination on earth. Section II of this appendix shows how this idea is directly adaptable to position determination in space. The use of near bodies in space navigation requires a technique for accurately locating their centers. Such a technique is described in Section III of this appendix. Some basic mechanizational and operational requirements are discussed in Section IV of this appendix. The principal recommendation made in Section V of this appendix is that an experimental program should be initiated to demonstrate the precision of the method and to develop an operationally efficient primary navigational instrument for spacecraft.

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* See References 1-19 and 1-20.

** Detailed analyses of the concepts outlined in this appendix are presented in References 1-1 to 1-6.

SECTION 1 OF APPENDIX 1-F
POSITION DETERMINATION ON EARTH

The technique for position determination can best be visualized by considering the problem of finding the coordinates of a zenith point directly above an observer standing on the earth. Knowledge of these coordinates and time is sufficient to determine position.

Because of the rotation of the earth about its polar axis, a local vertical line from the observer to the zenith point describes a cone in space, as shown in Figure F-1. The locus of zenith points on the celestial sphere, therefore, is a small circle. The problem is to identify a particular zenith point at a corresponding particular time.

Suppose that at this particular time two stars S_1 and S_2 were observed to lie in a plane containing the local vertical. The intersection of this plane with the celestial sphere would be a great circle passing through the zenith Z of the observer. Suppose also that at this same particular time a second pair of stars S_3 and S_4 were observed to lie in another plane containing the local vertical. The intersection of this plane with the celestial sphere would be a second great circle passing through the zenith. Hence the zenith coordinates are those of the intersection of the two great circles on the celestial sphere. This geometry is illustrated in Figure F-2.

Identification of the four stars and knowledge of their fixed positions on the celestial sphere constitute all of the information needed to calculate the coordinates of the zenith point on the celestial sphere. Knowledge of the particular time at which these star pairs were observed to lie in planes through the observer's local vertical is sufficient to relate the zenith point to the observer's position on the rotating earth.

In practice it is improbable that two pairs of stars can be observed simultaneously. Simultaneity is unnecessary. It is sufficient to record the times of observation of each star pair and to apply a correction to the coordinates of one pair of stars as indicated in Figure F-3 to account for the movement of the zenith point on the celestial sphere during the time interval between the two observations.

A simple application of this technique has been made using a plumb line, a star catalog and a watch. The plumb line establishes the local vertical from the observer to the zenith point. By observing the motion of the stars caused by rotation of the earth, the observer, sighting across the plumb line, can easily locate a pair of stars about to transit the plumb line. He records the time of simultaneous transit and searches for a second pair of stars some 90 degrees in azimuth from the first pair. Upon locating this pair, he records its simultaneous transit of the plumb line. From the star catalog he obtains the coordinates of the four stars he has observed, corrects the coordinates of one pair to account for the time lapse between observations, and solves for the zenith point. Finally, from his knowledge of the time to which the observations are referenced, he establishes his position on earth. With only this plumb line instrumentation an observer's position on earth has been computed to an accuracy of one mile. *

* See Reference 1-1 or Appendix F of Reference 1-2.

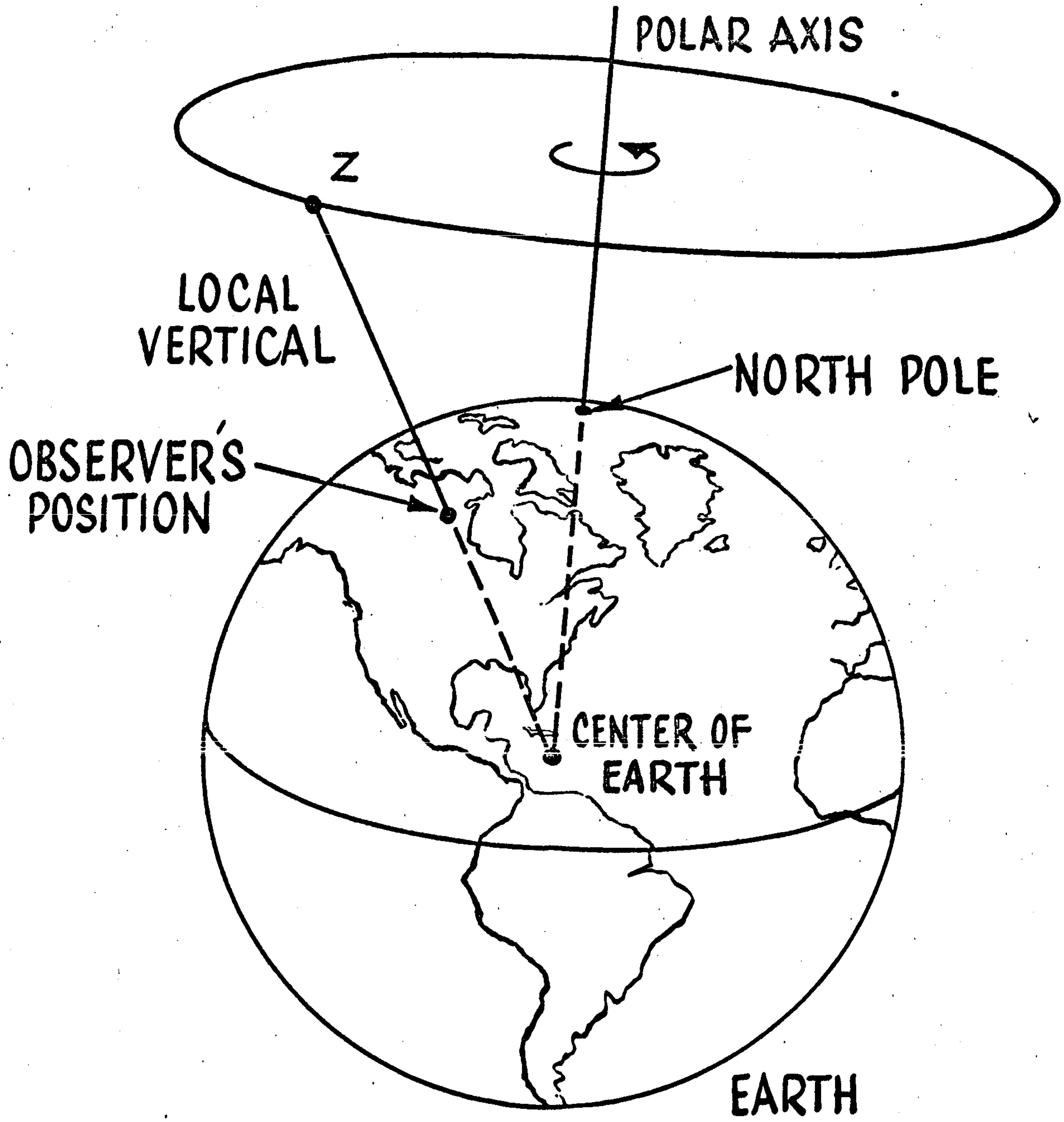

Figure F-1. Locus of Zenith Points on Celestial Sphere
Generated by Earth's Rotation

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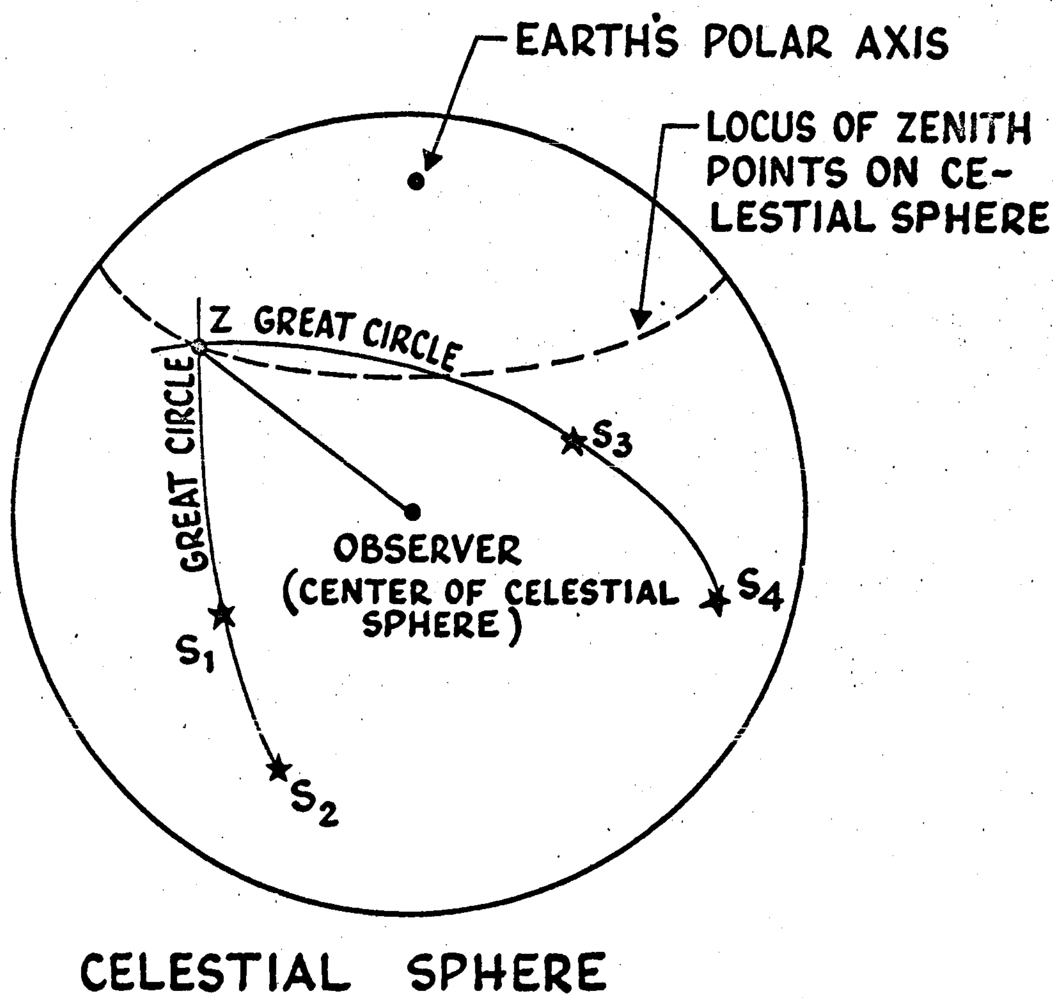

Figure F-2. Location of Zenith Point at Intersection of Two Great Circles on Celestial Sphere

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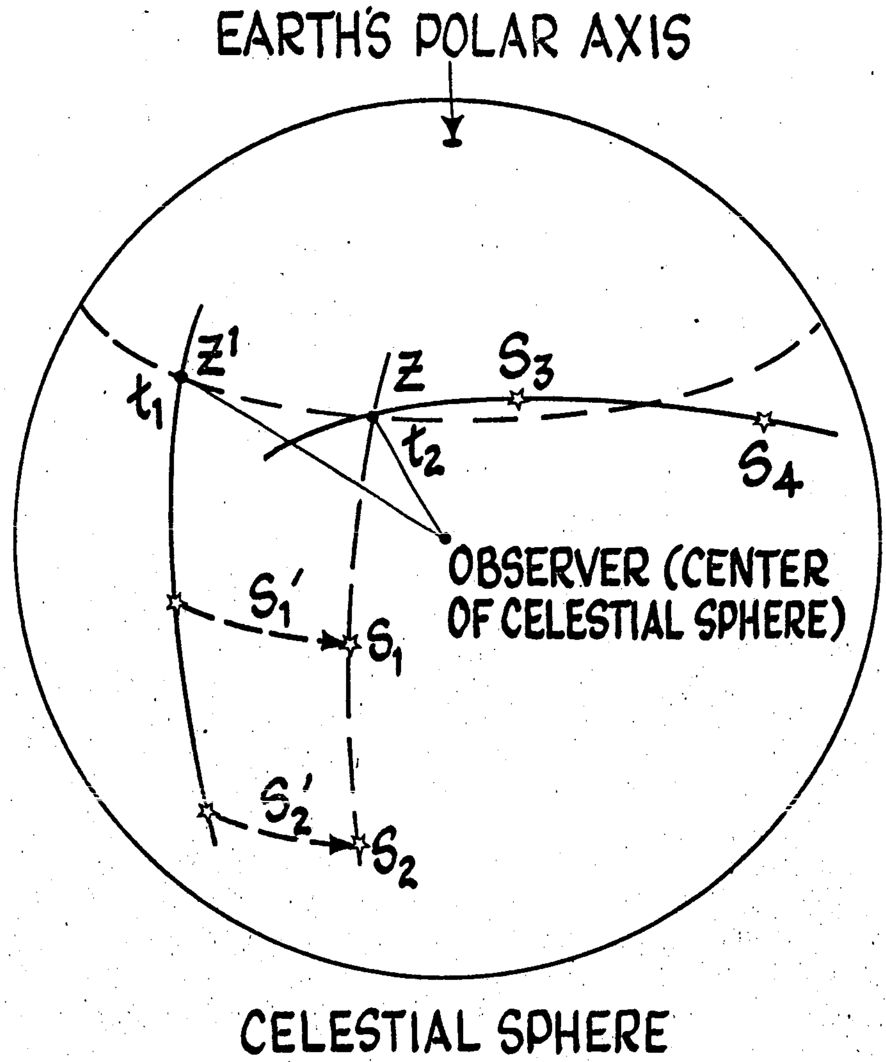

Figure F-3. Correction for Change In Right Ascension of Zenith Point During Time Between Observations of Star-Pairs

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SECTION 2 OF APPENDIX 1-F
POSITION DETERMINATION IN SPACE

In Section 1 of this appendix it was shown how earth's rotation sweeps a local vertical line drawn from an observer on the earth through his zenith in a cone about the polar axis, causing the zenith to describe a small circle on the celestial sphere. Further, it was shown how this moving zenith point may be observed to pass (generally at different times) across two great circles drawn through two pairs of known fixed stars. Finally, it was shown that the times of occurrence of these two events are sufficient data to permit calculation of the coordinates of the zenith and the position of the observer on the surface of the earth, assuming the constant angular rate of rotation of the earth is known.

It will now be shown how this same observational technique may be applied to determine the celestial coordinates of a planet as seen by an observer on board a spacecraft. Two such determinations yield two intersecting lines of position in space from the spacecraft to two planets (or other near bodies) whose positions in the solar system are known. The spacecraft's position may then be calculated as the intersection of these two lines.

On earth, the observer's local vertical is a line fixed to the earth and rotating with it. If the navigational concept is to be applied in space a similar rotating line is required. It can be defined as the collimating axis of a telescope which is fixed to a base rotating at a slow constant angular rate about a known space-stabilized polar axis. The precision of this base motion is probably the determining factor in establishing navigational accuracy.

On earth, the objective is to determine the coordinates of the zenith, to which the local vertical instantaneously points. In space the objective is to determine the coordinates of a planet (or other near body) to which the telescope points at only one instant. In space, therefore, it is necessary first to point the telescope at the planet and then to lock the telescope to its rotating base. The telescope's collimating axis at that instant begins to describe a small circle on the celestial sphere, and the planet passes out of the field of view.

Subsequently, the collimating axis is observed to pass through two great-circle planes defined by pairs of known stars. The times of these events are recorded. If these events are referenced to the time at which the telescope is locked to its rotating base, it is possible to calculate the celestial coordinates of the collimating axis at this instant. These are the coordinates of the planet at this instant as seen from the spacecraft.

Figure F-4 shows the three events which must be timed. They are the telescope locking, the first star-pair transit and the second star-pair transit. Corrections are made as in Figure F-3 for the time lapse between locking and the first transit, and locking and the second transit, so that the coordinates of the collimating axis at the instant of locking can be calculated.

Having obtained by this procedure the coordinates of a planet or other near body at a known time as seen from the spacecraft, the observer then repeats this procedure to obtain the coordinates of a second planet. To correct for the motion of the spacecraft during the interval between these two planetary coordinate determinations, the first planet may be observed again. Simple interpolation may be used to correct the observations of the first planet to the time of observation of the second one, as shown in Figure F-5.

Figure F-6 illustrates the geometry from which the spacecraft's position is computed. The positions of the planets with respect to the sun and each other are known. The coordinates of the planets as seen from the spacecraft are obtained by the method just described. The distances of the spacecraft from the two planets may then be obtained by evaluating explicit functions of these quantities.*

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* These functions are derived in Reference 1-2.

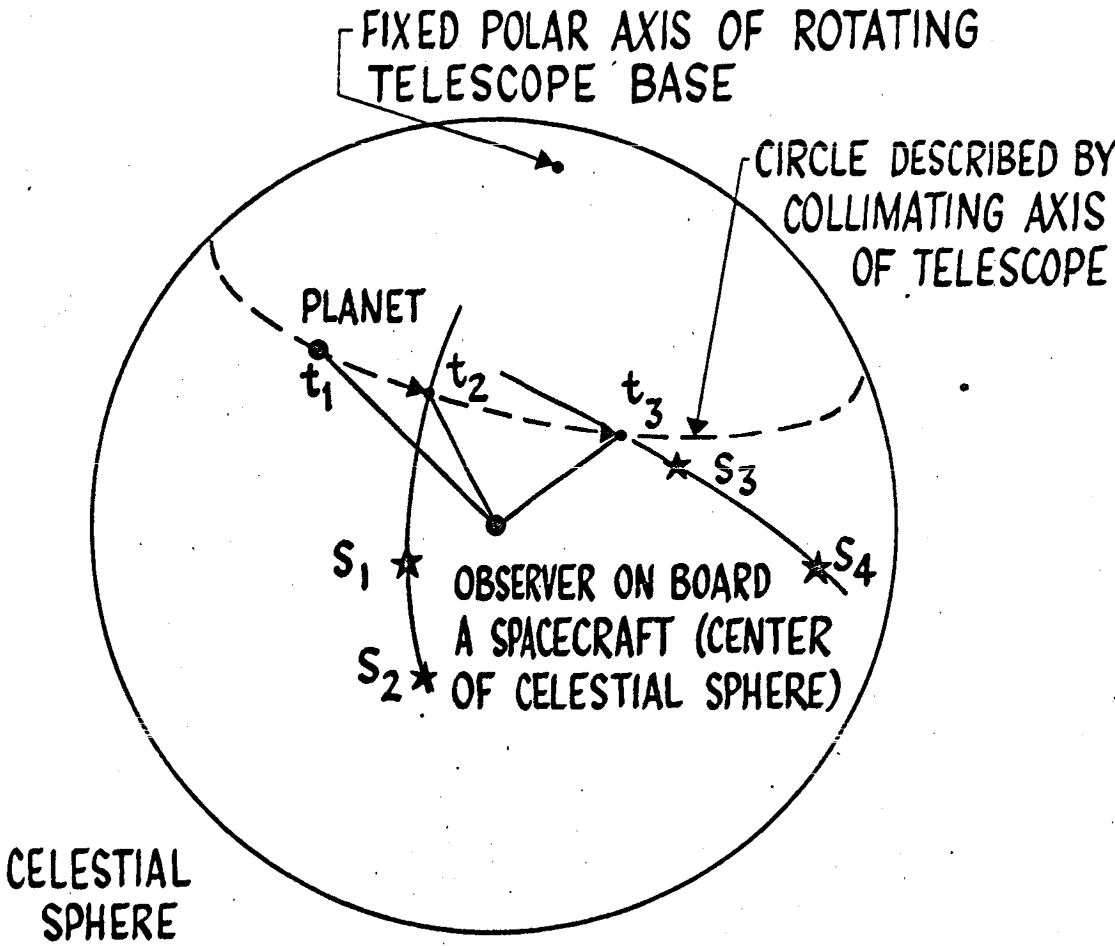

Figure F-4. Three Time Measurements Required to Determine the Celestial Coordinates of a Planet as Seen by an Observer on Board a Spacecraft

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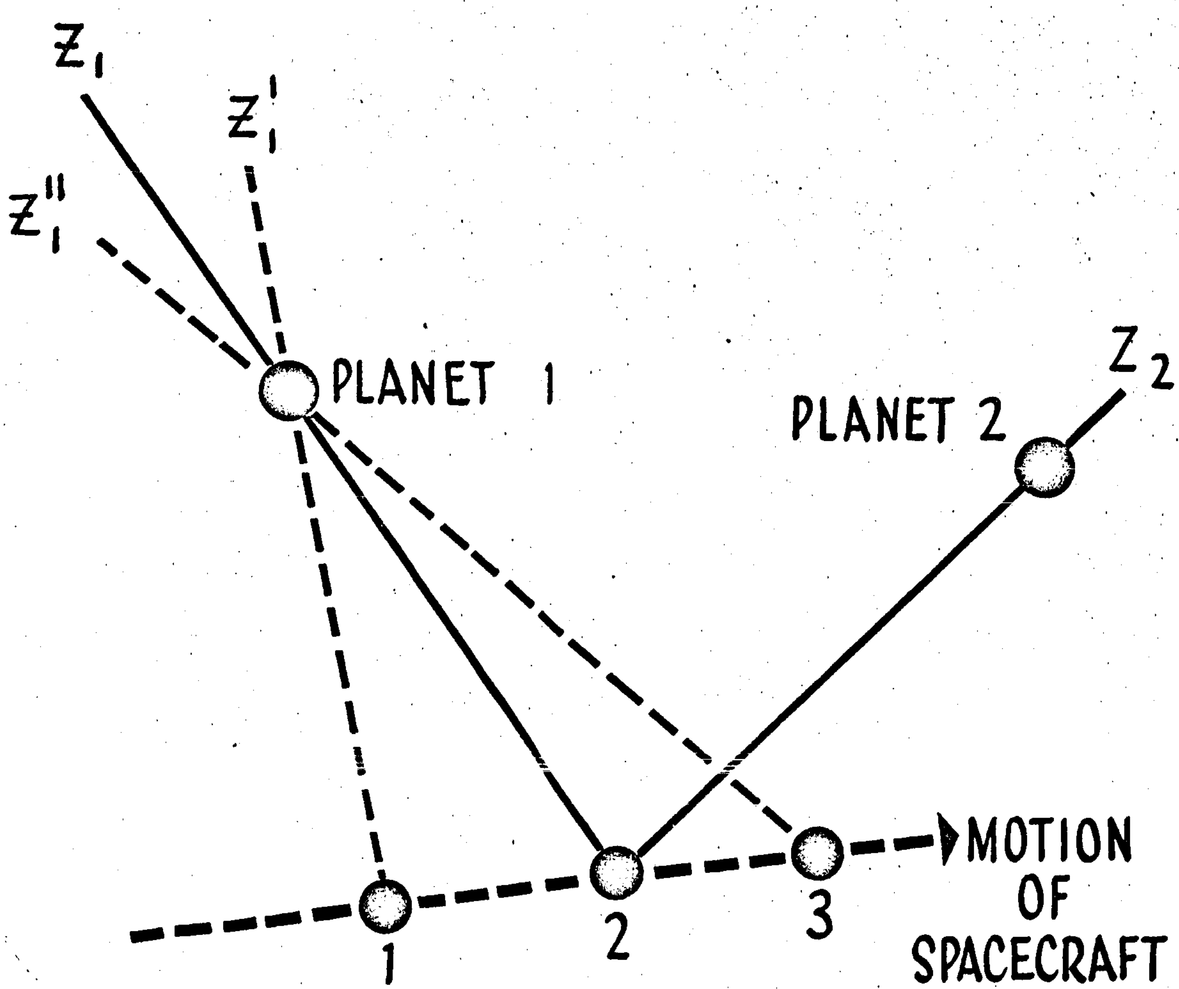

Figure F-5. Method of Referencing Planetary Observations to a Common Time

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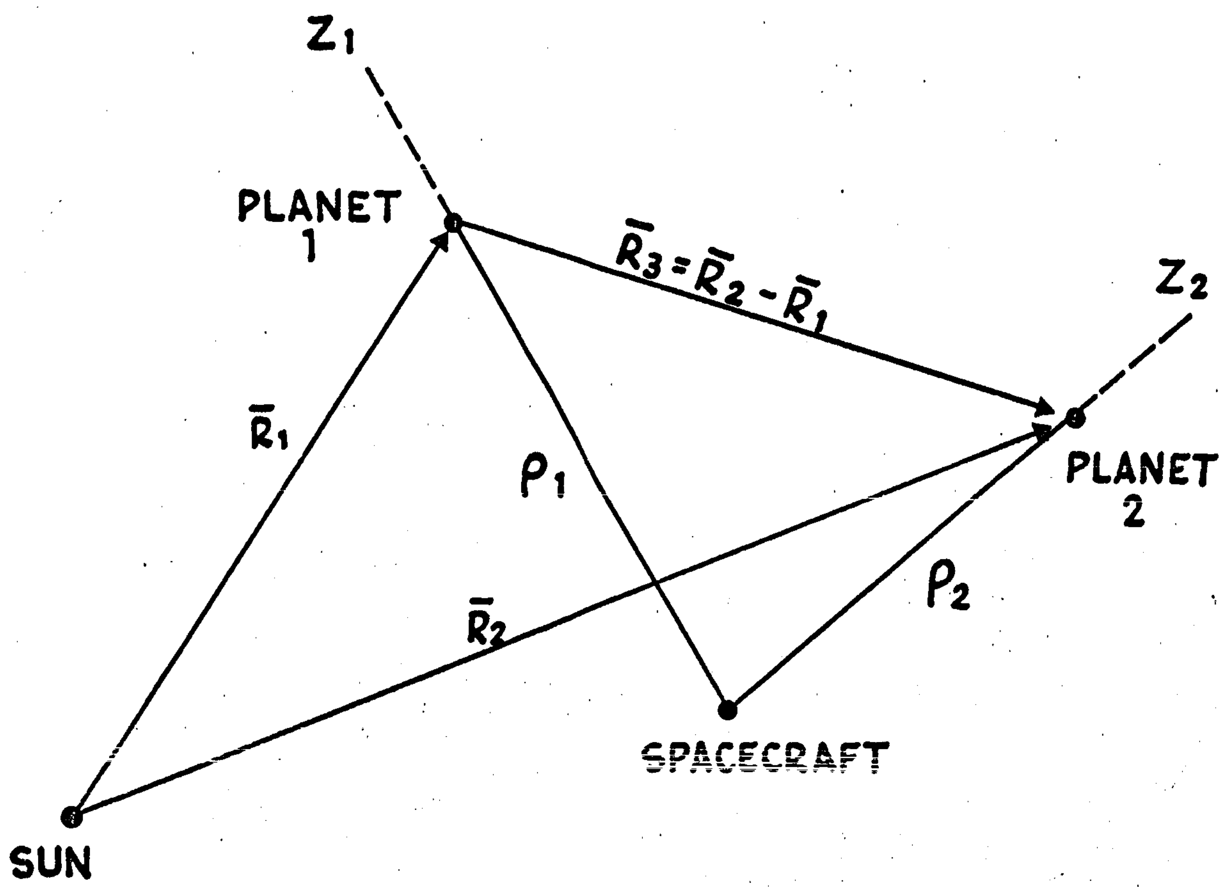

Figure F-6. The Triangulation Method for Position Determination

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SECTION 3 OF APPENDIX 1-F
POINTING TO NEAR BODIES

The navigational technique described in Section 2 of this appendix is dependent upon accurately established lines of sight to the centers of planets or other near bodies which may appear as partially illuminated disks. In this section an iterative technique of planetary observation is described which should permit the location of planetary centers to as great accuracy as may be required.* The technique potentially can reduce pointing error to a satisfactorily small value within a few iterations except when the spacecraft is very close to the planet being observed.

The iteration begins with an estimated value of the distance from the spacecraft to the planet. From this value and knowledge of the planet's diameter, the angular radius of the planet is obtained. Observations of the planetary limb are then made and are corrected using the angular radius to obtain the planetary center with respect to the collimating axis of the telescope. The navigational method described in Section 2 of this appendix establishes the coordinates of the collimating axis, which are then corrected to the planetary center. A determination of spacecraft position obtained from two such planetary sightings permits an improved estimate of the angular radii of the planets so that a new iterative cycle can begin. For navigation in cislunar space no more than two or three iterations are required.

Note that the iteration is entirely computational and does not require new observational measurements in each iterative cycle. After the initial determination of spacecraft position, improved estimates of planetary angular radii may be substituted into the calculations just performed to arrive at an improved position determination without making any further observations.

A planet-sighting technique appropriate to this method is to direct the telescope to a point near the planet, lock the telescope to its rotating base, and wait for the rotation to carry the telescope past the planet. As the planet crosses the field of view of the telescope, limb-tangency measurements are made. The time at which the center of the planet crosses a reference axis in the field of view is recorded and used as the reference time for the determination of the celestial coordinates of the planet. Figure F-7 shows the geometry in the celestial sphere and Figure F-8 illustrates the passage of the planetary image across the field of view.

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* Reference 1-3 contains a detailed discussion and analysis of this technique and considers mechanizational problems.

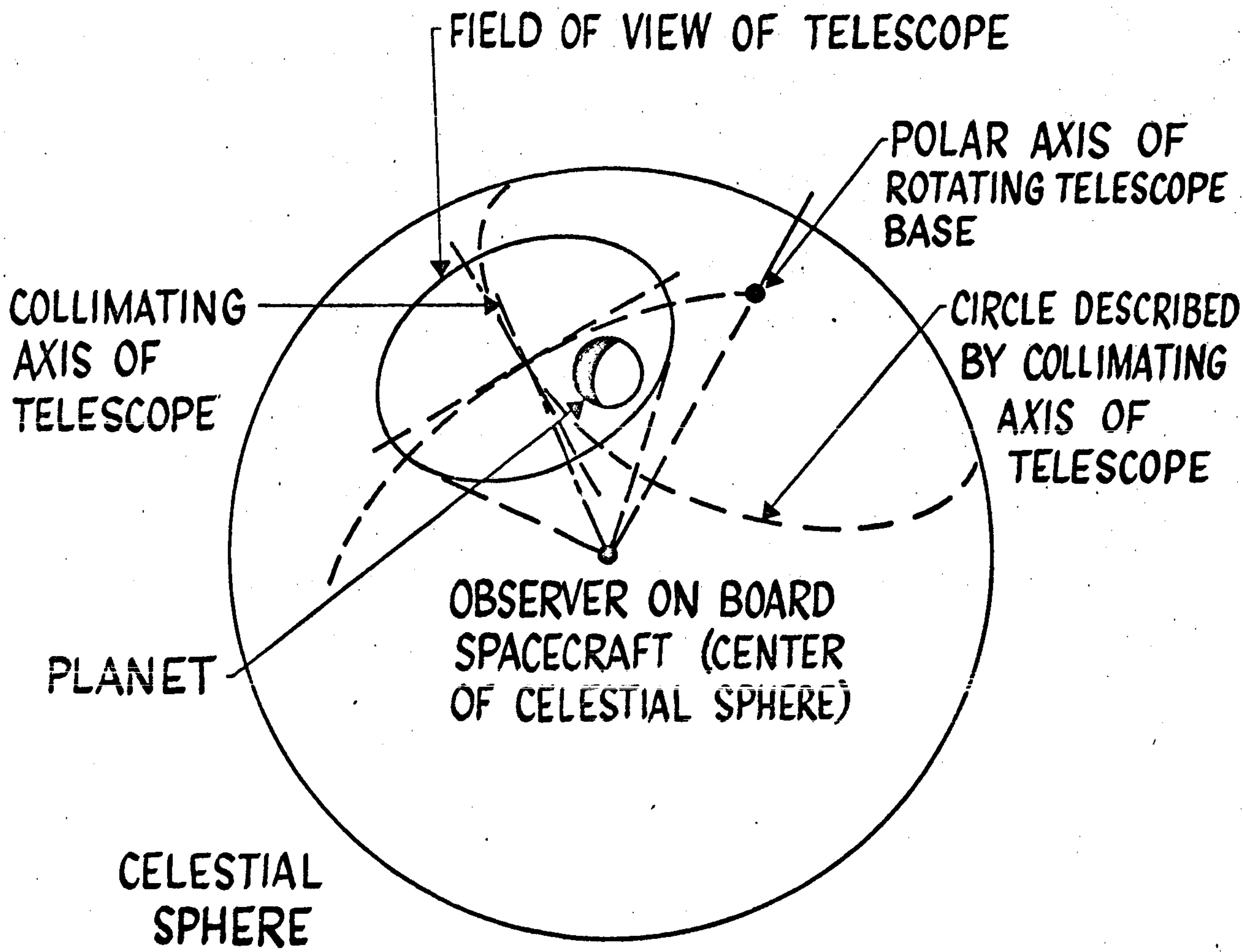

Figure F-7. Pointing to a Partially Illuminated Near Body

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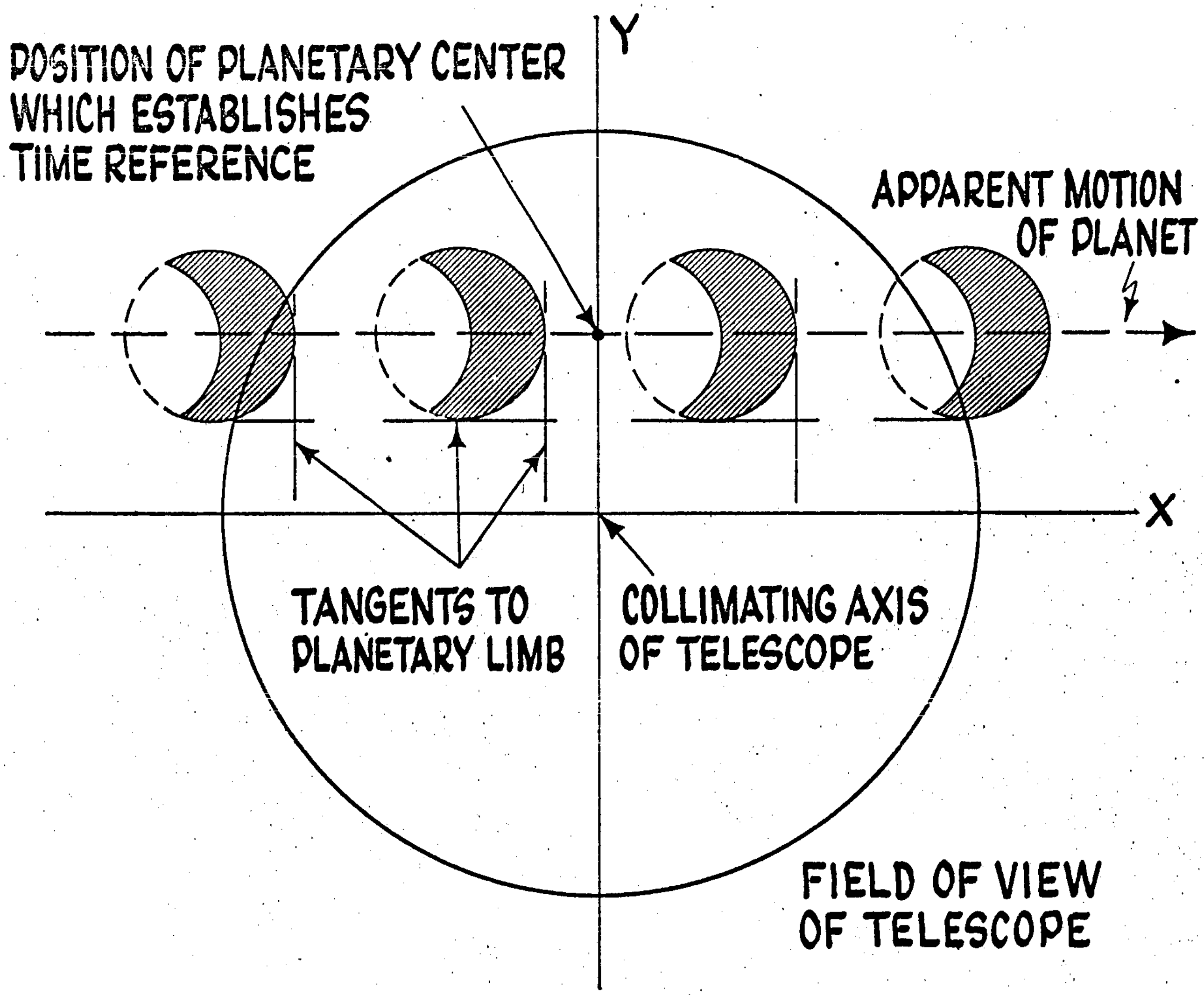

Figure F-8. Location of Planetary Center with Respect to Collimating Axis of Telescope as Planet Passes Through Field of View

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This planet sighting technique has several advantages. First, it avoids the practical difficulty of locking the telescope at the precise instant to be used as the reference time in the navigational calculation. The time of locking becomes unimportant and the angular errors associated with mechanical locking have no effect on the accuracy of the sighting. Also, since several measurements of the tangent lines can be made in the time interval during which the planet is visible in the field of view, statistical averaging may be used to improve accuracy.

SECTION 4 OF APPENDIX 1-F
MECHANIZATIONAL AND OPERATIONAL CONSIDERATIONS

There are many possible mechanizations of the navigational method described in this appendix. The mechanization may be automatic, or it may be semi-automatic with various degrees of participation by the human observer. In its simplest form an entirely manual system could be mechanized. Mission requirements will dictate the proper trade-offs between precision and simplicity. Whatever the details of the mechanization may be, however, it must have certain basic features.

The most fundamental requirement is a slow constant rate of rotation about a known space-fixed axis. A convenient way to establish this axis would be to point to a known star. The rate of rotation need be no more than a few degrees per hour, but could be several revolutions per day if desired, depending on the means available for establishing the rate and on the allowable elapsed time for completing a navigational fix. The desired rotation rate could be achieved by a precision frequency source controlling a synchronous motor which drives an equatorial mount, or by one of the other methods discussed in Reference 1-6. The base of the instrument mount must be accurately space-stabilized during observational periods.

A second requirement is an optical system consisting of a telescope and a means for observing a pair of stars simultaneously transiting the telescope's collimating axis. Provision must be made for pointing the telescope in a desired direction and locking it to its rotating base. A suitable reticle is needed to assist in accurate pointing and in observing star transits. Since these transits may occur in any plane containing the collimating axis, it is necessary in general to provide freedom to search around the collimating axis for star pairs about to transit. Mechanically, the scanning axis must be accurately aligned with the collimating axis. The optical elements which project star images onto the reticle must likewise be carefully aligned. Because of limitations on spacecraft window size, the star pairs must be chosen from a restricted area of the celestial sphere. Operationally, the observational stars must be identified and brought into the field of view of the optical system so that transit can be observed. In many applications it may be possible by pre-selection of the polar star and observational stars greatly to simplify the search procedure and to achieve a corresponding mechanizational simplification.

Third, an accurate clock is required so that the times of occurrence of observed events may be recorded and used in the navigational calculations.

Finally, a computational capability is required. A fully automatic system would include a digital computer with suitable sequencing and control logic. A semi-automatic system would employ a greatly simplified computer and would depend on the human observer to identify planets and stars, to point and lock the telescope, to bring transiting star pairs into the field of view, and to make appropriate timing measurements. In a manual system specially prepared tables and hand-operated calculation aids could entirely eliminate the need for a computer.

SECTION 5 OF APPENDIX 1-F
CONCLUSIONS AND RECOMMENDATIONS

An attempt to improve the precision with which position fixes may be obtained on board a spacecraft has resulted in the conception of a method of celestial navigation which entirely avoids direct angular measurements. Furthermore, it incorporates an iterative computational technique for determining the centers of partially illuminated near bodies being used as navigational references.

The analyses reported in References 1-2 to 1-6 lead to the conclusion that this new navigational technique merits a thorough evaluation. First, a preliminary design study should be made to work out mechanizational details and to evaluate the operational trade-off relationships for typical spacecraft missions. Next, a careful error analysis is required to establish predicted performance. Experiments should then be performed to solve critical mechanizational problems and to verify predicted performance.

Among these experiments might be one to determine the celestial coordinates of the zenith over a known point on earth. Data would be taken to show the accuracy with which simultaneous transits of star pairs across a vertical collimating axis can be observed and timed automatically or manually.

Another experiment might be to determine the celestial coordinates of a known star or planet from earth. The goal would be to develop and evaluate the basic optical system in a configuration closely resembling that which would be incorporated in a navigational instrument.

A critical experiment would be to determine the precision with which a constant angular rate in inertial space can be achieved. An equatorial mount could be placed on an air-bearing table to evaluate the stabilization of the polar axis of the mount with respect to a simulated star. The mount could then be driven about its polar axis to determine the most satisfactory method of achieving a constant rate, and to establish requirements for spacecraft stabilization during observational periods.

Pointing accuracy to near bodies using the technique outlined in Section 3 of this appendix could be investigated by observation of the moon from the earth.

Throughout the development program care should be taken to devise operational procedures which are compatible with human capabilities and limitations, and to obtain work-load data for use in human factors analyses. At the same time it should be shown how the critical functions performed by a human operator could be made automatic.

Finally, a navigational instrument should be built and evaluated for accuracy and operational characteristics in a spacecraft such as Gemini or Apollo.

SECTION 2
DESCRIPTION OF MECHANIZATION MODEL

2.0 INTRODUCTION

This section describes the mechanization model developed to perform an accuracy analysis. It is not the intent here to firmly define in detail a hardware system for any particular vehicle or mission; for this reason the descriptive drawings are limited to assembly and conceptual sketches brought to a level of detail sufficient to show operability, and to permit an engineer to proceed with a detailed design for a specific application.

2.1 OVERALL SYSTEM

2.1.1 Block Diagram

Figure 2-1 is a functional system block diagram showing information flow for a generalized, interplanetary mid-course navigation system based upon the PDT instrument.

2.1.2 PD Axis Angles

The basic measuring instrument is the 2^{23} bit dynagon mounted within the instrument table. It observes the angles from an arbitrary, initial reference (fiduciary pulse marker), which is manually entered at the beginning of each sighting series. This PD axis angle is recorded as a designation of the look direction of the optical axis at the time when a "transit pulse" opens the AND gates between the dynagon register and the computer. The angle is that between an arbitrary magnetic fiduciary upon the rotating table and another upon the frame referenced mount. The absolute or relative initial locations of these magnetic fiduciary markers is unimportant, since the angles recorded, and used in the calculations, are differences between: 1.) the initial angle between magnetic fiduciaries (when a fiduciary "transit" pulse is manually set in), and: 2.) the angle recorded in reponse to each subsequent transit pulse. The dynagon is described in paragraph 2.2.5. This instrumentation is the design choice of "rate generating apparatus" in response to items 2 and 4 of the basic statement of work (paragraph 1.2.3, of Section 1).

2.1.3 Star Transit

The basic observational instrument is the "transit detector" in the one degree field-of-view "transit telescope". A half-transparent, half-reflective reticle is used to switch the light from the star or planet image between a pair of photosensors (PM No. 1 and PM No. 2) at the instant of transiting the reticle line at the edge(s) of a reflective band (or bands). PM No. 1 receives the direct light, and PM No. 2 is arranged to receive the reflected light. In the case of a diffraction image crossing a reticle band edge from transparent toward reflective, the output from PM No. 1 decreases as the diffraction image is slowly occulted by the edge, and the output of PM No. 2 rises in approximately the same manner. Due to the extremely small star image size (about 0.0002 inch diameter) the transition of photosensors between their "yes" and "no" states is quite rapid, even at low pseudo-diurnal rates. Nevertheless, the transit electronics seeks to generate a sharp pulse precisely when the two sensors are about equally illuminated, representing the time at which the center of the diffraction image crosses the reticle line. The occurrence of this pulse gates the simultaneous readout of a digital electronic clock and the dynagon angle. A "read trigger" at the control panel is used to designate which transit events shall be recorded. This trigger merely applies a voltage to an AND gate at the output of the transit detector electronics. This manual sorting of significant transits (from the thousands which may occur in a series of sightings) is considered to be a function which only a human operator could effectively perform. The operator also must designate with a keyboard the names (or coordinates) of the body presently being transited. Only a human operator could so easily recognize the bodies in this manner.

Star transit events occur across one of two reference "great circle" projections inclined either +45 degrees or -45 degrees from the pole-star or "pseudo-diurnal" axis. Planet transit events occur across both axes (+ and -45 degrees) sequentially. This selection is arranged by properly pushing either the star or planet "read triggers" just prior to the observed transit event; and releasing the trigger upon its completion, as signalled by a flashing light (on the monitor kinescopes) and/or a buzzer (on the control panel) when the transit pulse occurs. The sequence for manually controlling these events, and their significance in establishing lines-of-position is shown in Figure 1-9 (of Section 1). The Optical Configuration is described in paragraph 2.2.4.

2.1.4 Monitors

The view through the wide-field "finder" telescope is presented upon one monitor kinescope, and the view through the one degree "transit" telescope is presented on another. A selector switch permits connecting

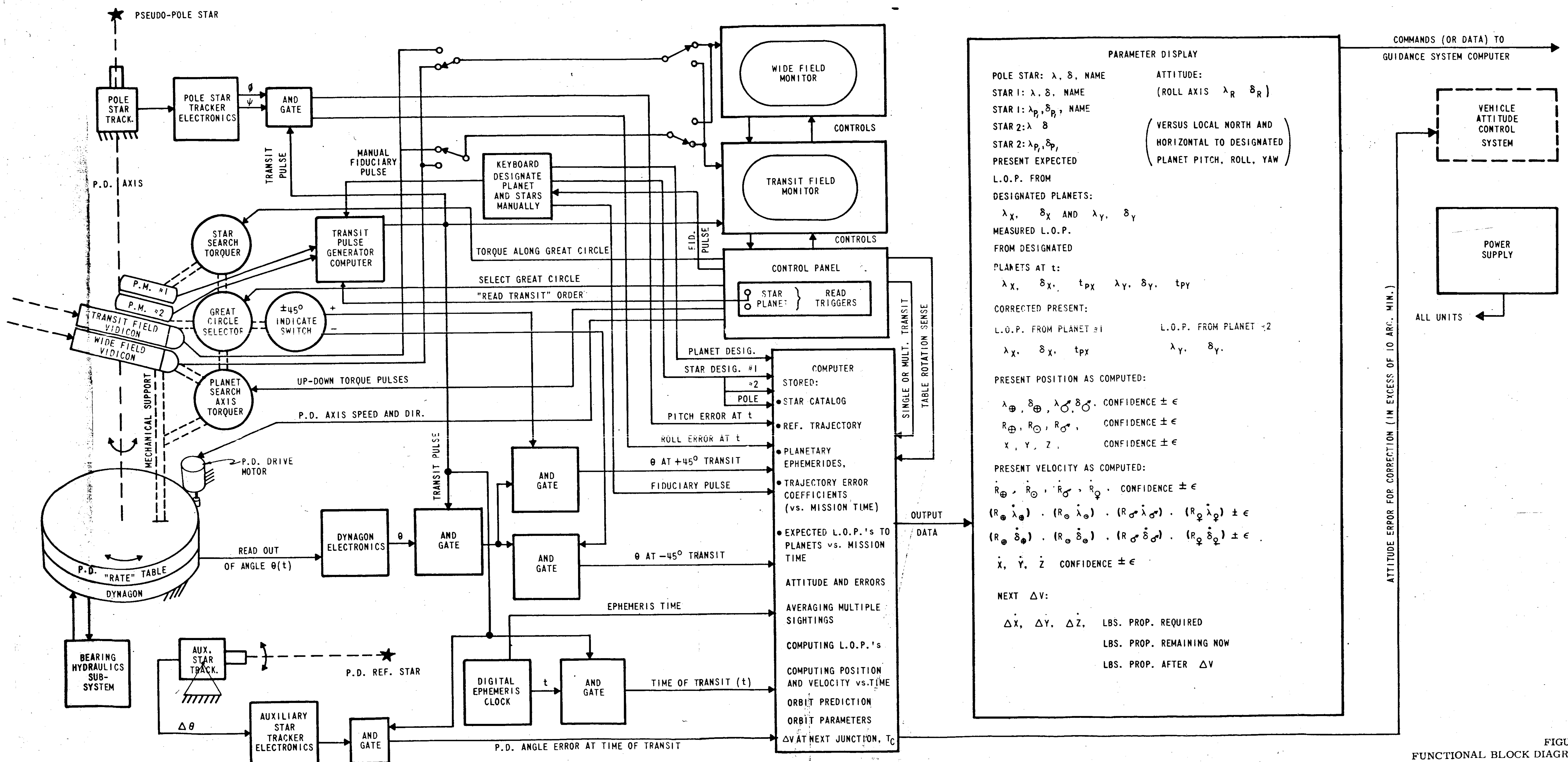


FIGURE 2-1
FUNCTIONAL BLOCK DIAGRAM PDT NAVIGATION SYSTEM

Honeywell Aero Report 2858-FR

either Vidicon to either monitor scope. In actuality one kinescope monitor can suffice for all abservations, but for high resolution of transit, convenience, and the sake of redundancy two monitors should be provided.

2.1.5 Attitude Reference

The pseudo-diurnal axis angles all are referenced to the inertial space attitude of a common, rigid mounting frame secured to the frame of the vehicle. Quite obviously the stability of the vehicle during a series of sightings enters directly into the accuracy of all observations. Also quite obviously it is too stringent a demand upon the ship's autopilot to require it to hold a fixed attitude in space for several minutes time to within a fraction of an arc second. The autopilot will hold attitude in three axes to within some pre-determined dead zone. Present state of the art practice would make the size of this dead zone one degree; for the purpose of this analysis an improvement by a factor of six over this was assumed for the 1970 to 1975 time period, and the dead zone was chosen as 10 arc minutes. During sightings the vehicle will wander around within this dead zone because of random shifts of angular momentum within the vehicle, due to men and machinery in motion. To overcome this random wandering of attitude, a pair of star "trackers" are provided which have the capability of reading deviations of attitude from the starting reference attitude (marked by the fiduciary pulse) to an accuracy of ±0.5 arc second. These star trackers are described in paragraph 2.2.6. The star tracker outputs are used by the computer to correct each transit-angle readout to what it would have been had attitude not changed during the sighting. One tracker traces the apparent wandering of the pseudo-pole-star in two vehicle axes (pitch and roll, for example) while the auxiliary tracker traces the apparent wandering of a "rate reference star" in the third vehicle axis (yaw, for example, if the PDT axis is assumed to be co-aligned with the vehicle's yaw axis). The auxiliary star tracker must be gimbaled for adjustment in the pseudo declination axis (for example in elevation above or below the yaw plane), in order to permit freedom in choice of some convenient rate reference star. This rate reference star should be as close as possible to the pseudo-equator, and bright enough for reliable tracking, but its coordinates need not be described except in terms of their deviation from a reference attitude at the time of the fiduciary marker. It would be convenient to provide ultradex detent gimbaling in two axes for the auxiliary tracker to make acquisition easier. This also opens up the possibility of applying the auxiliary tracker as a manual back-up device by using it as the observing instrument for the original PDT scheme.

Both star trackers are mounted on the rigid frame common to the main observing instrument, to minimize distortions from vibrations of the vehicle. It is believed that the amplitudes of such vibrations will be less than the 1/4 arc sensitivity of the star trackers, and would therefore be ignored by the trackers. At any rate, this conceivable error source should be minimized by judicious structural design. (This should be confirmed experimentally in Phase II of this study).

2.1.6 Calculations

The calculations required for data reduction were estimated to require about 1/8th of the capacity of a Honeywell ADEPT computer (Reference 2-1), or 1/4 of a Honeywell PICO computer (References 2-2, 2-3, 2-4). Despite the seeming complexity of the operations, the required total computer capacity appears to be well within the capabilities of the conventional aerospace guidance type of tally box, according to M. C. Zeiler. However, before serious specification of a particular machine, it will be necessary to write the complete data reduction program for some representative machine. It was decided that this effort was not pertinent to the goals of Phase I of this study (see paragraph 1.4.2.5.5 of Section 1). It is recognized that there will be some truncation and round-off errors in the computer. The ±1 bit doubt in the least significant place of input data can be propagated to become significant, where many iteration steps are involved. Without a complete, detailed program for a specific machine, this cannot be precisely estimated. There is, however, little unique significance to this error source for the purpose of this study, since similar errors would be involved, whatever type of digital computer or whatever observing means is employed for space navigation. Therefore, this error source is neglected in the tabulation in Section 3.

2.1.7 Keyboard and Controls. (Manual Inputs)

Only the functions essential to the PDT system are mentioned. It is realized that any actual hardware design for a specific space mission would have several navigational modes integrated with the navigation and guidance hardware. The form of the keyboard and control panel is visualized as being generally similar to that for Apollo.

Certain functions for which manual intervention is necessary are:

- Pole star choice and acquisition using vehicle attitude controls,
- Rate reference star acquisition by viewing through the optics of the auxiliary star tracker,
- Reference fiduciary mark pulse ("start"),
- Slewing in planet search axis and PD axis to acquire the planet,

- Designation of planet's identity (by keyboard buttons) for computer,
- Observation of planet transit across lines in the transit scope field,
- Trigger designation of planet transit pulse by holding down "planet read trigger" until both 45 degree "great circle tangencies" have occurred,
- Visual identification of two navigational stars,
- Keyboard designation of star No. 1 and star No. 2 to computer (Probably the right ascension and declinations would be entered into a conventional keyboard together with storage instructions),
- Choice of great circle (+ or - 45 degrees),
- Slewing of transit scope along great circle axis to prepare for star transit,
- Starting PDT drive motor to begin transit,
- Observing transit on kinescope screen,
- Holding "star read trigger" until first line transit has occurred (ignoring second),
- Repetition for both stars and both great circles,
- Compute order,
- Repetition for second, third, et cetera planets and later sights on the same planets,
- Choice of single transit or averaging reticle operation,
- System operational checkout,
- Maintenance,
- Repair of equipment failures,
- Decision as to whether output data displayed is to be relied upon, or whether more observations are required to increase confidence, and
- Application of back-up modes in event of catastrophic instrument failures.

2.2 DESCRIPTION OF INSTRUMENTATION

2.2.1 Observing Instrument Exploded View

Figure 2-2 is a conceptual, isometric, exploded view of the layout of the primary observing instrument. Most of the detail has been left out for simplicity. This drawing is chiefly for visualizing the physical relationships of the major components.

The observing instrument is mounted with its "Ultradex" gimbaling upon the rotating PDT "rate" table, which is supported within a fluid bearing casting together with the Dynagon transducer rings. The whole assembly is mounted beneath a dual, concentric, spherical astrodome.

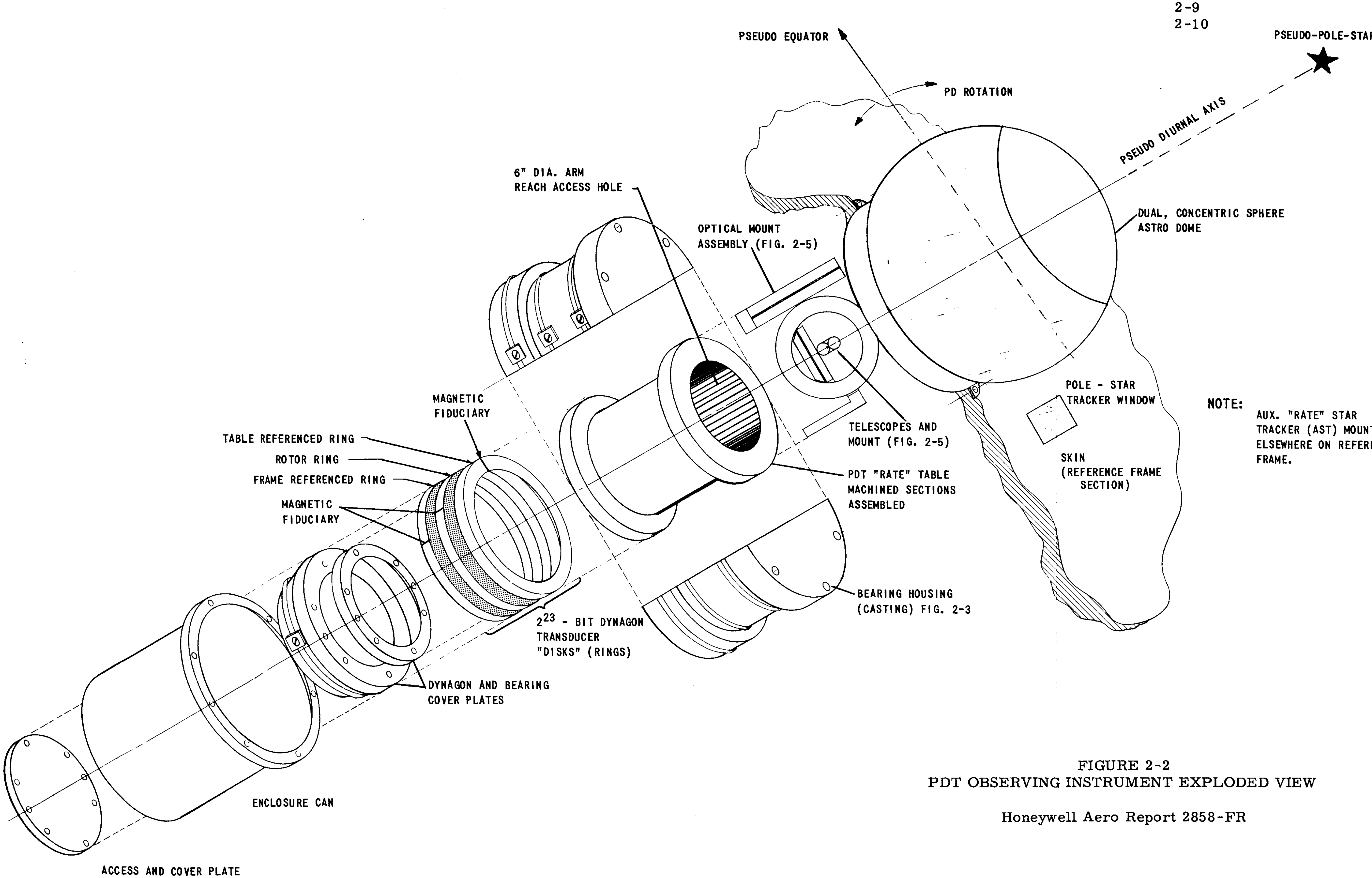
Slip rings are avoided by spiral cabling directly to the PDT table. Approximately six complete revolutions in one direction are allowable before an "unwind" operation is required. The Dynagon electronics logic can be arranged to trigger an "unwind" alarm light on the control panel whenever five complete revolutions have been algebraically accumulated. It is critically important that slip rings be avoided, since these are among the lower reliability components. A little invonvenience in winding and unwinding a flat, spiral cable coil is a small price for extending MTBF by thousands of hours. Few (if any) series of observations will require more than one algebraic revolution about the PD axis, so this is not considered to be a significant limitation.

2.2.2 PDT "Rate" Table Assembly

The key mechanical component is the stiff bearing assembly which suspends the observing instrument to the vehicle common reference frame. This bearing assembly fits into a cylindrical space 12-1/4 inches long by 16 inches in diameter. Figure 2-3 is the reduced scale assembly drawing (conceptual sketch).

Operation of the fluid bearing depends upon maintaining a heat balance between the liquid phase of the freon fluid (in the bearing clearance gaps of 0.0005 inches) and the gas phase outside the bearing surfaces. This balance is maintained by a refrigeration cycle driven by a sealed unit compressor. A mutual heat exchange takes place in the spiral coring of the Table Case Casting. Waste heat is radiated away from the surfaces of the casting. Heat may be added by a temperature control thermostat subsystem and heater coils (not shown) if this proves necessary in the experimental model.

The bearing table is designed to maintain the stability of the optical assembly versus the frame to within ±0.1 arc second about any axis other



NOTE: AUX. "RATE" STAR TRACKER (AST) MOUNTED ELSEWHERE ON REFERENCE FRAME.

FIGURE 2-2
PDT OBSERVING INSTRUMENT EXPLODED VIEW

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FIGURE 2-3
PDT "RATE" TABLE ASSEMBLY

than the rotation axis, and to provide full freedom of rotation for both table and Dynagon about the PD axis. The essentially frictionless support permits Dynagon rotor drive with low-power, two-phase torquing coils and table drive with a very small, reversible, variable-speed gear-motor. Since table rate stability is operationally of little consequence, there are no rigid requirements on this table drive motor. A brushless, dc (silcon-switched) motor probably would be applied to improve reliability and permit smooth speed control.

Design details shown on Figure 2-3 are not discussed at length here inasmuch as the design was undertaken merely to demonstrate operability. No doubt the experimental model to be built for phase II of the study will lead to many alterations in design detail, since the concept of a freon fluid bearing is relatively new. Some of the reasoning behind selection of this bearing is related in paragraph 1.4.3.1 of Section 1. Experimental confirmation of this general bearing design is deferred to phase II. A. R. Bock should be consulted at the time bearing table fabrication is undertaken.

There must be no displacement of the axis in either of the three rotational planes (pitch, roll and yaw) while rotation about the roll axis occurs. In order to accomplish this, deviation from the ordinary sleeve or ball bearing axis was necessary. Bearings containing coulomb friction sources are subject to wear and therefore depreciation of accuracy. They would require machining tolerances presently not practically obtainable.

Some of the bearing types considered and the negating reasons are as follows:

a. Sleeve - high probability of obtaining accuracy with existing machining abilities (only two diameters to be held). When dry, the bearing would have high galling and wear rates. Permanent lubrication would be extremely difficult due to the minute clearance that would be required for the accuracy of ±0.1 arc second. Vaporization effects would detract from the permanency of lubrication.

b. Ball Bearings (includes needle, taper, roller, barrel and the like). Many, low-tolerance dimensions must be tightly held (balls, inner and outer races, race O. D., counter-bore I. D. and concentricities). Lubrication problems and galling tendancies would be less severe, but still present.

c. Hydrostatic - Coulomb friction and lubrication problems would be eliminated (no wear or galling problems).

Air suspension was considered; however, the compressibility of the air would result in a vibration amplification effect that would be intolerable; the machining and temperature requirements would be very difficult to meet.

Oil suspension would be nearly ideal; however, the problem of recovery of the oil in a zero gravity field and the possibility of optical fogging caused rejection of oil bearings.

A firm decision was made to compromise upon use of freon, which can yield all the advantages of a liquid bearing with the recovery advantages of a nearly inert gas. The primary disadvantages in using freon are: 1) a compressor rather than a pump must be used (requiring more power), and 2) a temperature differential would be created in the bearing, requiring consideration of heat balancing controls.

Figure 2-4 shows a sketch to illustrate the reason for a six inch cylindrical hole through the center of the rate table. This permits arm reach access to the observing instrument without completely dismounting the instrument from the dome. Such operations as replacing vidicons or photosensors, wiping lenses and removing stray objects can be accomplished without the trouble of dismounting the whole device.

It also is possible to dismount the entire bearing table by unbolting it from the reference frame and withdrawing it from the astrodome.

2.2.3 Optical Instrument Mounting

The optical instrument package (including the 30 degree-field finder telescope, the one degree-field transit telescope and associated electr-optical sensors) is mounted to the PDT table surface as shown in Figure 2-5. Freedom in three axes with respect to the table surface is provided by "Ultradex" detented drives capable of moving in one degree steps, and settling to detented positions which are accurate to ±0-1/4 arc second from an initial position. The stability of the Ultradex detent, once latched, is approximately that of a solid metal member of equivalent dimensions. Latching is fail safe by use of an expander coil spring; unlatching is powered by solenoids. Drive about the axis in the unlatched condition is accomplished with reversible dc motors driving ring gears through small drive gears. The accuracy of the Ultradex is highly desirable in this application, but the repeatability and stability are essential.

Once the telescope is positioned for a transit it is essential that it not move with respect to the PDT table about any axis; that is, all motion during transit is due either to table rotation about the PD axis or to vehicle motion within its 10 arc minute dead zone, and both of these motions are measured to high precision by the dynagon and star "trackers" respectively.

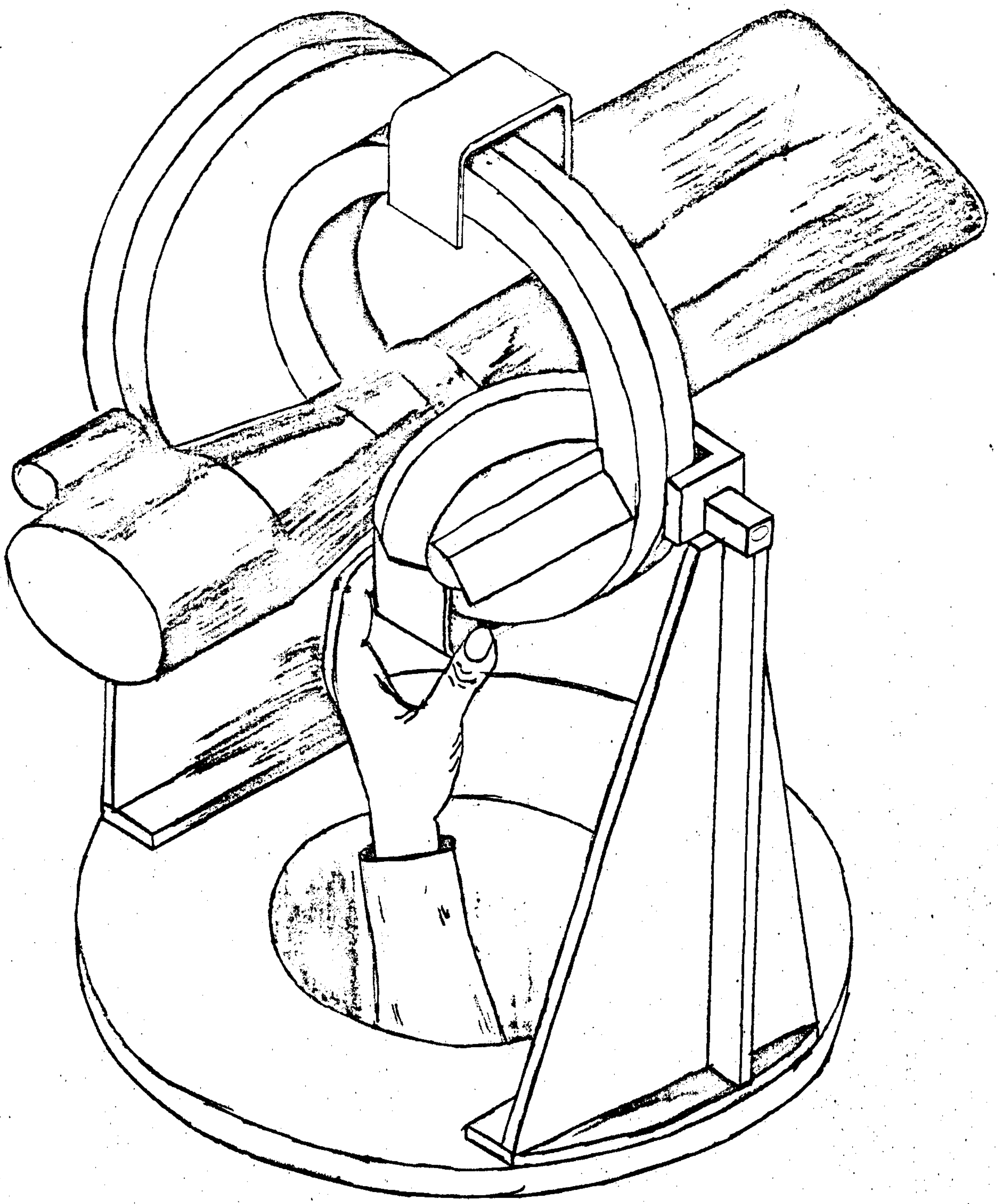

FIGURE 2-4
THE ARM REACH ACCESS HOLE

The PDT motion and the three Ultradexed motions with respect to the PDT table are noted (from the transit telescope outward) as follows:

1) The scope is free to step ±30 degrees from center about the "Star Search Axis", in one degree increments.

2) The Star Search Axis is Ultradexed to a second ring which is free to move about the optical axis of the transit telescope ("Great Circle Selection Axis") to one of two positions detented (by counting Ultradex detents) to be at either right 45 degrees or left 45 degrees, with respect to the PD or "pseudo-pole-star axis". The operational reasons for choosing these two "great circle positions" are discussed in paragraph 1. 4. 2. 1 and 1. 4. 2. 6. 6 of Section I. The result of this is that star search motion is constrained to be along one of these two "great circles", and star transit can only occur across one of these. (It is noted that a further requirement is that the transit detector reticle lines be precisely aligned with the star search and optical axes and rigidly held in this relationship).
Since the entire telescope assembly must pass through the ring-form Ultradex (about the optical axis), two latching solenoids (as a minimum) are required to detent the great circle selection axis. Future designs perhaps will employ three solenoids spaced 120 degrees around the optical axis to assure uniform pressure on the Ultradex.

3) The great circle "Ultradex" is suspended between a sleeve bearing and a planet search axis "Ultradex". This permits Ultradexing motion about the "Planet Search Axis" (motion in pseudo-declination).

4) Finally the entire assembly, when latched, is limitied to motion about the PD axis with the PDT table (motion in pseudo-right ascension) which sweeps the chosen great circle segment along a small circle about the PD axis.

Feedback from the Ultradex units will be required, since the drive motors can cause continuous rotation about any axis while the Ultradex detents are unlatched. This will require simple, one degree bit-size code disks on each axis to operate a display on the control panel. This is necessary, since no direct view of the instrumentation itself has been provided. The code disks are not shown in Figure 2-5.

2.2.4 Optical Instrument Assembly

Figure 2-6 is a conceptual sketch of the optical instrument assembly. This instrument has two main parts; 1) the 30 degree-field "finder telescope" with its vidicon, and 2) the one degree-field "transit telescope" with its vidicon and electro-optical transit detector.

The finder telescope is an ANGENIEUX 25 millimeter f/0.85 or equivalent 16mm format camera lens coupled to a one inch vidicon (tube type not specified). This subassembly is rigidly mounted to the barrel of the transit telescope and carefully pre-aligned to have its optical axis parallel to that of the transit telescope. There are rather loose requirements for accuracy on this instrument, since its primary function is to permit star identification. A small reticle just ahead of the vidicon designates the one degree transit field within the finder field for the convenience of the operator. This reticle causes a one degree circle to appear on the monitor. This reticle also carries a carefully centered "X" pattern, to permit use of the finder telescope for manually observing transits in a back-up mode and for reference, while positioning the transit telescope by Ultradexing orders, in the primary mode.

The transit telescope is the central observing instrumentality. One choice is a CARL MEYER three inch diameter, 14 inch focal length, f/4.5 telephoto, 35mm-format camera lens (or equivalent). The light from this lens is divided into three parts somewhat as shown in Figure 1-20 of Section 1. A pellicle just behind the lens system splits the light (approximately 50/50) and diverts half of it to form an image on the transit field vidicon face; the remaining light forms an image on a reticle just ahead of photo sensor no. 1 (Dumont 6365 or equivalent photo multiplier). Light transmitted by the reticle will illuminate sensor no. 1. Light reflected by the reticle is returned to the pellicle and about 50 percent of this is diverted through a re-imaging lens to illuminate sensor no. 2. The bias voltages on the two sensors are balanced to make up for the illumination differential between the two sensors. Preliminary calculations indicate that the light gathered by the three inch objective is more than adequate for the purpose here.

The reticle, Figure 2-7, is a glass plate with accurately plane parallel surfaces. Its thickness need only be enough for mechanical stability. It is divided into four quadrants as shown, alternately transmitting and reflecting. Light transmitted through the reticle falls on sensor no. 1. That reflected falls on sensor no. 2. The art of putting on reflecting strips is highly developed. A boundary can be accurately located to 0.0001 inch and its sharpness can be held to 0.00001 inches.

The plane of the reticle is accurately normal to the optic axis of the transit telescope. The optic axis also intersects the reticle at the common boundary intersection. The reticle is rigidly mounted with the telescope tube, and rotates with it as the telescope turns about the great circle selector axis, which is coincident with the optic axis.

The two quadrant edges of the reticle, called A and B, are themselves aligned with the two great circles, ±45 degrees, along which star search is made. In other words, edges A and B represent the projections of the ±45 degrees great circles. When the telescope is in the -45 degrees great circle position, motion of the telescope about the star search axis is along the circle whose projection is edge A. When the telescope rotates about the great circle selector axis or optic axis into the +45 degrees position edge A also rotates so that in the new position also motion of the telescope in star search is along edge A. Only this edge need be used for star transit determination. When transit of the planet is to be determined the telescope is normally in the -45 degrees position and the image moves first across edge A then across edge B.

A simple two edge reticle has been described, and the error for a single edge transit is discussed in Section III. In principle one could as easily read a sequence of transits across multiple parallel edges, and therefore decrease the expected error. Practically this could be accomplished either with a more complex reflecting-transmitting reticle or with a fiber optics array.

Image Considerations

The size of the spot formed by any optical system is theoretically limited to the size of the Airy disc produced. This may be calculated from:

$$d = \frac{2.44\,\lambda\, f}{D}\;.$$

where d is the diameter of the spot in inches, λ is the wavelength of the light in cm, f is the focal length of the lens in inches and D is the diameter of the collecting lens in cm. For the PDT System:

$$d = \frac{2.44 \times 5.6 \times 10^{-5} \times 20}{7.6} = 3.65 \times 10^{-4} = 0.000365 \text{ inches}$$

The size of the detector to obtain a one degree field of view is:

$$X = 2\, f \tan 1/2\ \alpha = 40 \times 0.00873 = 0.349 \text{ inches}$$

The apparent angular subtense of the star due to the size of the Airy disc is:

$$\beta = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 5.6 \times 10^{-5}}{7.6} = 0.0000182 \text{ Radians}$$

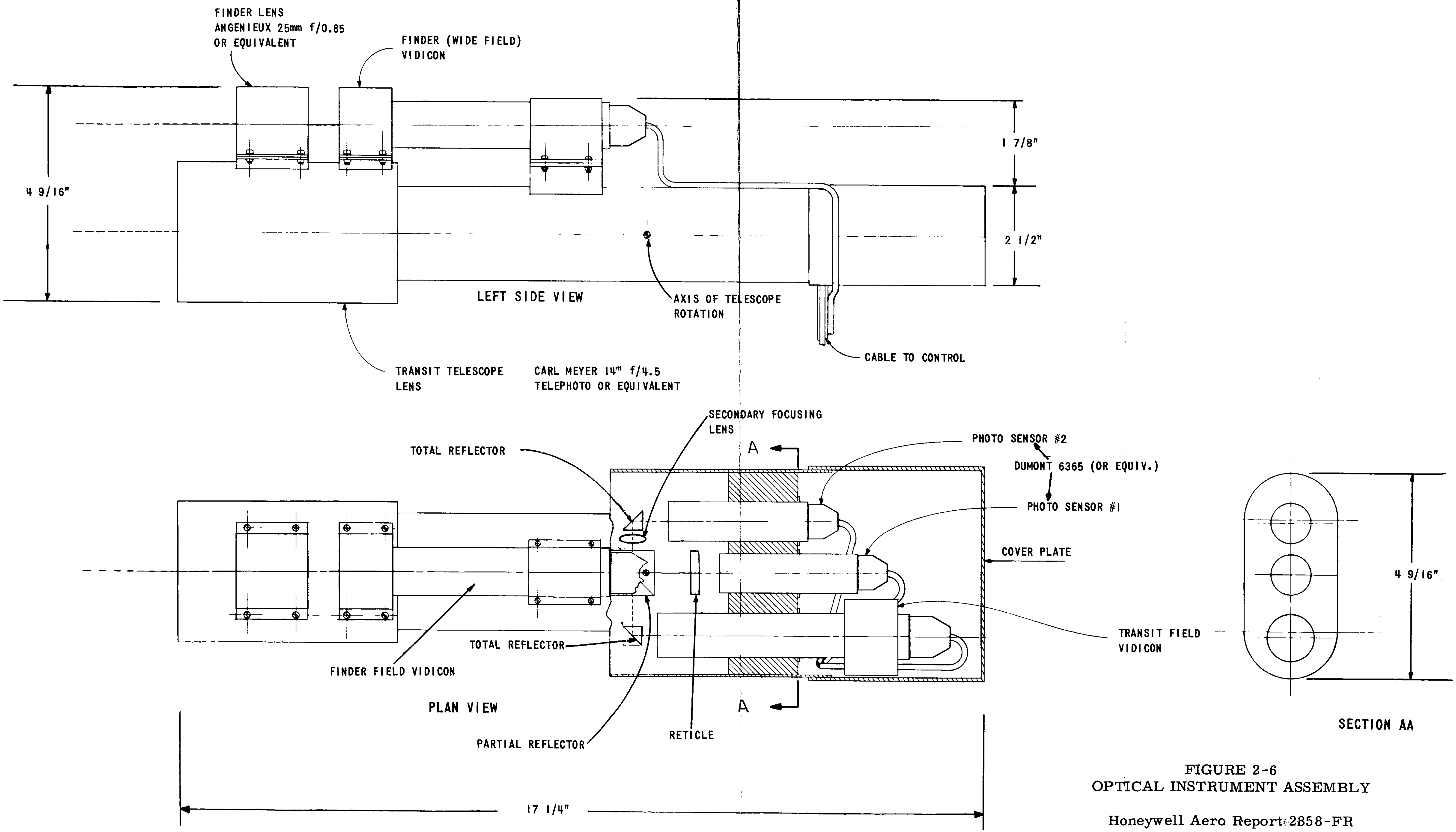

FIGURE 2-6
OPTICAL INSTRUMENT ASSEMBLY

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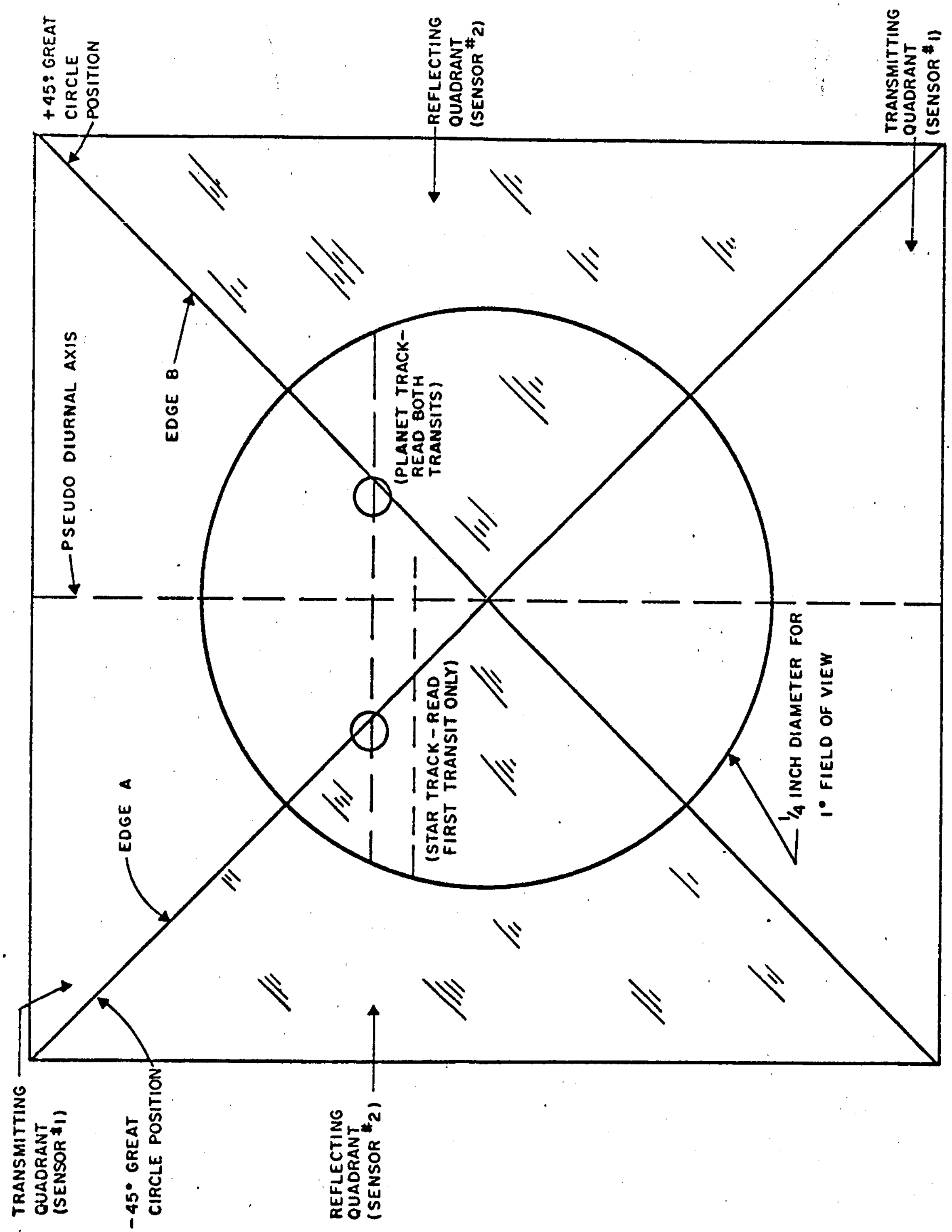


FIGURE 2-7
TRANSIT DETECTOR RETICLE

or 3.73 seconds of arc.

The linear speed at which the star image travels is:

$$\dot{X} = \frac{f\dot{\alpha}}{\cos^2\alpha}, \cong f\dot{\alpha} \quad \text{(for transit detection at the center of the field}$$

If the telescope moves at earth rate, $\dot{\alpha}$ = 15 arc sec/sec or 0.0000727 radians/sec:

$$\dot{X} = 20 \times 0.0000727 = 0.00145 \text{ inches/sec.}$$

The time it takes the image to traverse its own width on the detector is:

$$t = \frac{0.000365 \text{ in}}{0.00145 \text{ in/sec}} = \frac{3.65}{14.5} \text{ sec} = .252 \text{ sec.}$$

To obtain transits accurate to 0.1 arc second, the electronics must recognize transit to:

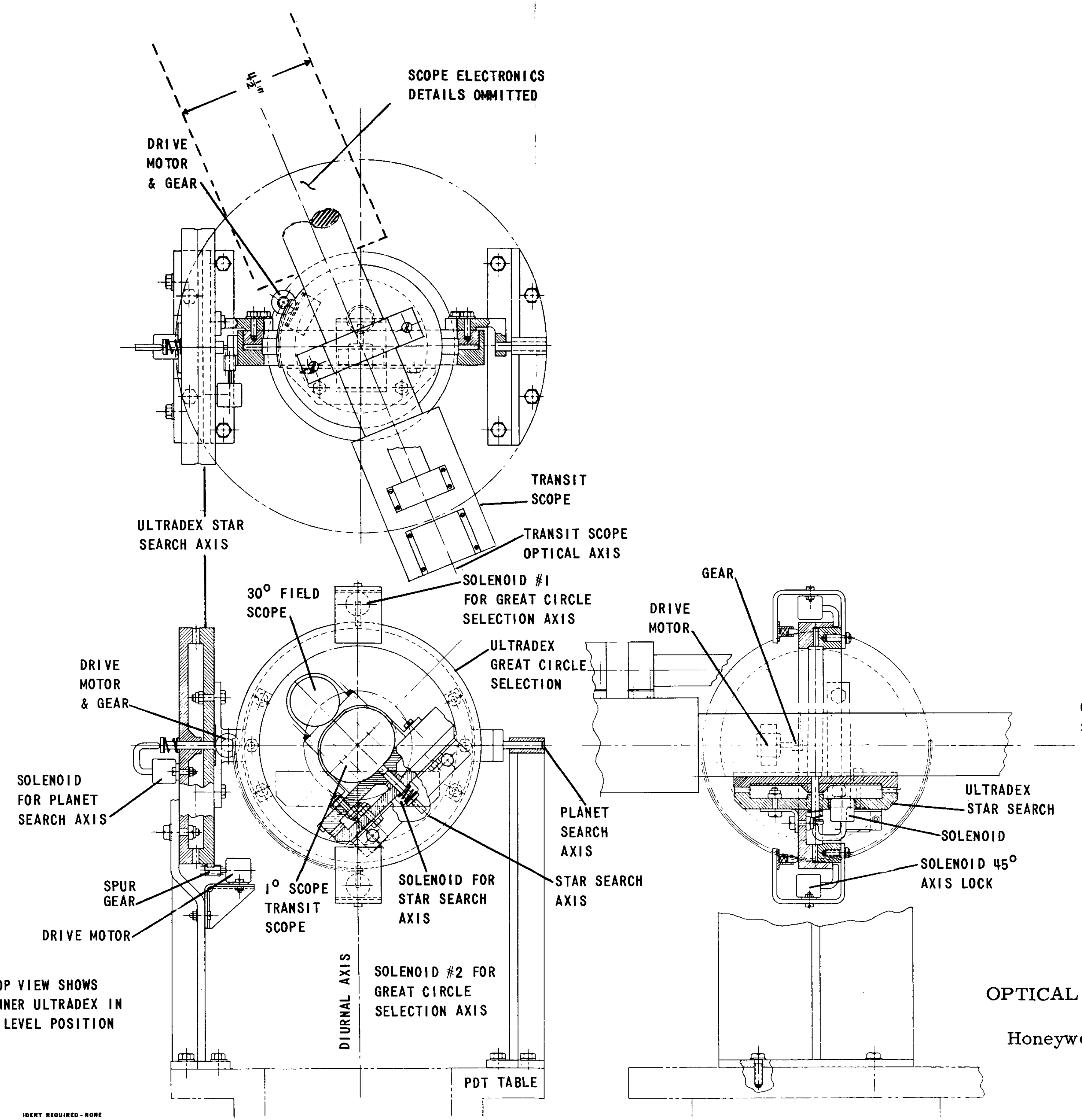
$$\Delta t = \frac{(0.252 \text{ sec})(0.1 \text{ sec})}{3.73 \text{ arc sec}} = 6.75 \text{ milliseconds}$$

Detector Considerations

The photo detector may be one of three types, photo-voltaic, photo-conductive or photo-emissive. Photomultiplier tubes are typical photo-emissive devices; as such they have a high current amplification (10^7) and a high sensitivity. A typical photo-multiplier used by astronomers is the RCA 1P21. Photo-voltaic and photo-conductive devices are typically small chips of silicon and lead sulphide respectively, and are used when small size and low voltages are more important than sensitivity and amplification.

The photo detector must have a minimum usable diameter of 0.35 inches. This is not difficult, as all types of detectors may be obtained at this or larger sizes. However, since detector noise is a function of detector area, some consideration should be given to limiting the detector size to reduce noise.

Detector sensitivity limitations, imposed by dark current, have been computed for photo-multipliers. These tubes have efficiencies of about 10 percent, that is, one electron is produced for every 10 incident photons.



NOTE: TOP VIEW SHOWS INNER ULTRADEX IN A LEVEL POSITION

IDENT REQUIRED - NONE

FIGURE 2-5
OPTICAL INSTRUMENT MOUNTING

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The dark current limitations in the best photo-multipliers at room temperature are 10 to 20 electrons per second per square cm of photo-cathode surface, dropping to one percent of that at dry ice temperatures (-80°C). With a detector diameter limited to the required size of 0.35 inches (0.89 cm) dark currents of 70 percent of the above can be expected. Light from a magnitude 2 star collected by the optics has been computed to be approximately 9×10^{-12} watts. With an optical efficiency of 33 percent, collected light on the photo detector will be 3×10^{-12} watts. Since there are 2.5×10^{18} photons/watt-sec, approximately 7.5×10^{6} photons arrive per second, corresponding to 7.5×10^{5} electrons produced by the photocathode. This number is sufficiently higher than the 10 to 20 electrons produced by dark current, to eliminate detector noise as a fundamental limitation. The situation is not quite the same with other types of detectors, notably photoconductive and photo-voltaic. However, studies have indicated that photovoltaic detectors, of the silicon chip variety, are usable down to stars of magnitude 3 (Reference 2-7). The same study showed that this type device was actually superior to photo-multipliers when very high background illuminations are involved (daylight sky). Although this is not the situation in this program, their extremely small size still makes them attractive. With photo-conductive devices, information from Reference 2-8 indicates that these devices have a noise equivalent power (NEP) of approximately 2×10^{-12} watts/mm and the noise varies with the square root of the area. Thus the NEP for a 0.9 by 0.9 cm cell would be about 1.4×10^{-11} watts, which indicates that the expected noise would be about five times the signal from a magnitude 2 star. Photo conductive devices thus are not recommended for this transit detector.

Background Noise

The limit on system performance is noise. Three types of noise are present, optical or background noise, detector noise and electronic noise, In most applications, the circuit designer can ensure that the electronics contributes little noise in comparison to the detector itself. Thus the major tradeoff is between the star background noise and the detector noise. The brightness to be expected from the star background is considered in the following paragraphs.

The average brightness of the night sky has been computed (Reference 1-21) to be equal to 1.27 magnitude 5.4 stars per square degree. However, peak backgrounds of six times this amount are present in some areas of the sky. This yields a "worst-case" consideration of about 7.6 magnitude 5.4 stars or 6.2 magnitude 5.0 stars. Converting this to equivalent 2nd magnitude stars, shows that for worst-case in the field of view of interest there is one magnitude 2 star which requires transit detection, and 0.386 magnitude 2 star as background. (Note that this is a pessimistic estimate.)

This background will be divided by the reflecting reticle into two parts, one on each of the two photo-detectors. If these parts were equal in in intensity, that is, if the background were absolutely uniform in intensity, they could be cancelled out by the electronics. Unfortunately, they are not uniform, and this poses a limit on the accuracy of detecting the transit. For example, with a 10 percent difference in brightness on the two photo detectors due to brightness fluctuations of the background, there is a 2 percent error in transit of the desired star (see Figure 3-1, Section III). The exact limit which this background brightness poses on the system is dependent on the differences in this background which may be received between the two photo detectors (page 1 - 10 of Reference 1 - 21 shows regions where this difference exceeds 25 percent). Note that the situation is worse if only one sensor is used, since now the uniform nature of the background cannot be cancelled out, unless a computer were programmed for this purpose. In any event, however, the fluctuations would be impossible to take out in this manner.

Star Transit Detector Circuits

Figure 2-7(A) illustrates three types of detection methods. The first method uses only one photo device, and relies on determining the half amplitude point of the signal. The presence of noise, unfortunately, results in an output at the half amplitude of signal plus noise, unless some method for subtracting out the noise is utilized. Since the noise depends on what area of the sky the star is in, this would require computer assistance. Method two utilizes circuitry to follow the amplitude of the larger signal, and then relies on sharpening the resultant dip in signal output at the time of transit. As a result, this approach has the same drawback as the first with a more difficult detection problem. The recommended approach is shown as method three. In this approach, a differential amplifier is used to offer a resultant signal which crosses the zero axis at the instant of star transit.

A typical circuit for differential amplification of photo-voltaic cells is shown in Figure 2-7(B). Two balance adjustments are shown, to balance out differences in the photo device as well as the amplifier.

2.2.5 The 2^{23} bit Dynagon

The dynagon rotor, suspended in its freon-fluid-bearing support, and its corresponding upper and lower code "disk" patterns, all are arranged to have the 12 inch nominal O. D. pattern circle divided into 2^{13} (=8192) sector elements. Application of the identical electronics developed for the 2^{20} bit dynagon (Reference 1-24) permits interpolation between elements to one part in 2^{10} ($=\frac{1}{1024}$ or 9.765625×10^{-4} of an element mark); this represents resolution of the whole circle into 10^{23} parts, where each "bit"

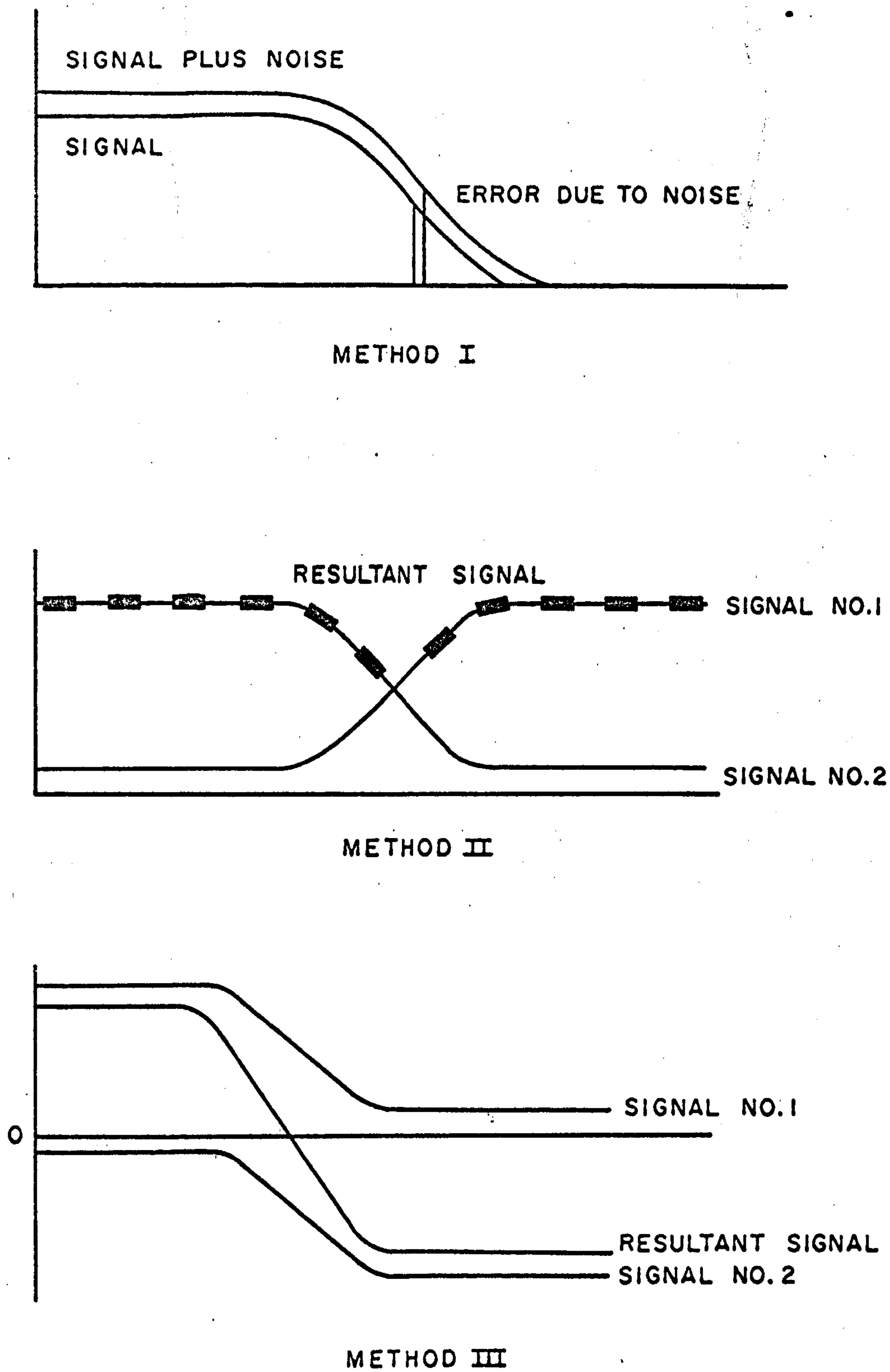


FIGURE 2-7A
METHODS OF DETECTING STAR TRANSIT

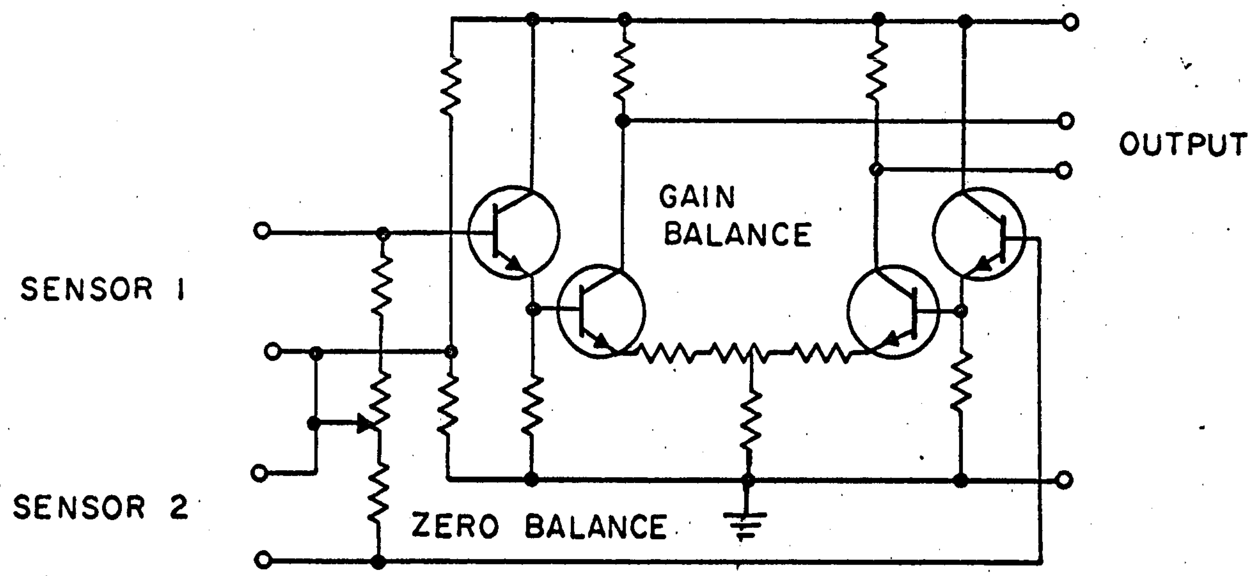


FIGURE 2-7B
TYPICAL BALANCED AMPLIFIER CIRCUIT

represents a circle fraction of 1/8,388,608 or $1.1920928955078125 \times 10^{-7}$. The 1.296×10^6 arc seconds per full circle are thus divided into bits, each of which represents 0.154495239192 arc seconds or roughly 0.15^+ arc second or less than 1/6th of an arc second. The subsystem has an estimated RMS accuracy of 0.25 arc second comparable to the Ultradex detents.

This transducer is in essence merely a physical scaling-up from the present six inch, 2^{19} bit angle encoder. The present 2^{10} interpolating and read out electronics design is not affected by this scale up of the actual transducer.

The 8192 elements or sectors dividing the circle on the two code "disks" would be disposed into three annular arrays such that symmetrical signals would be generated for the balanced input of the electronics. Specifically, an inner and outer band of sectors would be electrically connected such that the combined capacitance, and the combined signal, and the combined space-rate-of-change-of-signal from the two bands would equal the same characteristics of the third band. Physically, the sectors would be nearly 250 microinches wide and the gap between disk patterns would then be set at 125 microinches.

With a 75 RPM (1-1/4 RPS) rotation rate of the fluid bearing rotor "disk" (annulus or ring), the basic frequency generated is 10.24kcs, with an absolute, new angle-datum generated every 0.8 seconds (once per revolution).

The standard Dynagon signal electronics occupying about 10 cubic inches would be used to produce the electrical signals for use in the interpolation-readout system.

The patterns would be processed onto the three, metal-coated ceramic disks (about 3/4 inch thick) with flatness held to within 1/2 wavelength of light (about 11 microinches) and mechanically centered within 25 microinches.

Fluid bearings would be required having tolerances approximately as follows:

Radial < 50 microinch TIR,

Axial < 10 microinch (with maximum settle distance 1/2 mil),

Tilt < 15 microinch in six inches.

The rotor is spun by a rotating magnetic field produced by torquing coils in the PDT table assembly.

The Dynagon principle involves subdividing the circle physically and very precisely in a pattern-form that can be used to generate electrical signals on a physical one-one correspondence, then electrically subdividing the generated signals either on a time basis or physical basis as desired.

Simultaneous averaging of all the pattern elements enhances the accuracy and reduces the error contributions of eccentricity and axial displacements. Two such patterns have sandwiched between them a rotating pattern to generate the electrical signals by capacitance changes. One pattern configuration of each pair is uniquely identified once each revolution as the initial and as the terminal side respectively of the angle to be measured--which is the relative angular position of the two outside pattern plates. This once-per-revolution identification is done by a tiny magnet and a pickoff. The patterns and their electrical equivalents establish a scale factor for the angle; however, if the signal indicating the terminal side of the angle fails between the primary scale signals, then interpolation is necessary, but only for that minute portion of the complete circle. For reasonably constant rotational speed of the central disk, time interpolation becomes practical and especially so if the time length of the interpolation interval is used for automatic calibration.

Any measuring system requires coincidence of marks (fiducials) and a scale. Any complete angle measuring device (goniometer) must provide the initial and terminal fiducials for the angle and some convenient scale that is generally based on the subdivision of a circle into many parts. The accuracy of a goniometer is limited by practical sizes of a circle and of the physical subdivision, together with the inherent difficulties of establishing coincidence of the center of the circle with the pivot points of the fiducial arms. Accuracies can be improved by averaging multiple readings and also by combining data from opposite sides of the circle. In distinction to accuracy, resolution is often added by interpolation between the smallest physical subdivision of the reference circle, but resolution without accuracy is like empty magnification-- it may make the job easier but has added no new information. Hence, interpolation must be used intelligently if both accuracy and resolution are preserved.

For any successful design of a goniometer, the concepts of averaging, fiducials, subdivision of circle, and interpolation must be balanced against the goal specified. In general, the extremes are to be avoided such as few subdivisions of a circle combined with extraordinarily difficult interpolation scheme and, of course, the converse. The coincidence of centers (bearings) of the circle and the fiducials is readily controlled in the former case while the latter (converse) has a severe bearing problem to offset its simpler interpolation.

The Honeywell concept is to subdivide the circle into as many parts as compatible with reproductive technique and to interpolate only as far as stability and accuracy will allow without requiring an unreasonably costly or complex system.

As in Figure 1-14B, Section I, if a set of physical marks can be made to define a field on two adjacent members, and if there is relative motion of the two members, then variations of the field between the two members occur. Appropriate means will sense these variations and give rise to a signal, the wave form of which will be dependent on the control of the field exercised by the physical marking or configuration-- whereby it is possible to effect a one-to-one correspondence of the signal and the pattern configuration. By disposing the field delimiters or pattern in a circular array with each element possessing accurate angular position and form, relative rotational motion of two such array systems now gives rise to signals corresponding to a physical subdivision of the circle. In addition there is gained an accuracy from simultaneous averaging of all the minute errors contributed by each element individually. Moreover, slight shifts or misalignments of axes of the two pattern configuration cause no error in the resultant signal, since opposite sides of the circle compensate. Provided that the patterns are accurate, regular and closely aligned, the resultant signals correspond to an accurate, faithful subdivision of a circle, or in terms of measurements-- there is now an angular scale-- but without any identifying marks or zero.

For the n elements on each circle there are n successive pairings per revolution, none of which can be uniquely identified and used as a fiducial. Use of an auxiliary, once-per-revolution signal (such as might be generated by a tiny magnet and a pickoff) is sufficient to "mark" or identify the fiducial.

With two such pattern pairs on a common axis and with the middle patterns rigidly connected, two signals are generated, the relative phase of which exactly corresponds to the relative angular positions of the extreme disks. (Note also that if there is an angular rate between the extreme disks, the frequencies generated will be different in proportion to the magnitude of the angular rate.)

This combination now has the essence of the Dynagon or any measuring system. There are two fiducials and a scale (one being redundant). Figure 1-14A, Section I, shows the two fiducials and a scale which are directly related to the two "angle" disks with their elements that subdivide the circle. An angle determination consists of counting the number of elements or sectors between the reference fiducial and the input fiducial. Interpolation may be performed if the basic signals have the necessary accuracy for higher resolution.

As the interpolation is only required during one $\underline{n}$ th part of the circle, and as mechanical inertia of the rotor prevails, time can be used as an interpolation means. Time interpolation determination consists of counting the time elements between limits of the sector interval (that is, calibrate the the interval) and simultaneously counting the time elements between the input fiducial mark and one of the interval limits. The ratio of the counts determine the fraction of the sector to be included in the integral sector count to make up the total angle digitally. The ratio or calibration of the interpolation interval makes the result independent of the exact frequency of the generated signals or the rotor speed.

Note now that all factors of measurement are present: two marks or fiducials and a scale. With appropriate electronics that have been developed, an accurate signal (which is averaged from all the physical marks simultaneously) is present for the initial, the terminal, and the scale values of an angle. A tallybox with some simple logic accumulates the number of sectors between the initial sector and the terminal sector of the system: the tallybox must also indicate the count for any interpolation scheme. Consequently, an angle is determined digitally with the usual digital resolutions error of ±1 count in the least significant place.

Some of the important characteristics of the 2^{23} bit Dynagon are:

- Unlimited angular range.
- Self calibrating each 0.8 seconds.
- Absolute readout, non-incremental, new independent datum every 0.8 seconds.
- $<1/6$ arc second resolution (10^{-23} of a circle).
- Digital output, parallel or serial, no analog to digital converter required.
- Better than 0.25 arc second RMS accuracy.
- Total reliability of data (non-incremental and hence non-volatile with respect to power interruptions).
- Small physical size for the accuracy involved.
- Yields both angle and angular rate signals with the standard electronics.
- Rate ranges 1800 degrees per second to as little as 0.0001 degrees per hour, between input and output "shafts".

- Provides a signal at the instant when the angle is determined.
- Incremental data also is available at a 5kcs rate (in one readout system) or continuously available for computer. interrogation with an alternate system.
- Electronics fully transistorized capable of being custom packaged into a volume less than a three inch cube (27 in^3).

Exclusive Practical Factors

Two important factors that determine the practicability of the particular principle and version chosen for development have been successfully considered in the Dynagon design.

The first has been the development of a combination of techniques to produce by photo-reproductive processes, a precise, accurate and regular relief pattern. This pattern subdivides the circle into many fine parts (sectors) having a particular relief profile suitable for generating an electrical signal from impedance variations when moved relative to a complementary pattern. Figure 2-8 is a photomicrograph of the profile section of such a pattern.

The second factor has been the design and execution of special signal processing circuits in which the main emphasis has been to preserve the angle information (phase) regardless of other interfering factors. (Stability of phase has been demonstrated to at least one part in 8,000.) The signal processing can withstand relatively large distortion and noise since only the cross-over portion of the signal is used. This two-points-per-cycle requirement is less stringent than all points of a waveform per cycle so often employed.

Figure 2-9 shows a gross, functional block diagram of the PDT Dynagon electronic system for reading out angles and ephemeris time of transit to the navigation computer. Greater detail is to be found in Reference 1-24, Section I.

Reliability Prediction For Dynagon

A preliminary reliability study has been performed to determine the mean time between failures of the Dynagon configuration. The results of this study show that the Dynagon and its associated signal electronics (exclusive of the logic circuitry associated with the "absolute" readout feature) will demonstrate a mean time between failure of 76,700 hours, even with welded module electronics.

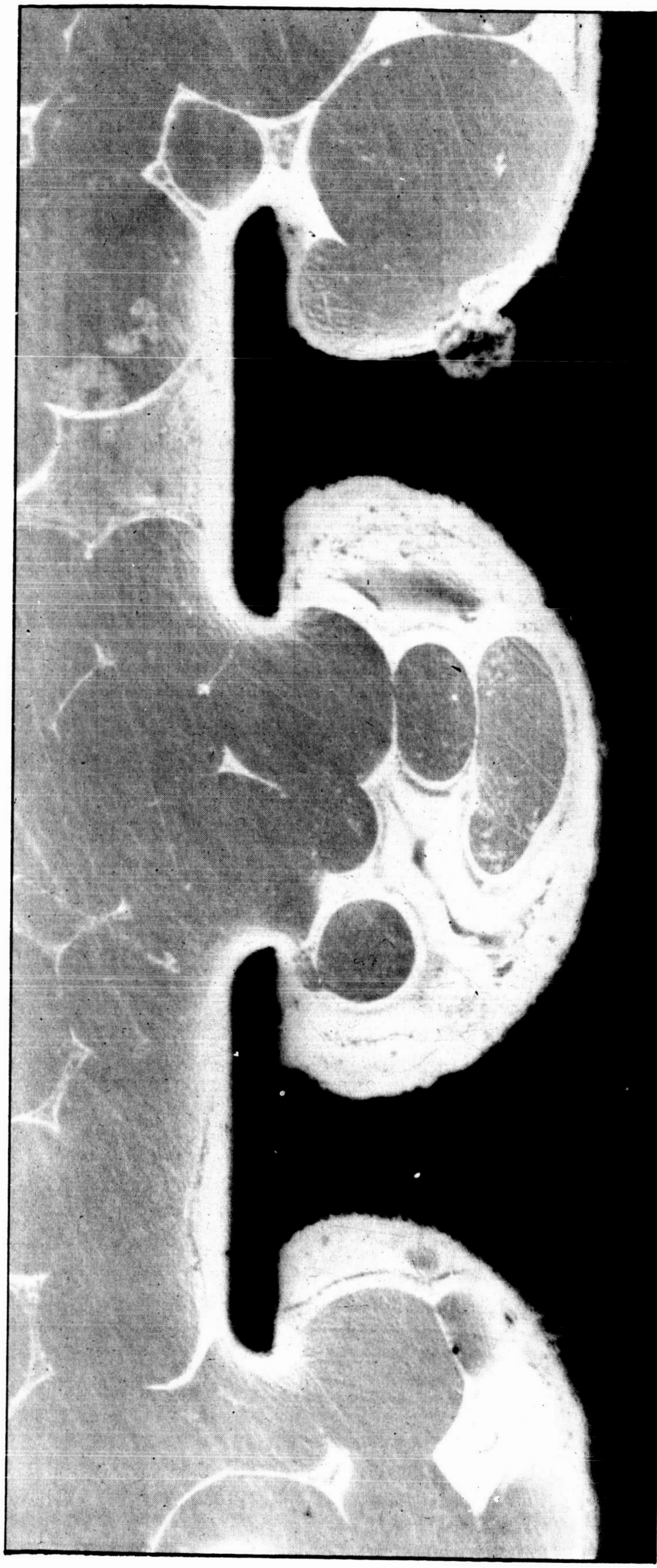


FIGURE 2-8
PHOTOMICROGRAPH OF RELIEF PATTERN BEING USED
(BLACK AREA IS METAL, GREY AREA CAVITY)

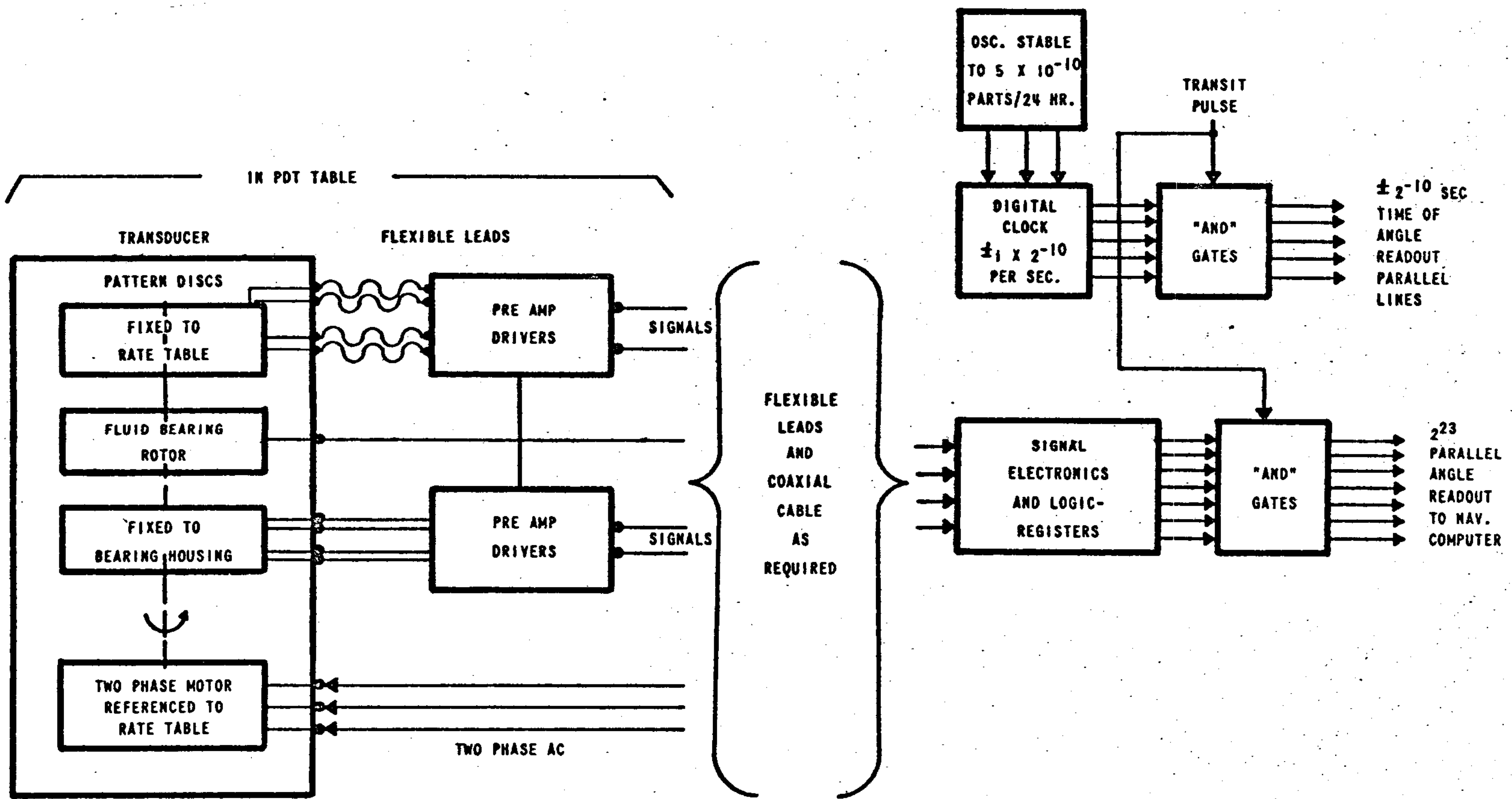


FIGURE 2-9
BLOCK DIAGRAM OF DYNAGON
ANGLE AND TIME READOUT

The reliability study further indicates that the predominant failure mode of the Dynagon configuration will be electrical as opposed to mechanical. Due to the low speed of the Dynagon rotating mechanism and use of a fluid bearing, wear will not contribute significantly to the over-all failure rate. Exhaustive tests conducted by Honeywell on precision bearings for inertial components indicates a Dynagon bearing MTBF of 19.2 x 10^6 hours with a normal distribution. Due to the normal distribution, 90 percent of the bearing will last at least 9.6 million hours. It is obvious, therefore, that the bearing failure rate within the 76,700 hour MTBF of the complete Dynagon configuration is, for all practical purposes, zero. Failure of the fluid bearing could, of course, be coincident with loss of fluid or compressor failure which are repairable conditions in flight.

The following table shows the reliability apportionment among the various functional elements of the Dynagon.

	Total F/R %/1000 Hours	Qt'y	Total F/R %/1000 Hours	MTBF (Hours)
Preamplifier (C)	0.035	2	0.070	1.43 x 10^6
Preamplifier (F)	0.085	2	0.170	588,000
Mag. Sig. Crossover Det.	0.142	2	0.284	352,000
Stat. Sig. Crossover Det.	0.208	2	0.416	241,000
Filter	0.094	2	0.188	532,000
Motor (Elec. Failures)	0.075	1	0.075	1.33 x 10^6
Magnetic Pickups	0.050	2	0.100	1.00 x 10^6
TOTAL			1.303	76,700

2.2.6 The Pseudo-Pole-Star Tracker

The pole star tracker is rigidly referenced to the common frame of the main PDT instrument bearing. This is not a star tracker in the sense that something is physically torqued in two axes to point at a particular star. Rather, a 40-inch focal length 1.0 degree field of view telephoto lens, is arranged with its optical axis precisely co-aligned with the PD axis. The field of view image is dissected by electromagnetic scanning of a sensitive spot (image of a mask aperture in the photo-multiplier tube) over the surface of the photo-emissive cathode in an ITT type FW 118 image dissector phototube. The rms spot resolution claimed is on the order of 7200 lines (by assuming an 800 cps clock rate and filtering) with error on the order of ±0.5 arc second. The efforts of General Precision, Inc., are recognized for having constructed a working model of this type of tracker (the instrument is described in an article in Electronic Design News, May 1962, reprinted as Appendix 2-A in this report).

Reference 2-5 discusses a similar development at the Bendix Corp. Eclipse Pioneer Division in Teterboro, New Jersey. The Bendix tracker has a claimed 15 arc second resolution in a four degree field of view, but is chiefly designed for stabilization systems. Narrowing the field optically to 0.5 degree should give a resolution on the order of two arc seconds. Honeywell's star tracker program has considered image dissector designs also (reference 1-21, pages 8-57 to 8-66).

The image dissector tracker was not actually designed as a part of this phase of the study. Knowing that its design is expected to be within reason in the 1970-75 time period permits one to assume a ±0.5 arc second error in stabilization reference. If necessary, a two stage tracker (1.0 degree field gross track and 0.1 degree field fine track) could be designed for applications where weight and size of the required optics is allowable.

It is recognized that in the PDT system mode being considered, the accuracy of line-of-position determinations hinges largely upon the stability and readout accuracy of the attitude monitor star trackers. In fact, one can say quite safely that if star tracker reference accuracy is no better than "±X" arc seconds, then a line-of-position could not be taken to an accuracy better than "±X" by the method assumed in this report. Adding further to this, if X exceeds 12 arc seconds, one would be as well off to use a sextant and a set of star co-altitude curves. The importance of this problem has resulted in heavy stress upon star tracker development at Honeywell Aero, as evidenced by Reference 1-21 and the follow-on efforts to that contract.

2.2.7 Auxiliary Star Tracker (AST)

In order to provide signals to the computer for correcting the measured angles for minor vehicle rotations about the pseudo-diurnal axis (causing minor variations in pseudo-right-ascension, during a series of sightings) an auxiliary star tracker must be provided; this subsystem is mounted at some convenient location, near the PDT observing instrument dome, and to a rigid frame member common with the PDT bearing table mounting ring. Its "zero, zero" optical axis is nominally at 90° to that of the pseudo-pole-star tracker. Since there is no guarantee that a suitable pseudo-right-ascension reference star will exist at precisely 90° from the pseudo-pole-star chosen, it is necessary to gimbal the auxiliary star tracker in pseudo-declination. For convenience in selecting the stabilization reference star, the auxiliary tracker also is gimbaled with limited freedom (±40 degrees) in pseudo-right ascension.

The required electronics of the auxiliary tracker is practically identical to that of the pseudo-pole-star tracker.

The auxiliary star tracker is basically a detented gimbal-mounted image-dissector photomultiplier (PM) tube and associated electronics designed to track first magnitude stars and reject less bright stars. The instantaneous field of view is ±1. 0 degree in each of two orthogonal axes and the gimbaled field of view is ±40 degrees in both axes. Figures 2-10 and 2-11 show conceptual sketches of the auxiliary star tracker unit.

Rather than gimbal the photomultiplier tube and image dissector, wire twisting is avoided by Ultradex detent gimbaling of a front surfaced tracking mirror in two axes. The primary function of the Ultradex detents is to provide ±1/4 arc second repeatability to look directions; once set stability is estimated in the thousandths of an arc second. This repeatability and stability are bought at the price of a one degree minimum stepping increment. Solenoids are provided for latching and unlatching the detents. In the unlatched position the mirror is movable by pull-turn hand cranks in two axes over a maximum range of ±40 degrees. This permits manual acquisition of a first magnitude star. The ±10 arc minute dead zone steps of the vehicle's altitude control system (assumed) may be used to center the star visually upon the cross hairs of the eyepiece reticle, which is pre-aligned with the center point of the photomultiplier, image dissector, tracker. (It is possible also, of course to use a manually torqued reaction wheel as a vernier attitude control for the same purpose).

Having manually set the reference star upon the nominal reticle center, the deviations of this star from its location at the beginning of a series of measurements are read out by the computer from the image dissector tube, and any change (in pseudo-right-ascension) is applied as a correction to the Dynagon measurements being made. To this extent, the star is kept track of (rather than being actively tracked) during any series of observations. If the star wanders more than 10 arc minutes from its reference location, commands are sent to the attitude control system to bring it back toward center (only during observations).

Stepping of either the PD or declination axis solenoid causes the mirror to rotate in one degree increments repeatable to ±1/4 arc second. Once set, the Ultradex is effectively rigid as a solid member. Stepping may be in either direction and can be commanded remotely by the computer as a discrete signal or manually by button keys (or selector switches).

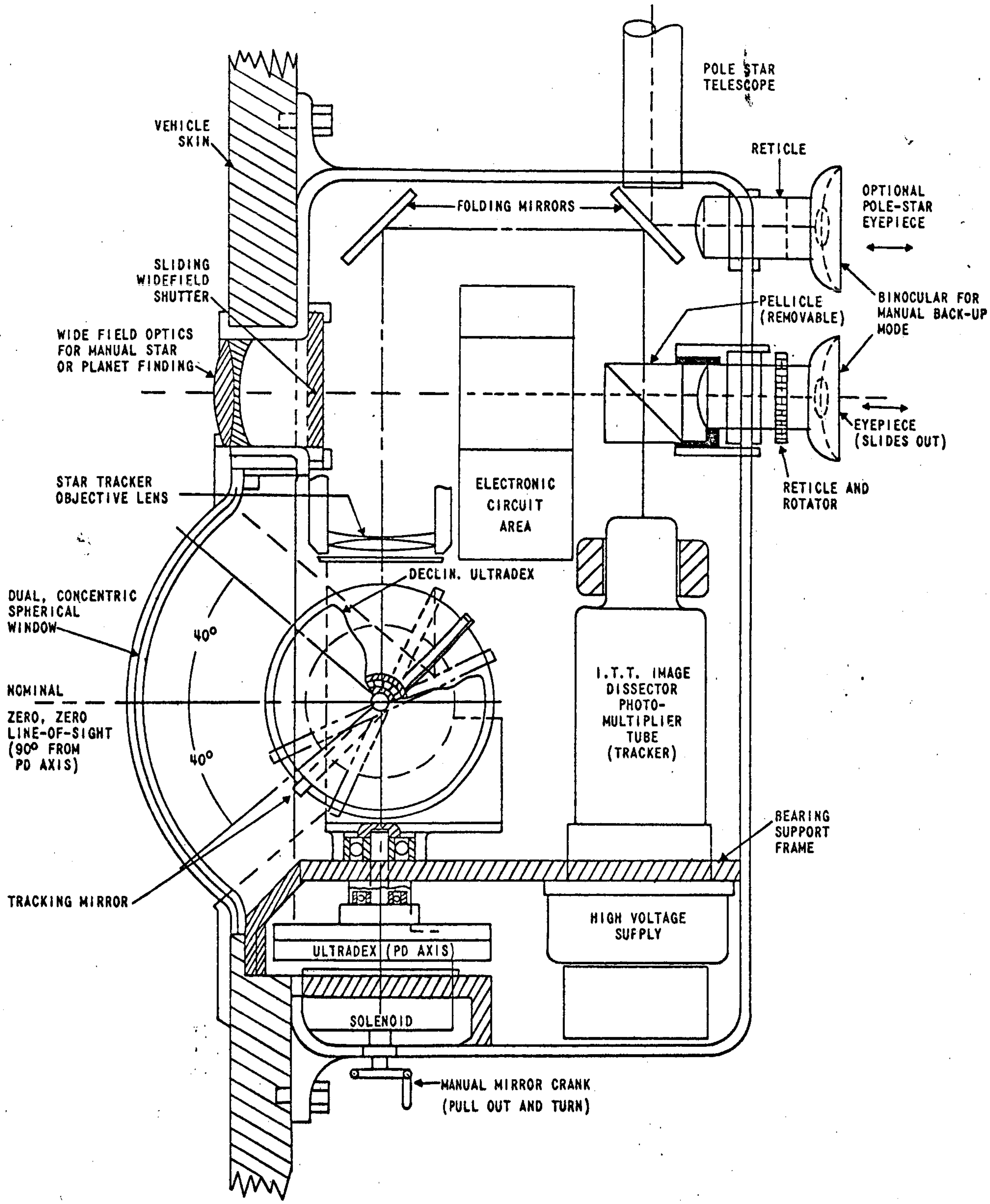


FIGURE 2-10
AUXILIARY STAR TRACKER (SIDE VIEW)

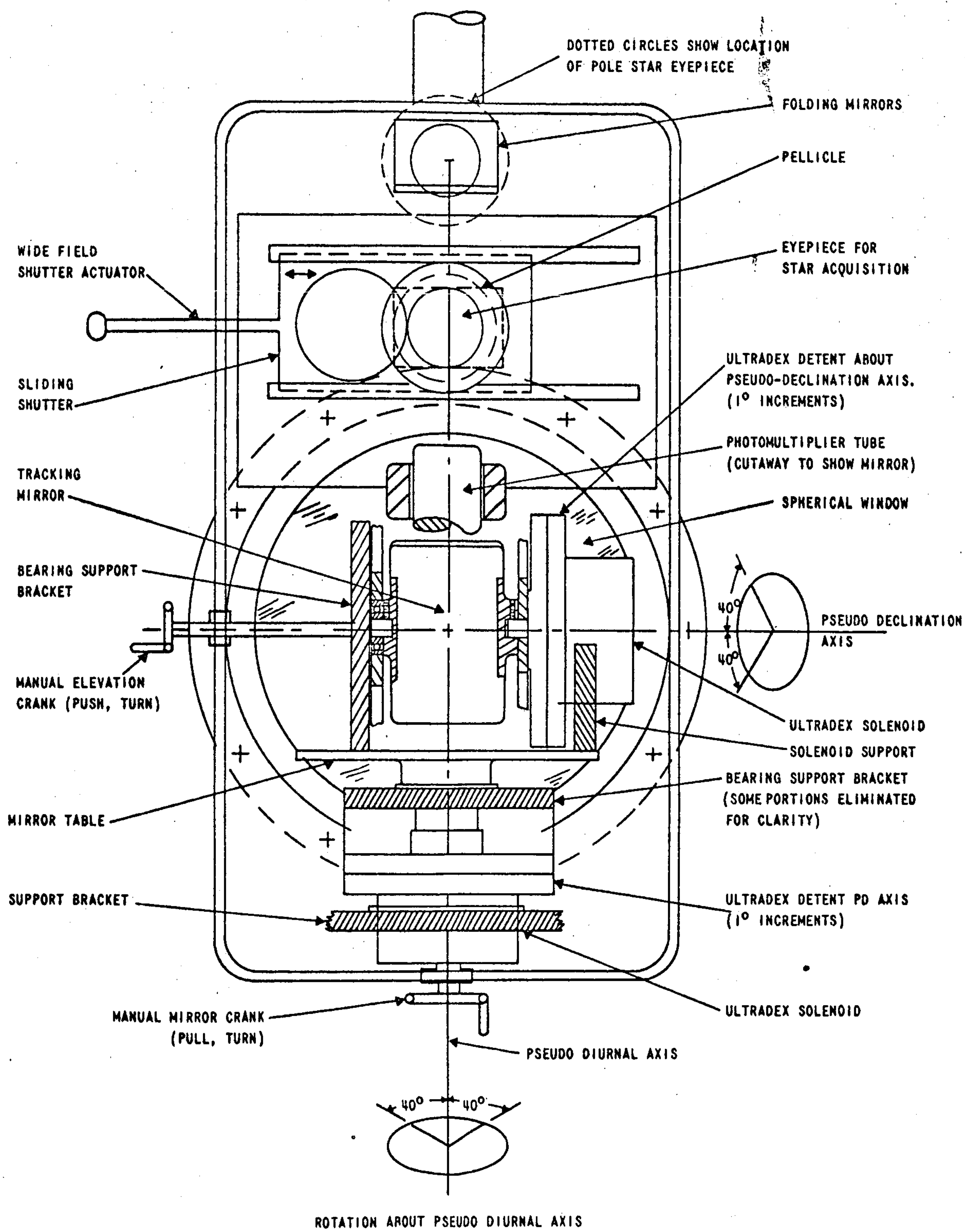


FIGURE 2-11
AUXILIARY STAR TRACKER (REAR VIEW)

The main eyepiece is arranged to permit either a direct, wide-field viewing telescope, or view through the star-tracker-optics, one degree telescope, or both in the manual mode. Moving a sliding shutter from the wide field objective permits it to couple directly to the eyepiece through a hole in the electronics package. Sliding the pellicle into its normal position permits superimposing the one degree field upon the wide field. Closing the objective shutter removes the wide field view. Sliding out the eyepiece removes the pellicle from the star tracker optics path for automatic (normal) tracking. A reticle marker circle on the pellicle designates the one degree field when it is superimposed upon the wide field. However, aside from mirror angle settings, no means exists for knowing where (in the wide field) the one degree field is pointed.

2.2.8 The Manual Back-Up Optics

It is to be noted that the auxiliary tracker unit conceivably can be used as a manual transit detector in a "fall-back" mode; the method of operation would be quite similar to that described by Kardashian in his original disclosures (Appendix I-F). A manually rotated spider reticle in the eyepiece would be applied to observe "iso-azimuthal transits" using only the auxiliary star tracker optics and gimbals, a stop watch, manual calculation aids and a gimbaled reaction wheel. To produce the pseudo-diurnal rate, the chosen pole star is centered upon the reticle of the pole-star-eyepiece using the gimbaled reaction wheel to "jockey" the star. With the star centered the reaction wheel is aligned by trial and error to have its axis parallel to that of the pole star, whereupon the vehicle altitude rate is established by cranking to store angular momentum in the wheel, setting up a slow, counter-rotation of the vehicle about the pole star axis. The rate is then repeatedly measured by timing the passage of known stars across the auxiliary tracker's reticle. Note that wide field, search optics are applied when the pellicle is in place for manual operation. Rotating (or sliding out) motion effectively removes the pellicle to avoid interference with star tracking in the automatic mode. A shutter is applied to mask the wide field in this mode also. It is felt that this general, manual method should be regarded only as a back-up in the event of total failure to the semi-automatic system. Lines-of-position by this method eventually might be obtained to accuracies as good as ±3 arc seconds and as poor as ±27 degrees depending upon rate sensing accuracy, distances from the planet and torque disturbances within the vehicle at the time of observation (see paragraph 3.5). In this manual, back-up mode, it is interesting to note the remarkable similarity between the auxiliary star tracker optics and the usual, single-star, space sextant. This leads one to suggest that proper design of manual or automatic space sextants would permit eventual, in-space trials to be made of the back-up PDT technique without additional hardware; possibly an experiment of this type could be planned for a Gemini, Apollo or Dyna-Soar flight with no increase in required payload.

REFERENCE FOR SECTION 2

2 - 1 "ADEPT" Computer Series 7 October 1963.
Honeywell Aero Document R-ED 24063

2 - 2 "Honeywell PICO Subminiature Digital Computer"
Honeywell Aero Document U-ED 29062-A

2 - 3 "Designing A Spaceborne Computer"
W. A. England, pages 12-18 of April 1963 issue Computer Design.

2 - 4 "Honeywell PICO Computer Presentations" 1963
(Copy supplied to NASA Langley)

2 - 5 "Bendix Star and Planet Tracker Program Locks on New Detector Device"
Michael Getler, missiles and rockets October 21, 1963, pages 24 and 25.

2 - 6 "Inertial Navigation"
R. H. Parvin - D. Van Nostrand 1962

2 - 7 "Advanced Celestial Tracking Instruments for Navigation"
Honeywell Aero Report No. 11694-IR-2 (Confidential)

2 - 8 "Lead Sulfide Photo Conductors: A State of the Art Report"
Infrared Industries, Inc., Technical Bulletin No. 2.

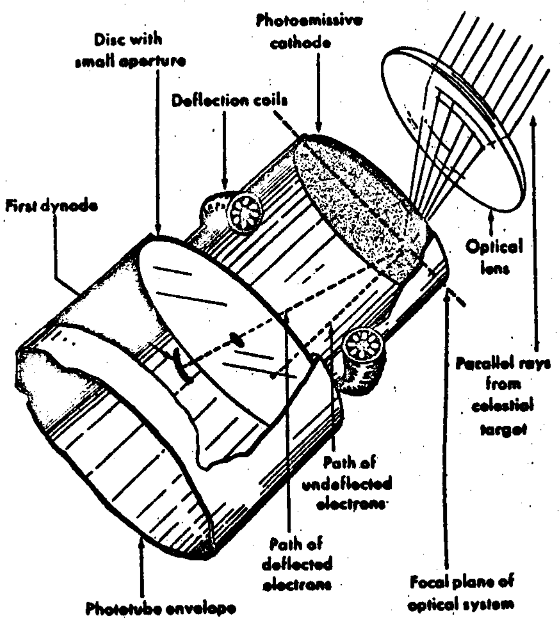
APPENDIX 2-A

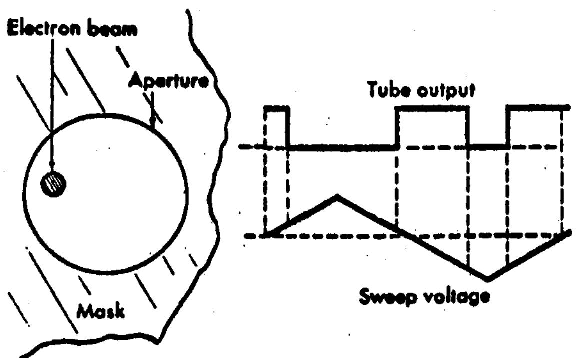
TO SECTION 2

PHOTOMULTIPLIER TUBE,
DUAL SCANS PINPOINT CELESTIAL TARGETS

REPRINTED FROM ELECTRICAL DESIGN NEWS - MAY 1962

OPTICAL SYSTEM receives light from star in its field of view and focuses rays on photocathode. With no coil signal, electrons emitted from photocathode leave in paths parallel to centerline of tube. If path is not deflected, electrons are blocked by disc which masks first dynode. Now, with judicious application of current through coil windings, electron path may be directed through aperture, and output from phototube results. Amount of current necessary in X and Y coils is measure of coordinates of image on photocathode.





A

B

GLENDALE, CALIF.—The flat, circular photocathode of a photomultiplier tube is in the image plane of an optical system. Electrons resulting from secondary emission of the photoemissive surface leave the photocathode in parallel streams. Near the first dynode is a disc with a small aperture. Deflection coils which control the space between the photocathode and first dynode, if properly excited, can cause the electron stream emitted from any point on the cathode to pass through the small aperture.

A variety of sweep schemes may be used to energize the coils and obtain information about the position of points of light on the photocathode. The most satisfactory combination has been a spiral sweep for gross examination of the photocathode and a cross-shaped sweep to provide precise error information for centered targets.

This basic scheme is being used in an electronic star tracker developed by General Precision, Inc. The tracker, when using a 40-inch focal length optical system with a 1-deg field of view, is capable of pointing accuracies of better than 1/2 sec of arc. The only mechanical devices necessary are those used to reorient the vehicle in which the tracker is mounted. Phototube is ITT type FW 118.

TWO OPERATING MODES use two sweep schemes. During **SEARCH MODE,** sweep is expanding, then contracting spiral. It appears on scope as pulsating circle. With sine function applied to X deflection coils and cosine function applied to Y coils, an open Lissajous pattern is formed. These are modulated by triangular wave so that scope pattern diminishes to dot, then expands to circle alternately. Z axis modulation of oscilloscope trace shows position of image on phototube in PPI fashion.

For **LOCK-ON MODE,** deflection coil sweep is in form of cross. Assume that tracker has been driven so that light from target is nearly parallel to optical axis. Electron stream then may be passing through aperture as shown in Fig. A during quiescent sweep conditions.

If beam is swept left, back to rest then right, and back to rest, in equal amounts, phototube will have output shown in Fig. B because beam is masked earlier on sweep left than on sweep right. Phototube output contains information on left-right position of image. Sweep up and down similarly provides information on vertical position of image. Output of phototube is phase inverted and gated for processing (see block diagram).

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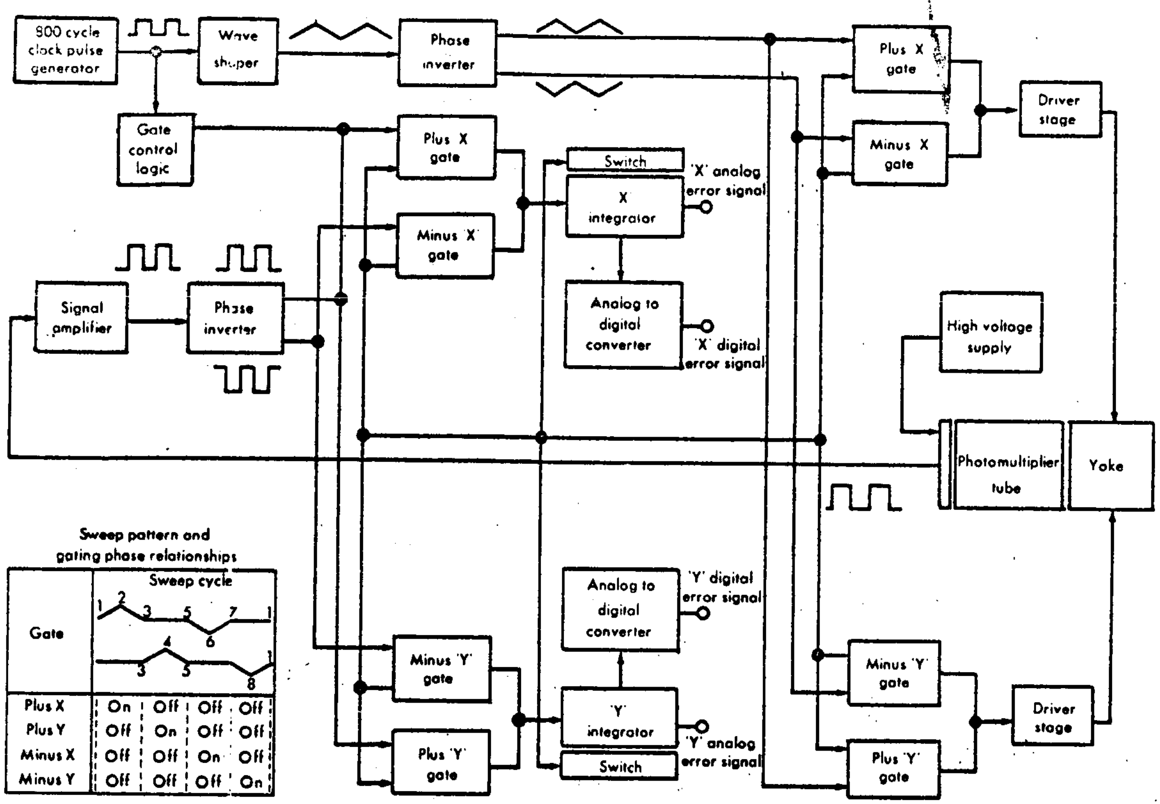

BLOCK DIAGRAM shows 800-cps master clock, sweep controls and information processing. Clock drives wave shaper and gate control logic. Gates determine whether positive or negative signals from inverters are passed. If sweep is going in positive direction, signal is positive; signal is inverted during negative-going sweep. Signals passing through gates drive integrators which provide analog error signals.

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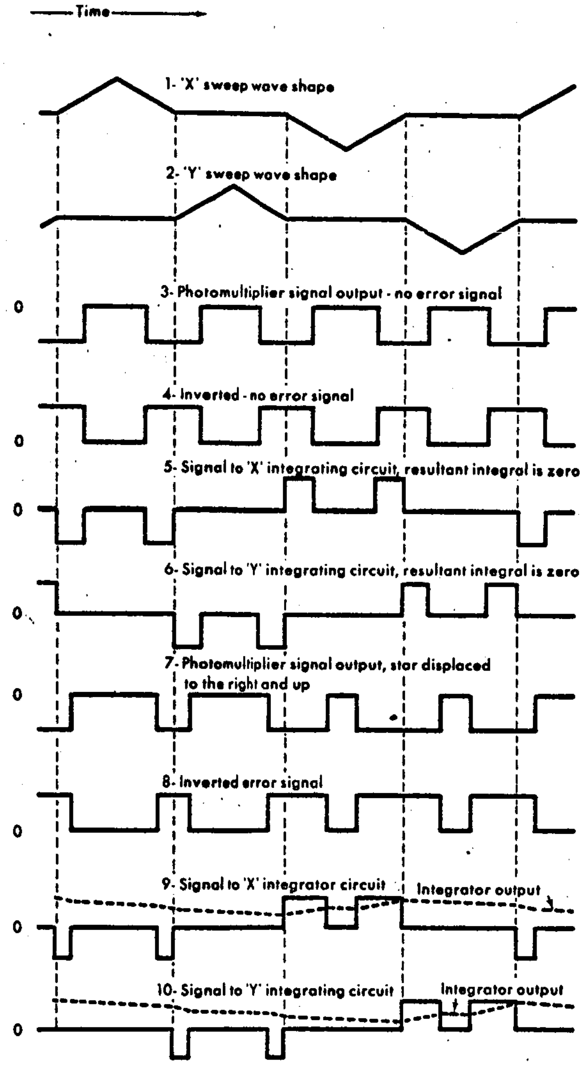

SYSTEM WAVEFORMS explain error signal generation. With star image in exact center of cathode, phototube outputs during positive and negative X sweeps are equal. Since output is inverted during negative portion of sweep, integral is zero. Same is true for Y sweep. If image is now displaced up and to right, positive and negative portions become unbalanced and integrators produce error signals proportional to coordinates of image displacement.

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SECTION 3

3.0 ACCURACY ANALYSIS

This section discusses the error sources examined and estimates their contributions to transit angle readout accuracy and accuracy of a single line of position. The estimated error in a single line of postion (using the instrumentation described in Section 2) is 3.6 arc seconds. The error sources are tabulated and individually discussed.

An approximate analysis of the original PDT manual scheme is included in paragraph 3.5.

3.1 TABULATION OF ERROR SOURCES

Table 3-1 is a tabulation of the individual error sources and arrives at "root-sum-square" estimates of the error per transit in three cases (first star pair, planet and second star pair). There are certain errors present in transiting the star pair a second time which are not present the first time due to the necessary rotation of the optics to the second "great circle" reference; therefore these had to be considered separately.

3.2 ESTIMATE OF LINE OF POSITION ERROR

Table 3-2 presents the analysis of the effects of individual transit errors upon a single line of position observation and concludes that the estimated error in a single, mid-course line of position would be ±3.6 arc seconds(RSS).

3.3 ESTIMATE OF EXTREME ERROR

Table 3-3 merely sums up all the extreme error estimates, times the number of times they could apply, in the worst of all possible cases (where all errors are of the same sign and are additive). It should be recognized that the probability of this event occurring approaches zero quite closely. Nevertheless, for the sake of reference only the maximum conceivable error in a single line of position for the stated conditions is 24.729 arc seconds.

TABLE 3-1
TABULATION OF ERROR SOURCES PER BODY TRANSIT
PDT NAVIGATION SYSTEM ACCURACY ANALYSIS

ERRORS PER TRANSIT	First Star-Pair Transits		Planet Transit		Second Star-Pair Transits	
	Error ~ Arc Sec	Square ~ Arc Sec^2	Error ~ Arc Sec	Square ~ Arc Sec^2	Error ~ Arc Sec	Square ~ Arc Sec^2
A. MECHANICAL						
1) Stabilization:	0.74	0.5476	0.74	0.5476	0.74	0.5476
2) Table Axis	0.14	0.0196	0.14	0.0196	0.14	0.0196
3) Telescope Gimbals						
a. Planet Search Axis	0.05	0.0025	0.05	0.0025	0.05	0.0025
b. Great Circle Selection Axis	0.009	0.000081	0.009	0.000081	0.009	0.000081
c. Star Search Axis	0.10	0.01	--		0.10	0.01
4) Dynagon Error	0.25*	0.0625	0.25*	0.0625	0.25*	0.0625
B. OPTICAL						
1) Star (Electronic)	0.20*	0.04	--		0.20*	0.04
2) Planet (Electronic)	--		1.06*	1.1236	--	
3) Planet (Radius Ratio)	--		1.0	1.0	--	
4) Image Aberration (Star)	0.30	0.09	--		0.30	0.09
5) Reticle Alignment	0.20	0.04	0.20	0.04	0.20	0.04
6) Axis Alignment (a, b, c and d mechanical)						
e. Optical versus mechanical	--				1.40	1.96
f. Mech. axis versus reticle	--				1.20	1.44
7) Reticle Edge Roughness	0.20*	0.04	0.20*	0.04	0.20*	0.04
C. METHOD						
a. Relative motion of Vehicle and Planet	0.12	0.0144			0.12	0.0144
b. Ephemeris Errors (neglected)						
SUMS:	2.309	0.866681	3.649	2.835881	4.909	4.266681
Averages (Rounded off):	0.21	0.0788	0.405	0.3151	0.378	0.3282
Square roots of avg. of squares (Rounded):		0.2807		0.5613		0.5729
Square roots of sums of squares (Rounded):		0.93096 Arc Sec		1.68401 Arc Sec		2.06559 Arc Sec

* Indicates Errors Subject to Improvement By Multiple Transits.

TABLE 3-2
ESTIMATE OF EFFECT OF TABULATED ERRORS UPON OBSERVATION OF A SINGLE LINE-OF-POSITION AT MID-COURSE BY THE MODIFIED PSEUDO DIURNAL TRANSIT TECHNIQUE

Operational Source	Per Transit RSS Contribution (Arc Sec)	Times Applied	Squares	Listed Square Contribution
Planet Transit	1.68401	1	2.83589	2.83589
First Star-Pair Transits	0.93096	2	0.86669	1.73338
Second Star-Pair Transits	2.06559	2	4.26666	8.53332
Sum of Squares:				13.10259
Square Root of Sum of Squares:				3.61975
Estimated Error Per Line of Position:				3.6 arc sec

TABLE 3-3
ESTIMATED EXTREME ERROR IN DETERMINING A SINGLE LINE-OF-POSITION AT MID-COURSE BY THE MODIFIED PSEUDO-DIURNAL TRANSIT TECHNIQUE. WORST POSSIBLE CASE (ALL ERRORS ADDITIVE)

Error Source By Paragraph Code	Max. Est. Value Arc Sec.	Times Applied	Totals Arc Second
A. 1.)/(3.4.1.1)	0.74	5	3.7
A. 2)/(3.4.1.2)	0.14	5	0.7
A 3) a/(3.4.1.3)	0.05	5	0.25
A 3) b/(3.4.1.3)	0.009	5	0.045
A 3) c/(3.4.1.3)	0.1	5	0.50
A 4) /(3.4.1.4)	0.25	5	6.25
B 1)/(3.4.2.1)	0.2	4	0.80
B 2) a/(3.4.2.2)	1.06	1	1.06
B 2) b/(3.4.2.2)	1.2	1	1.20
B 3)/(3.4.2.3)	1.0	1	1.0
B 4)/(3.4.2.4)	0.3	4	1.20
B 5) a /(3.4.2.5)	0.2	4	0.80
B 5) b $\delta\theta_0$ /(3.4.2.5)	0.5	1	0.5
B 5) b $\delta\beta$/(3.4.2.5)	0.5	1	0.5
B 6) e /(3.4.2.6)	1.4	2	2.8
B 6) f /(3.4.2.6)	1,2	2	2.4
B 7) /(3.4.2.7)	0.2	5	1.0
C 1)/(3.4.3.1)	0.012	2	0.024

Total of Extreme Error Estimates	24.729 arc sec.
1/7 of Extreme	3.5327

3.4 RECAP AND DISCUSSION OF ERROR SOURCES

3.4.1 Mechanical Errors

3.4.1.1 Star Tracker Error

A.1.) Vehicle attitude stability accounted for by error in star trackers is ±0.5 arc seconds on each axis (with a 1.0 degree field of view). Design for a 20 arc minute field of view conceivably could reduce this to ±0.2 arc second per axis. This would yield 0.74 arc second maximum for the 1.0 degree field or 0.28 arc second maximum for the 20 minute field. As applied to any single angular measurement, this could be ±1.48 arc seconds (extreme) for the 1.0 degree field or ±0.56 arc seconds (extreme) for the 20 minute field. The likely error in determining a single angle is (RSS) ±0.98 arc seconds for the 1.0 degree field and ±0.396 for the 20 minute field.

3.4.1.2 Table Axis

A.2.)
- a. Wobble ≅0.1 arc second maximum
- b. Radial displacement ≅ 0.1 arc second maximum
- c. Axial displacement ≅0.1 arc second maximum

The maximum error contribution on a single transit is estimated as ±0.14 arc second. The maximum possible effect for any one angle measurement is ±0.28 arc second.

3.4.1.3 Telescope Gimbaling

A.3.)
- a. Planet search axis due to estimated Ultradex variability: ±0.05 arc second.
- b. Great circle selection axis due to estimated Ultradex variability: ±0.05 arc second. This error is calibratable and the error influence as shown by I. G. Foster's 45 degree line analysis is a 60:1 approximate ratio. This axis can be calibrated to 1/2 arc second easily, so the error could be something less than 0.009 arc second and is considered negligible.
- c. Star search axis: ±0.1 arc second. Some hardware evaluation will be required to prove this error. The Ultradex is guaranteed to ±0.25 arc second in rotational accuracy, and its plane repeatability, which is the only factor of interest here, should be considerably better than ±0.25 arc second, since it is the

repeatability function of 360 teeth nested into 360 teeth. The estimated effect upon a single angle measurement is ±0.14 arc second.

3.4.1.4 Dynagon Error

A.4.) ±0.25 arc second per transit or an RSS for a single angle measurement of ±0.354. (See paragraph 2.2.5 of Section 2.)

3.4.2 Observing Instrument Errors

3.4.2.1 Star Transit Error

B.1.) Electronic accuracy is a function of amplifier drift, photo-cell stability, photo-cell tracking and zero crossing error. The major difficulty is in the differential amplifier.

Since feedback cannot be employed in this type of amplifier, it will tend to drift after being balanced. Experience indicates that the difference in amplification may be as high as 5 to 10 percent. Long term stability on photo-voltaic cells is in the order of 1 to 2 percent. A second problem with these cells is getting them to track over the input range of interest.

Typical unbalance over this limited range is in the order of another 2 percent. The circuitry for zero crossing will contribute about 0.5 percent error, depending on the signal levels involved. The RMS of all these errors will contribute a variation of 7 to 11 percent in voltage, or 3.5 to 5.5 percent error in timing. Since the time of transit is approximately 0.252 second, an error in timing of 0.01133 second will result. Converting this to star angular error yields about 0.168 arc second.

This electronic error must be coupled with the error from the star background brightness (and the mechanical errors involved, such as positioning the reticle). The background error is much the larger; it depends upon the section of sky where the navigation star is located and can reach 0.2 arc second (extreme).

Figure 3-1 shows the manner in which background noise errors contribute to transit timing error.

The error noted above is considered only once per angle determination.

3.4.2.2 Planet Transit Error

B.2.) Refer to Figure 3-2. Edges A and B represent reticle edges (See paragraph 2.2.4 of Section 2.) The two edge transits are represented

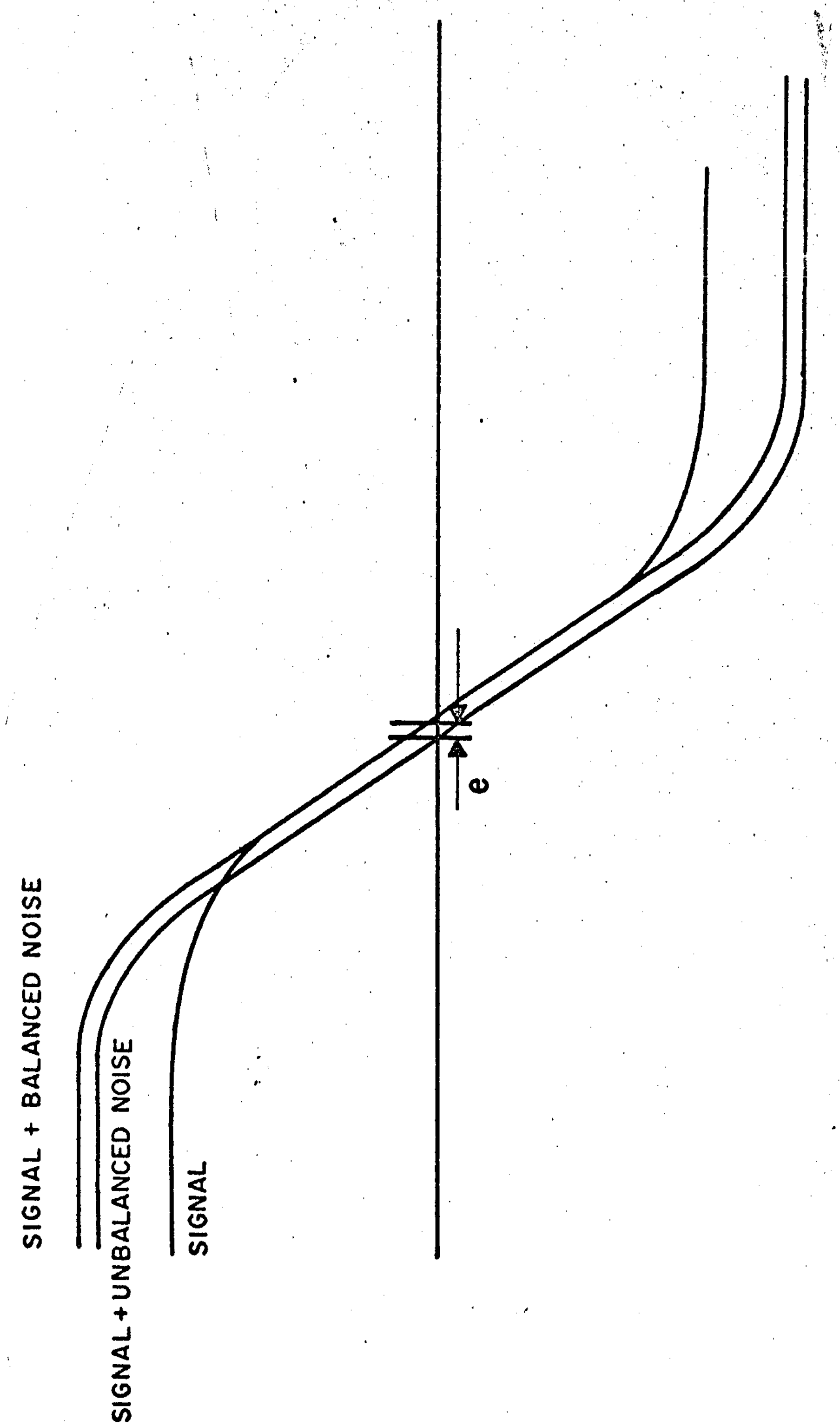


FIGURE 3-1
EFFECT OF ERRORS CONTRIBUTED BY BACKGROUND FLUCTUATIONS

on Figure 3-2 for explanatory purposes. As the planet moves across edge A to the position R_1, sensor No. 1 will respond with a signal when the total light from area No. 1 reaches a preset value. Similarly, sensor No. 2 responds to the light from area No. 2. Ideally, areas No. 1 and No. 2 are equal, so that R_1 will equal R_2 and the planet could be replaced by a fictitious circle of radious $R_1 = R_2$. (The method of detection required here makes it mandatory that separate planet and star "read triggers" be used to designate gating of transits to the computer.) Practically, there are two sources of error:

a. The planet disk is non-uniformly illuminated, so that equal amplitude signals arise when areas No. 1 and No. 2 are unequal and R_1 does not equal R_2. By properly choosing the planet, the pseudo-pole-star and the direction of pseudo-diurnal rotation, this effect can be minimized (human judgement is essential here), but a 10 percent difference in brightness over the areas in question leads to a 10 percent difference in R_1 and R_2. Assuming a planet diameter of 30 arc seconds gives an estimate of 1.06 arc second effect.

b. As the system is now defined, signals from edges A and B go to different photosensors which may themselves have different sensitivities, complicated further by long term drift effects. It is not difficult to design a system with an added optical link or "switch" by means of which all "equals" are referred to the same photosensor. (This usually would involve spinning a wheel in the optics, and would present reliability and vibration problems.) A second solution would only involve a different reticle pattern. If this is done, the only problem is short term drift, which readily can be held under 1 percent for the observing times in question. In general this would mean a 1 percent difference between R_1 and R_2. Since the only requirement here is a recognition of two signals of a predetermined amplitude occurring at nearly the same time from the same point in the sky, the problem of estimating error is just the problem of determining the effect on the line-of-position due to the fact that R_1 does not equal R_2.

If we assume that $\frac{\Delta R}{R} = 0.1$, it appears that the error in "θ.", the angle of planet center transit is 1.2 arc seconds and the error in $\Delta\beta$ is 1.2 arc seconds, also.

It should be remembered that in neither case (ideal with zero error, or the case just discussed) is the numerical value of R actually known. It was pointed out in Reference 1-3 that the computing procedure would have to include an iterative computation in which a value of R would be assumed on the basis of a "dead-reckoning" position, that the distance of the vehicle from

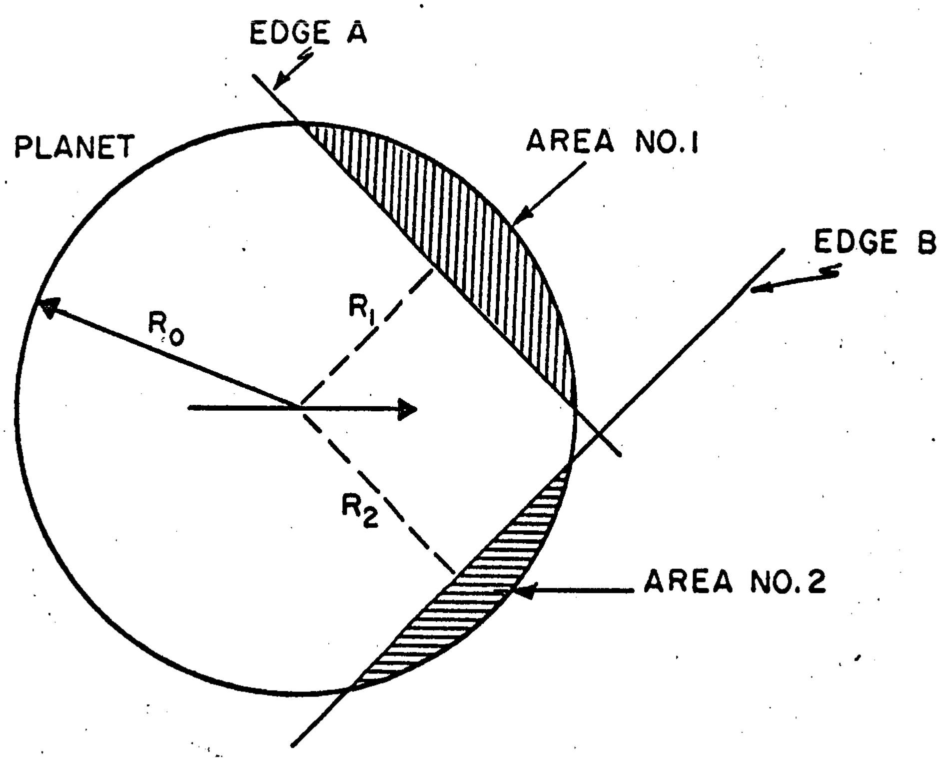


FIGURE 3-2
PLANET TRANSIT SITUATION

the planet would then be determined and that from this the value of R would be known and compared with the originally assumed value. This iterative convergence process still is necessary and may well result in decreasing the error in θ_o and $\Delta\beta$.

The situation with planet crossing is entirely different than for star crossing. In star crossing a second signal will be available from a second photocell to subtract from the first, thus giving exact time of transit. A problem with star crossing is the low amount of light available to the photocell. With planet crossings, a large amount of light is available, but edge crossing is desired (Figure 3-3) thus the same type of detection circuit cannot be used. The electronics can be set for a given amplitude signal, and when this occurs, planet crossing can be said to be accomplished. Of course, the distance from the edge will vary with distance to the planet (Figure 3-4) albedo of planet, and other factors, but will be a constant (subject to limitations discussed below) for the same conditions (i.e., when planet transit is attempted in the other direction). Two limitations in this constancy exist, and these affect the accuracy to which the planet center can be known. The first is from the non-uniformity of the surface - besides planet phases, there is also partial cloud cover, dark markings, etc. The second is electronic accuracy, primarily repeatability. The former will cause major problems, as a 10 percent difference between portions of the planet will cause a corresponding error in readout, the second is minor, since long-term drift doesn't hurt, and short-term drift can be held under 1 percent.

3.4.2.3 Effect of the Planet Center Location Technique

B. 3.) It has been shown in the foregoing paragraphs that a method of locating a planet center (by allowing an appreciable sector of the planet to cross the reticle edge before reading the signal) gives relatively small electronic error (particularly for distant planets) whereas recognition of tangency is quite unsatisfactory. (An error is introduced into the data by doing this which cannot be ignored). Essentially, using the sector signal method involves taking readings on a "planet" of smaller apparent radius R' (See Figure 3-5) than appears (as R_o) in the telescope. To determine θ_o, the reference angle at planet crossing, one must know the value of R' A mean value of γ the angle subtended by R is 40 arc seconds so that the term used in the calculation of θ_o is 56 arc seconds for $R' = R$. If $R' < R$, the θ_o term in question is less than 56 arc seconds.

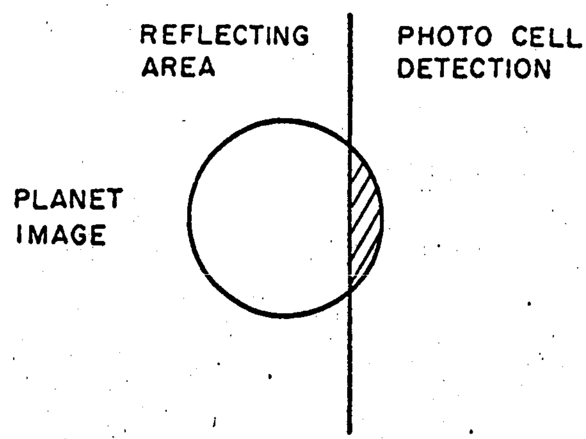


FIGURE 3-3
PLANET TRANSIT

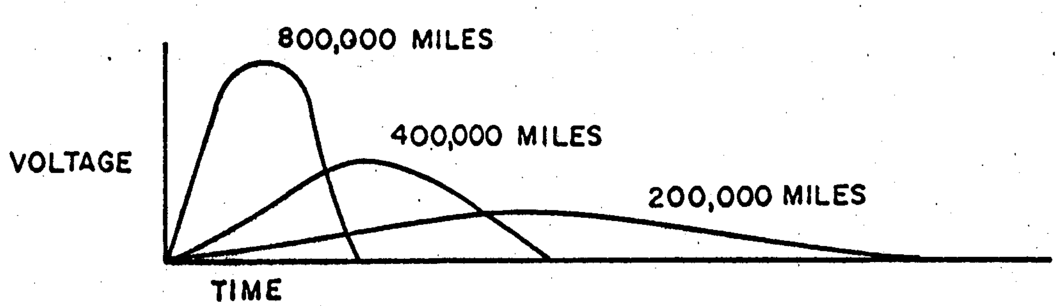


FIGURE 3-4
COMPARISON OF PHOTO-CELL SIGNAL OUTPUT
AS A FUNCTION OF DISTANCE FROM PLANET

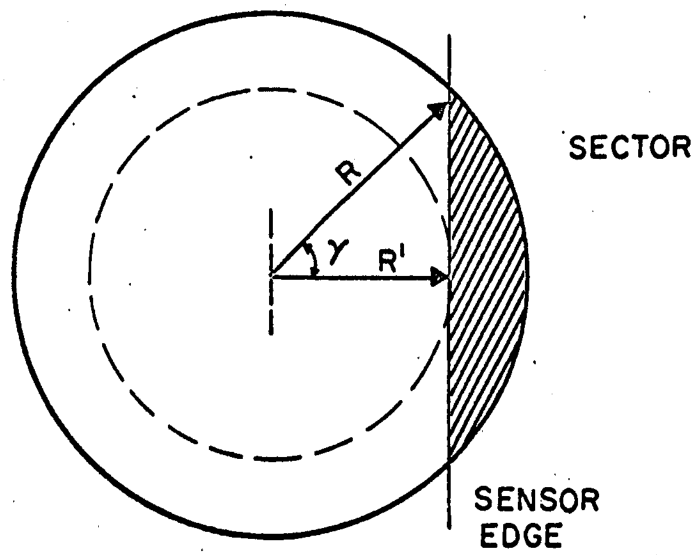


FIGURE 3-5
THE SECTOR SENSING TECHNIQUE

In order to realize the accuracy possible in this method it is necessary to hold the error to one arc second. The value of R'/R must be determined to within 2 percent. That this can be done electronically by comparing the signal from the sensor with a predetermined voltage level versus the R'/R ratio is suggested as follows:

$$\text{Sector area} = \left(\frac{2\,\gamma}{2\,\pi}\right)\left(\pi R^2\right) - \frac{2R^2 \sin\gamma\cos\gamma}{2} \tag{3-1}$$

$$= R^2\left[\gamma - \frac{\sin 2\,\gamma}{2}\right] \tag{3-2}$$

$$= \frac{R^2}{2}\left[2\,\gamma - \sin 2\,\gamma\right] \tag{3-3}$$

Assuming the light intensity of the sector to be uniform, the signal strength can be plotted versus γ as in Figure 3-6.

Since $R'/R = \cos\gamma$, signal can be plotted versus $\cos\gamma$ as in Figure 3-7.

A computer comparison, of the signal from the sensor and the predetermined curve of signal versus R'/R ($= \cos\gamma$), should permit readout of the ratio R'/R to 2 percent quite readily.

3.4.2.4 Image Aberrations (Stars)

B.4.) With careful choice of lens system and best possible alignment of optical axes of the optical components, such aberrations as exist will lie radially symmetric. Hence, the centroid of the image brightness will be on the radial line passing through the center of the aberration free diffraction disk. With the optics specified (in Section 2) the diffraction disk has a diameter of three arc seconds. If the brightness centroid is displaced 10 percent of the diameter from the disk center, an error of 0.3 arc second can occur in the transit angle determination method which involves signal "nulling" between two sensors.

This error would apply once per angle measurement.

3.4.2.5 Alignment of Reticle Lines with ±45 Degree "Great Circles"

B.5.) a. Star transit error due to reticle alignment. This problem is discussed in paragraph 2.2.4 of Section 2. If the reticle can be oriented to a tolerance of ±30 arc seconds (edge versus great circle), then the error in determining the transit angle and pseudo-declination correction will be ±0.2 arc second.

This error would apply for each star transit, or once per angle measurement.

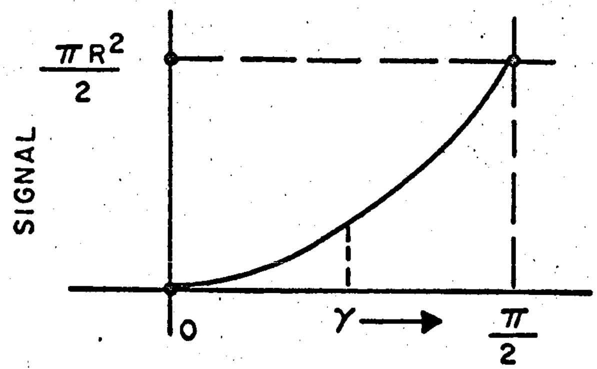


FIGURE 3-6

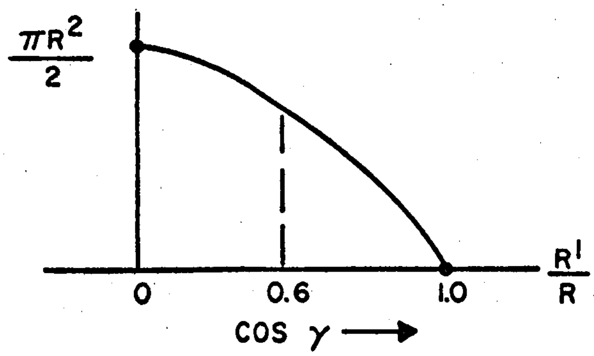


FIGURE 3-7

b. <u>Errors Introduced in locating the planet center if the reticle "crosshairs" are misaligned.</u> Planet transit-single reticle line.

The difficulty of locating the planet center and obtaining its coincidence with the center of the telescope is great; accordingly the location of the center has to be done indirectly. In the following paragraphs, a single, reticle-line (crosshair) method is discussed.

In order to separate errors, certain assumptions are made:

- The crosshairs are precisely 90 degrees apart.
- They are oriented in the telescope so that each makes exactly an angle of 45 degrees with the direction of pseudo-diurnal motion.
- Relative motion of planet and vehicle can be neglected.

This third assumption results in a linear path of the planet in the one degree telescope's field, and the path is parallel to that established by the pseudo-diurnal motion. Angles measured along this direction (line PP in Figure 3-8) are in pseudo-right ascension and those along GG (Figure 3-8) are in pseudo-declination. Any linear distance in the objective field of view of Figure 3-8 is related to the corresponding angular displacement on the celestial sphere by the simple expression:

$$f \cdot \Delta\alpha = d \tag{3-4}$$

where: f is the focal length of the objective lens,

$\Delta\alpha$ is the angular displacement (radians),

d is the linear distance in the field.

Figure 3-8 represents the objective field and three positions which the planet consecutively occupies during the pseudo-diurnal rotation. An angular scale associated with the rotating table reads out these positions of the telescope for the two "tangency" conditions. The distance between any two identical points in the two image positions is fixed by the angular difference and is:

$$T = f(\theta_2 - \theta_1) \tag{3-5}$$

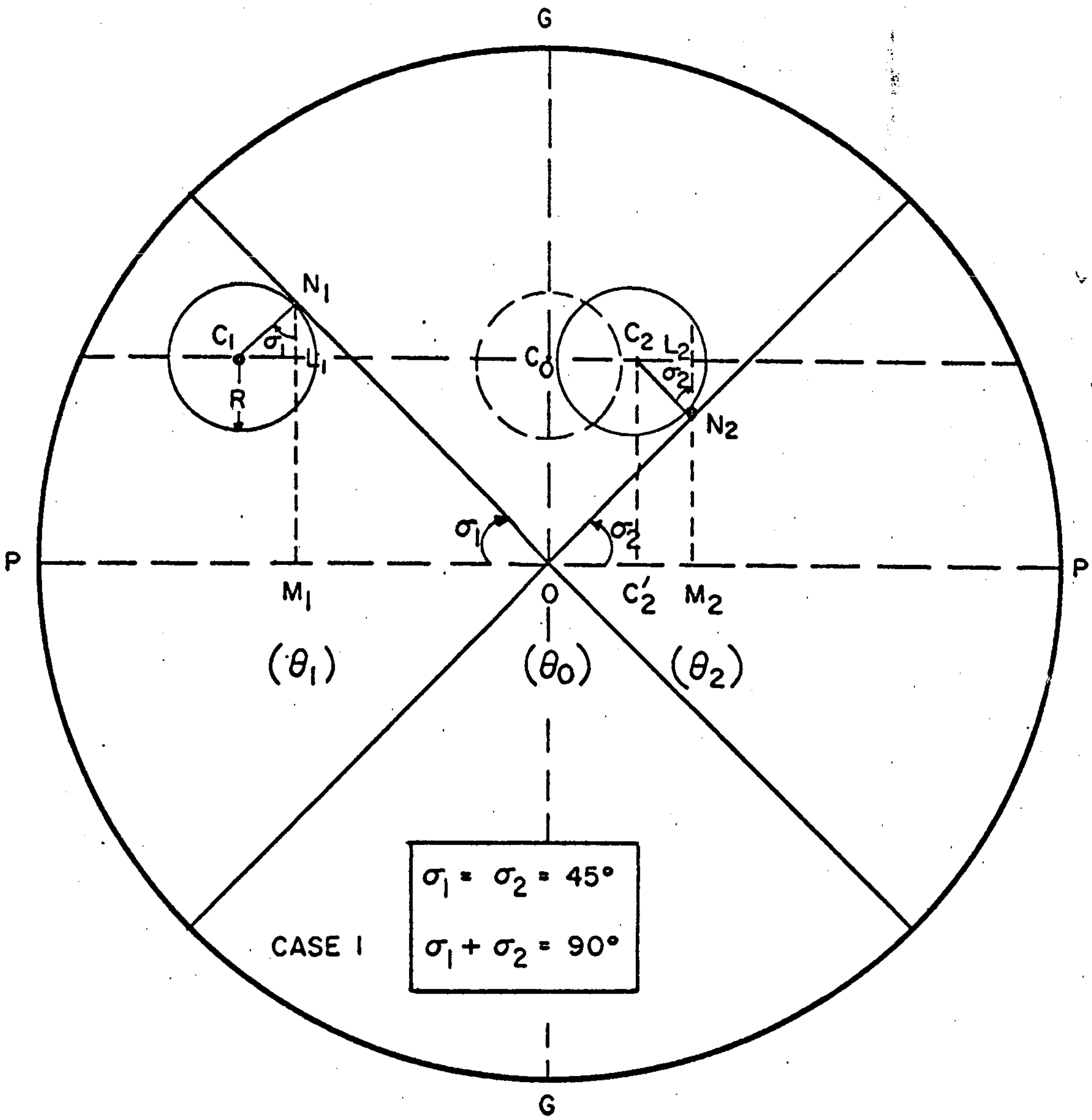


FIGURE 3-8
PLANETARY CROSSING IN ONE DEGREE FIELD

It is apparent that:

$$T = \overline{M_1 N_1} \cot \sigma_1 + \overline{M_2 N_2} \cot \sigma_2 \quad (3\text{-}6)$$

$$T = 2\, \overline{M_1 L_1} = 2\, \overline{M_2 L_2} = 2\, S \quad (3\text{-}7)$$

therefore:

$$S = \frac{f\,(\theta_2 - \theta_1)}{2} \quad (3\text{-}8)$$

θ may now be found by considering

$$OC_2' = f \cdot \Delta\theta \quad (3\text{-}9)$$

Inspection shows that:

$$\overline{OC_2'} = \overline{OM_2} - \overline{C_2 L_2} \quad (3\text{-}10)$$

$$\overline{OC_2'} = (S - R \cos \sigma_2) \cot \sigma_2 - R \sin \sigma_2 \quad (3\text{-}11)$$

So:

$$\Delta\theta = \frac{S - 2\, R \sin 45°}{f} \quad (3\text{-}12)$$

where R is the apparent diameter of the planet.

Since:

$$\theta_o = \theta_2 - \Delta\theta, \quad (3\text{-}13)$$

$$\theta_o = \frac{(\theta_1 + \theta_2)}{2} + \frac{\sqrt{2}\, R}{f} \quad (3\text{-}14)$$

Now from Equation 3-8, the difference in pseudo-right ascension between telescopic optical axis and planet center is:

$$\Delta\beta = \frac{S}{f} = \frac{(\theta_2 - \theta_1)}{2} \quad (3\text{-}15)$$

The angular positions of the planet center when it lies on the pseudo-right-ascension circle through "O" is given by Equation 3-14. In the form given, it cannot be used, because R is not known. An iteration procedure has been developed to overcome this difficulty (as previously mentioned).

Note that one could as easily use a method in which chordal lines rather than tangent lines are referred to. As long as these chordal lines are equidistant from the center, they could serve exactly as do the tangents, just as though a planet of smaller diameter were being observed.

Consider next a general case in which the "crosshairs" (reticle edges) are neither orthogonal nor symmetric, and derive expressions for θ_o and $\Delta\beta$. This is the most general case. Referring to Figure 3-9, define:

$$T_S = \overline{M_1 M_2} = \overline{M_1 N_1} \cot \sigma_1 + \overline{M_2 N_2} \cot \sigma_2 \tag{3-16}$$

$$T_S = (S + R \cos \sigma_1) \cot \sigma_1 + (S - R \cos \sigma_2) \cot \sigma_2 \tag{3-17}$$

Also,

$$\overline{C_1 C_2} = f(\theta_2 - \theta_1) \tag{3-18}$$

so that,

$$T_S = f(\theta_2 - \theta_1) - R \sin \sigma_1 + R \sin \sigma_2 \tag{3-19}$$

solving gives;

$$S = \frac{R(\sin \sigma_1 - \sin \sigma_2)}{\sin(\sigma_1 + \sigma_2)} + \frac{f(\theta_2 - \theta_1) \sin \sigma_1 \sin \sigma_2}{\sin(\sigma_1 + \sigma_2)} \tag{3-20}$$

To find θ_o, define:

$$\Delta \theta = \frac{OC_2'}{f} = \frac{(S - R \sin \sigma_2) \cot \sigma_2}{f} - \frac{R \cos \sigma_2}{f} \tag{3-21}$$

$$\Delta \theta = \frac{S \cot \sigma_2}{f} - \frac{2 R \cos \sigma_2}{f} \tag{3-22}$$

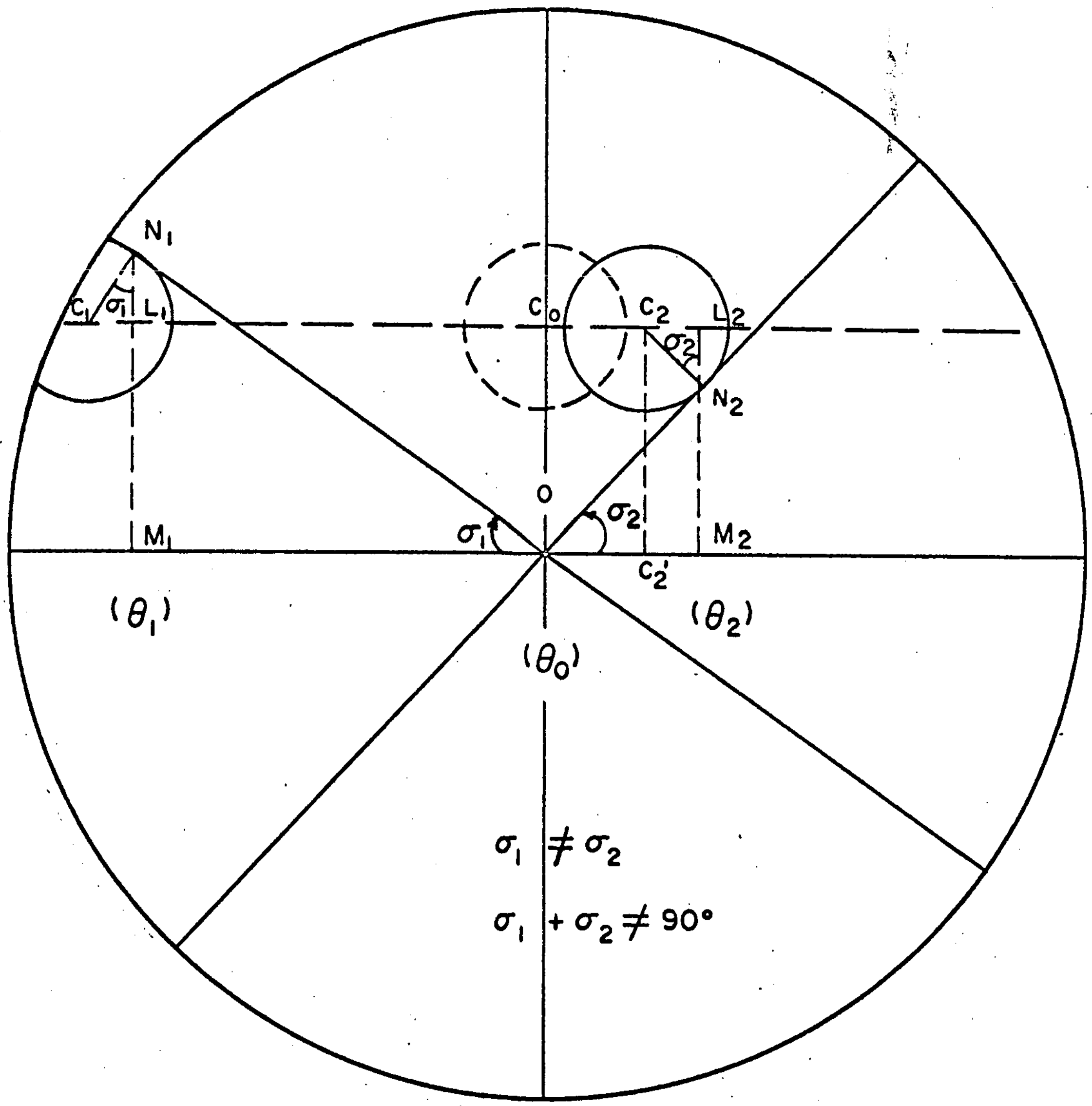

N₁
C₁
σ₁
L₁
C₀
C₂
L₂
σ₂
N₂
O
M₁
M₂
σ₁
σ₂
C₂'
(θ₁)
(θ₂)
(θ₀)
$\sigma_1 \neq \sigma_2$
$\sigma_1 + \sigma_2 \neq 90°$

FIGURE 3-9

Since,

$$\theta_o = \theta_2 - \Delta\theta, \tag{3-23}$$

$$\theta_o = \theta_2 - \left[\frac{(\theta_2 - \theta_1)\ \sin\sigma_1 \cos\sigma_2}{\sin(\sigma_1 + \sigma_2)}\right] - \frac{R}{f}\left[\frac{(\sin\sigma_1 - \sin\sigma_2)\cot\sigma_2}{\sin(\sigma_1 + \sigma_2)} - 2\cos\sigma_2\right] \tag{3-24}$$

where R is unknown.

The altitude correction:

$$\Delta\beta = \frac{S}{f} \tag{3-25}$$

is:

$$\Delta\beta = \frac{R}{f}\left[\frac{\sin\sigma_1 - \sin\sigma_2}{\sin(\sigma_1 + \sigma_2)}\right] - \left[\frac{(\theta_o - \theta_1)\sin\sigma_1\sin\sigma_2}{\sin(\sigma_1 + \sigma_2)}\right] \tag{3-26}$$

Also, look at the expressions for θ_o and $\Delta\beta$ which occur for the two special cases indicated below. These expressions are immediately derived from Equations 3-24 and 3-26:

- <u>Single crosshair, non-orthogonal</u> conditions are:

$$\sigma_1 + \sigma_2 \neq 90°, \ \sigma_1 = \sigma_2 \tag{3-27}$$

Set $\sigma_1 = \sigma_2$ in Equations 3-24 and 3-26 to get:

$$\theta_o = \frac{(\theta_1 + \theta_2)}{2} + \frac{2\ R\cos\sigma_2}{f} \tag{3-28}$$

$$\Delta\beta = \frac{(\theta_2 - \theta_1)\sin^2\sigma_2}{\sin 2\sigma_2} \tag{3-29}$$

- Single crosshair, non-symmetric conditions are:

$$\sigma_1 + \sigma_2 = 90°, \quad \sigma_1 \neq \sigma_2 \tag{3-30}$$

Set $\sigma_1 + \sigma_2 = 90°$ in Equations 3-24 and 3-26 to get:

$$\theta_o = \theta_2 \cos^2 \sigma_1 + \theta_1 \sin^2 \sigma_1 - \frac{R \tan \sigma 1}{f} \left[\sin \sigma_1 - 3 \cos \sigma_1 \right] \tag{3-31}$$

$$\Delta \beta = (\theta_2 - \theta_1) \sin \sigma_1 \cos \sigma_1 + \frac{R}{f} (\sin \sigma_1 - \cos \sigma_1) \tag{3-32}$$

Considering the basic Equations 3-24 and 3-26, take the differentials $\delta\theta_o$ and $\delta(\Delta\beta)$ which occur for variations $\delta\sigma_1$ and $\delta\sigma_2$. These variations in θ_o and $\Delta\beta$ are errors due to small changes in σ_1 and σ_2.

Taking the differential of Equation 3-24

$$\delta\theta_o = -(\theta_2 - \theta_1)\Bigg[\left[\sin(\sigma_1+\sigma_2)\right]\left[\delta(\sin\sigma_1 \cos\sigma_2)\right] - \left[\sin\sigma_1 \cos\sigma_2\right]\left[\delta \sin(\sigma_1+\sigma_2)\right] \qquad (3\text{-}33)$$

$$-\frac{R}{f}\left[\frac{(\sin\sigma_1 - \sin\sigma_2)\left[\delta(\sin[\sigma_1+\sigma_2])\right] - \sin(\sigma_1+\sigma_2)\left[\delta(\sin\sigma_1 - \sin\sigma_2)\right]\cot\sigma_2}{\sin^2(\sigma_1+\sigma_2)}\right.$$

$$\left. + \frac{(\sin\sigma_1 - \sin\sigma_2)}{\sin(\sigma_1+\sigma_2)}\ \delta(\cot\sigma_2) + 2\sin\sigma_2\,\delta\sigma_2\right]\Bigg]$$

$$\delta\theta_o = -(\theta_2-\theta_1)\left[\frac{\sin(\sigma_1+\sigma_2)\left[-\sin\sigma_1\sin\sigma_2\,\delta\sigma_2 + \cos\sigma_1\cos\sigma_2\,\delta\sigma_1\right] - \sin\sigma_1\cos\sigma_2\left[\cos(\sigma_1+\sigma_2)(\delta\sigma_1+\delta\sigma_2)\right]}{\sin^2(\sigma_1+\sigma_2)}\right]$$

$$-\frac{R}{f}\left[\frac{(\sin\sigma_1 - \sin\sigma_2)\cos(\sigma_1+\sigma_2)(\delta\sigma_1+\delta\sigma_2) - \sin(\sigma_1+\sigma_2)[\cos\sigma_1\,\delta\sigma_1 - \cos\sigma_2\,\delta\sigma_2]\cot\sigma_2}{\sin^2(\sigma_1+\sigma_2)}\right.$$

$$\left. + \frac{(\sin\sigma_1 - \sin\sigma_2)}{\sin(\sigma_1+\sigma_2)}\ (-CSC^2\sigma_2\,\delta\sigma_2) + 2\sin\sigma_2\,\delta\sigma_2\right] \qquad (3\text{-}34)$$

Evaluating this error for conditions of $\sigma_1 + \sigma_2 = 90°$ and $\sigma_1 = \sigma_2 = 45°$:

$$\delta\theta_o = -(\theta_2-\theta_1)\left[\left(-\frac{\delta\sigma_2}{2} + \frac{\delta\sigma_1}{2}\right) - 0\right] - \frac{R}{f}\left[-\left(\frac{\delta\sigma_1}{\sqrt{2}} - \frac{\delta\sigma_2}{\sqrt{2}}\right) + \sqrt{2}\,\delta\sigma_2\right] \qquad (3\text{-}35)$$

or

$$\delta\theta_o = -\frac{(\theta_2-\theta_1)}{2}\left[\delta\sigma_1 - \delta\sigma_2\right] - \frac{R}{f}\left[\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)\delta\sigma_2 - \frac{1}{\sqrt{2}}\,\delta\sigma_1\right] \qquad (3\text{-}36)$$

In order to estimate order of magnitude errors, assume that the angle between tangent points is $\theta_2 - \theta_1 = 30$ arc minutes and that the angle subtended by the planet, $\frac{2R}{f} = 4 \times 10^{-4}$ radians (or 80 arc sec.)

This gives at once

$$\delta\theta_o = -900\,(\delta\sigma_1 - \delta\sigma_2) - 40\,(2.12\,\delta\sigma_2 - 0.71\,\delta\sigma_1) \tag{3-37}$$

$$\delta\theta_o = -872\,\delta\sigma_1 + 815\,\delta\sigma_2 \tag{3-38}$$

In the same fashion operate on $\Delta\beta$ to get

$$\Delta\beta = \frac{R}{f}\left(\frac{\sin\sigma_1 - \sin\sigma_2}{\sin(\sigma_1+\sigma_2)}\right) + \frac{(\theta_2-\theta_1)\sin\sigma_1\sin\sigma_2}{\sin(\sigma_1+\sigma_2)} \tag{3-39}$$

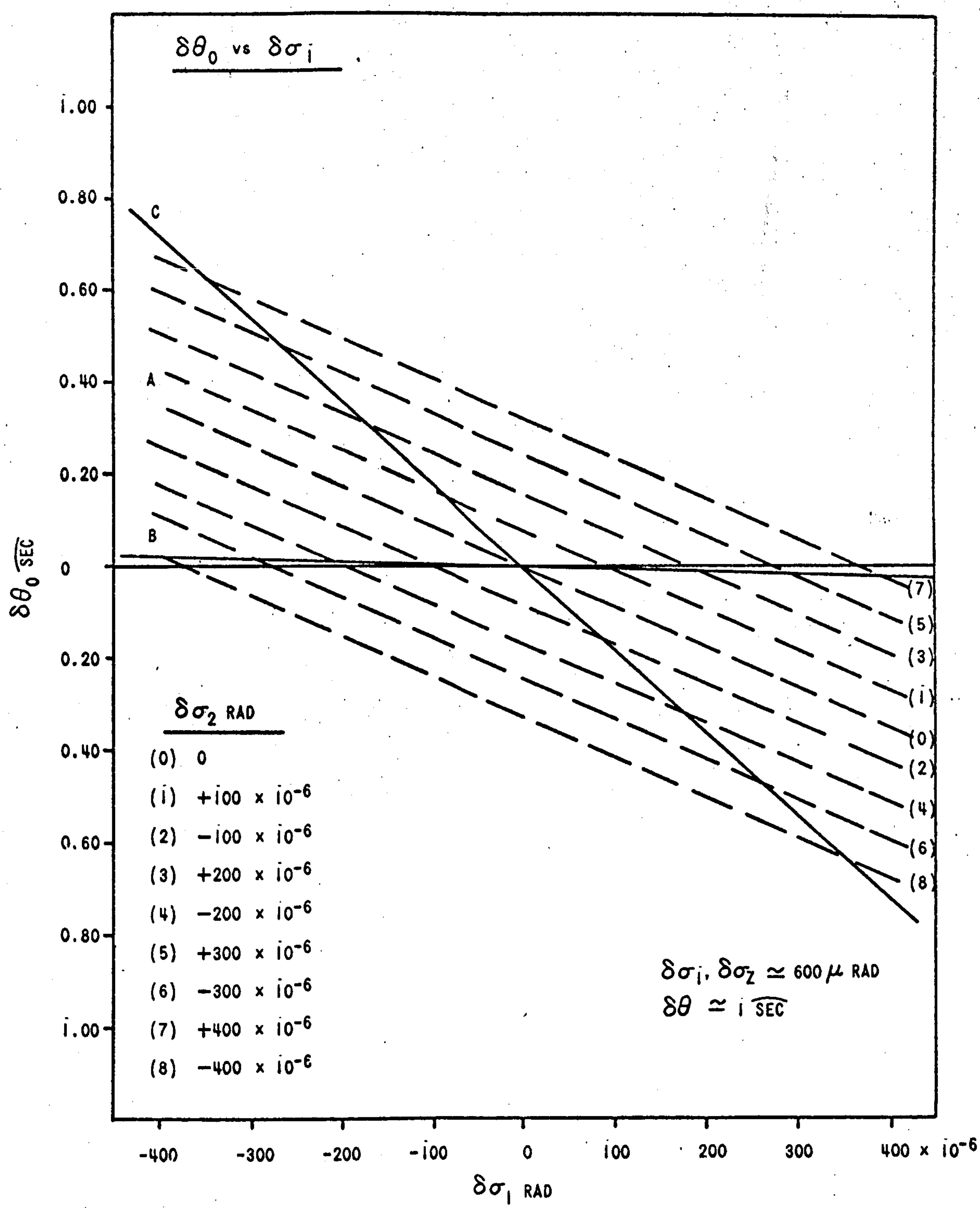
$$S(\Delta\beta) = \frac{R}{f}\left[\frac{(\sin\sigma_1 - \sin\sigma_2)\cos(\sigma_1+\sigma_2)(\delta\sigma_1+\delta\sigma_2) + \sin(\sigma_1+\sigma_2)(\cos\sigma_1\delta\sigma_1 - \cos\sigma_2\delta\sigma_2)}{\sin^2(\sigma_1+\sigma_2)}\right]$$
$$+ (\theta_2-\theta_1)\left[\frac{\sin(\sigma_1+\sigma_2)\left[\sin\sigma_1\cos\sigma_2\,\delta\sigma_2 + \cos\sigma_1\sin\sigma_2\delta\sigma_1\right] - (\sin\sigma_1\sin\sigma_2)\cos(\sigma_1+\sigma_2)(\delta\sigma_1+\delta\sigma_2)}{\sin^2(\sigma_1+\sigma_2)}\right] \tag{3-40}$$

Substituting here for $(\theta_2 - \theta_1)$ and $\frac{R}{f}$ gives

$$\delta(\Delta\beta) = 928\,\delta\sigma_1 + 872\,\delta\sigma_2 \tag{3-41}$$

One could easily find $\delta\theta_o$ and $\delta(\Delta\beta)$ for cases B and C also.

The results of these calculations are shown on Figures 3-10 and 3-11.



δθ₀ vs δσ₁
1.00
0.80
0.60
0.40
0.20
0
0.20
0.40
0.60
0.80
1.00
δθ₀ SEC
C
A
B
(7)
(5)
(3)
(1)
(0)
(2)
(4)
(6)
(8)
δσ₂ RAD
(0) 0
(1) +100 x 10⁻⁶
(2) −100 x 10⁻⁶
(3) +200 x 10⁻⁶
(4) −200 x 10⁻⁶
(5) +300 x 10⁻⁶
(6) −300 x 10⁻⁶
(7) +400 x 10⁻⁶
(8) −400 x 10⁻⁶
δσ₁, δσ₂ ≃ 600 μ RAD
δθ ≃ 1 SEC
-400
-300
-200
-100
0
100
200
300
400 x 10⁻⁶
δσ₁ RAD

FIGURE 3-10

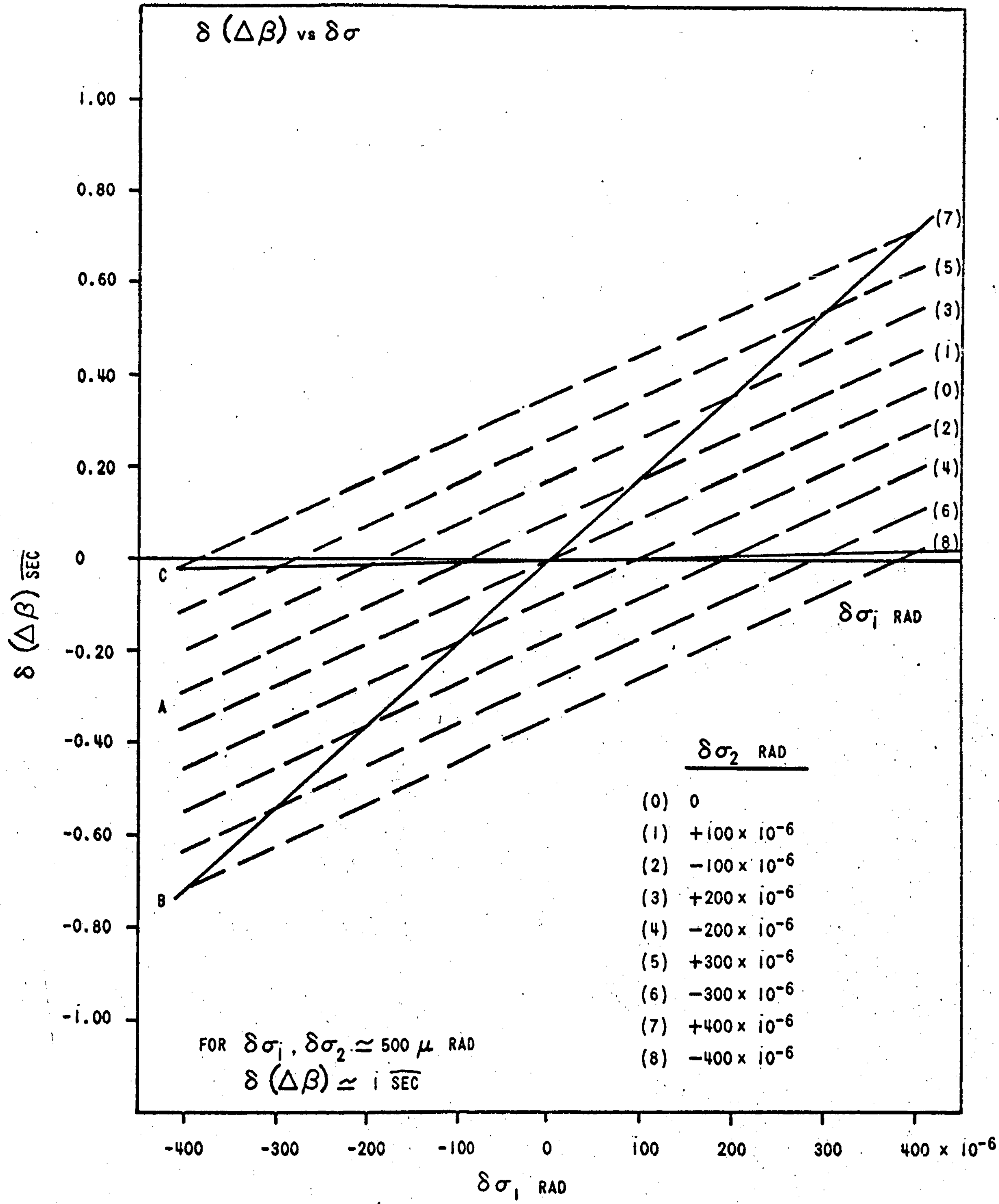

$\delta(\Delta\beta)$ vs $\delta\sigma$
1.00
0.80
0.60
0.40
0.20
0
-0.20
-0.40
-0.60
-0.80
-1.00
$\delta(\Delta\beta)$ SEC
C
A
B
(7)
(5)
(3)
(1)
(0)
(2)
(4)
(6)
(8)
$\delta\sigma_1$ RAD
$\delta\sigma_2$ RAD
(0) 0
(1) $+100 \times 10^{-6}$
(2) -100×10^{-6}
(3) $+200 \times 10^{-6}$
(4) -200×10^{-6}
(5) $+300 \times 10^{-6}$
(6) -300×10^{-6}
(7) $+400 \times 10^{-6}$
(8) -400×10^{-6}
FOR $\delta\sigma_1, \delta\sigma_2 \simeq 500\ \mu$ RAD
$\delta(\Delta\beta) \simeq 1$ SEC
-400
-300
-200
-100
0
100
200
300
400×10^{-6}
$\delta\sigma_1$ RAD

FIGURE 3-11

It may be concluded that if σ_1 and σ_2 can be held to tolerances of one minute of arc (300 micro radians) the error in θ_o is 0.5 arc second and in $\Delta \beta$ is also about 0.5 arc second, both representing maximum errors. Better control of σ_1 and σ_2 will give correspondingly better results for θ_o, $\Delta \beta$.

3.4.2.6 Common Intersection of Optic and Mechanical Axes

B.6.) Failure of the optic axis to intersect the common point of the mechanical axes can introduce errors as follows:

a) If the telescope does not rotate about the center of the spherical dome, astigmatic and other aberrations are increased, because the line of sight is no longer radial with respect to the dome. Such errors are included in category 1), above.

b) No error is introduced by the failure of the telescope axis to intersect the pseudo-diurnal axis.

c) The planet search axis misalignment is of no consequence since this motion is not used in obtaining angle readings.

d) The effect of great circle wobble and wander as the star search is made is considered under another heading.

e) The failure of the telescope optic axis to be set in coincidence with the axis of the ±45 degrees rotation does introduce error, since the fiducial point for measuring θ will not stay fixed in this case. As the telescope is rotated about the great circle selector axis, the optic axis of the telescope describes a cone centered on this mechanical axis. The maximum error possible in the measure of the angular position of the second star is 1.4 x (error in alignment). Assuming that the alignment error is one arc second, the error in angle determination is 1.4 arc seconds.

f) In somewhat the same way the failure of the mechanical axis to intersect the center of the reticle will introduce errors also, although now there is no pointing error. The reticle lines are in the "wrong" place so that transit angle readings are incorrect. In placing the reticle in the telescope tube alignment to 0.0002 inch should be possible which is 1.2 arc seconds in a 1 degree field. This is the maximum error introduced when the second great circle star search is started.

3.4.2.7 Variation in Reticle Edge

B.7.) Variation in reticle edge definition can be held to 0.00001 inch, which is equivalent to 0.2 arc second.

3.4.3 Method Errors

3.4.3.1 Effect of Relative Motion of Planet and Vehicle

C.1.) In the presently assumed PDT method of navigation there is a time lapse between the planet transit and the start transits which follow. It is of course assumed that the vehicle is fixed in space during this period, but this is only approximately true in interplanetary transfer.

For example, suppose a vehicle is traveling from earth to Mars on a minimum energy transfer ellipse. This is a 259 day journey. Near mid-course the radius vector from vehicle to planet is rotating at about 10 degrees per day or 0.02 arc second/second. Over a 10 minute observing interval the total change in apparent angular position of the planet is 12 arc seconds.

On a predetermined course this error is known and can be accounted for. The only residual error will be that due to deviations from the preplanned velocities. If these are to be held to one part in 10^4, for example, the error in the correction factor would be only 0.0012 arc second, if to one part in 10^3, 0.012 arc second. Thus, it would seem that the effects of such an error could be satisfactorily compensated and need not be considered here.

3.4.3.2 Errors in Basic Information

C.2.) There are certain errors which will be introduced into the computations because of our lack of precise knowledge (ephemeris errors).

Latitude and longitude values of the stars, the distance between planets, and the linear diameter of the planets are examples of this ephemeris information which is known with only limited precision.

All methods of navigation must necessarily suffer from such inadequate knowledge, of course. Since this ephemeris data is put through a long computational process involving use of trigonometric functions, only analysis of the method itself would give meaningful results. Furthermore, this analysis involves the effect of the computer on the results. Therefore, ephemeris errors are not included in this analysis.

3.5 ERROR ESTIMATES FOR ORIGINAL PDT CONCEPT WITH INERTIAL RATE SENSING

For the sake of reference, let it be assumed that the original Kardashian concept is to be employed for determining a single line-of-position. The equations of paragraph 1.4.1.4 (Section 1) apply.

Assume that rate measurement is to be inertial, and that rate gyros have advanced, by 1975, to such a remarkable state that their thresholds are ±0.01 degrees per hour (presently ±0.01 degree per second). Then $\delta\omega$ could be assigned a partial value due to this source of ±0.01 arc second/time second. Let it further be assumed that rate changes greater than this value, due to motions of men and machinery within the vehicle, can be observed and calculated out. Therefore, two values can be assigned tentatively to $\delta\omega$: 1) Present ≅ 36 arc second/time seconds, 2) 1975 ≅ 0.01 arc second/time second.

Next let it be assumed that a man can visually time star and planet transits to an accuracy of ±0.027 second; this value is not likely to change in ten years and is already optimistic. Then the value assigned to δt would be 0.027 second.

Let it be assumed that a body rate of 15 arc second/time second is used for the observations and that 20 minutes or 1,200 seconds are occupied by each observation.

Five transits are required per line-of-position (one planet, plus four star transits), whether two, three or four reference stars are used.

For each transit, we have the angular errors:

1) Present:

$$\delta\alpha = (1200)(36) + (15)(0.027)$$

$$\delta\alpha = (43,200) + (0.405)$$

$$\delta\alpha \cong 43,200 \text{ arc seconds}$$

$$\delta\alpha \cong \pm 12 \text{ degrees per transit!}$$

Reducing the observing time to two minutes, (rather than 20) would still give an error per transit of 1.2 degrees! To be optimistic, suppose this smaller value be assigned, also. Increasing the PDT rate by a factor of 10 (in order to observe the required angles faster) and reducing the required observing time by a factor of 10 to cover the same sweep angle about the PD axis would yield for the present:

$\delta\alpha$ = (120)(36) + (150)(0.027)

$\delta\alpha$ = (1080) + (4.05)

$\delta\alpha$ = 1084 arc seconds

$\delta\alpha \cong \pm 0.31$ degree per transit.

2) 1975: Assuming 20 minutes (1200 seconds) per observation and a $\delta\omega$ of 0.01 arc second/second, at a PDT rate of 15 arc second/ time second

$\delta\alpha$ = (1200)(0.01) + (15)(0.027)

$\delta\alpha$ = 12 + 0.405

$\delta\alpha \cong \pm 12.4$ arc seconds per transit.

Increasing the rate by a factor of ten to 150 arc seconds/time second and reducing the observation time to 2.0 minutes (120 seconds) to cover the same angle yields for 1975 an estimate of:

$\delta\alpha$ = (120)(0.01) + (150)(0.027)

$\delta\alpha$ = (0.12) + (4.05)

$\delta\alpha \cong \pm 4.17$ arc secondsper transit.

Now, if 5 transits per line-of-position are involved, then a fairly rough estimate of the L.O.P. error ($\delta\theta$ or $\delta\phi$) can be had by taking the square root of the sum of the squares of five transit observations:

TABLE 3-4
ORIGINAL PDT SCHEME WITH INERTIAL RATE MEASUREMENT

PDT Rate	Time Per Transit Observation	R.S.S. L.O.P. Error 1963	R.S.S. L.O.P. Error 1975 Est.*
15 deg/hr.	20 minutes	±26.83 degrees	±12.41 arc sec.
	2 minutes	2.68 degrees	± 2.77 arc sec.
150 deg/hr.	2 minutes	±0.694 degrees, or ±41.64 minutes or ±2500 arc sec.	± 9.32 arc sec.

* Assuming a factor of 3600 improvement in rate measurement by 1975!

It is easily seen from the preceding that a factor of 3600 improvement in the state of the art of rate sensing would just begin to make the original concept (with inertial rate measurement) competitive with marine sextant observations! Arguments of this type led to serious consideration of the alternate methods for applying the modified PDT concept as reported in this document. Note that integrated rate (angle) can be obtained also by closing a loop around a viscously damped (integrating) gyro. Uncalibrated (random) drift of such gyros can be on the order of 0.01 degrees per hour, and angles could be measured at fairly high rates (i.e. 150 degrees per hour), but rate threshold limitations would apply in this case, as with ordinary rate gyros; static friction of gimbal bearings is the chief limiting factor.

In defense of the original concept, let it be noted that Earth's rate has been remarkably constant throughout recent history, and that its value has been determined from thousands of filtered observations to be 15.041069 degrees per hour (reference 2-6). The accuracy with which this has been determined is now thought to be $\pm 0.000027^{+}$ percent, which when compared to atomic clocks has permitted observation of an unaccounted for variability (in either Earth's rate or the atomic clocks) such that calendar corrections may be necessary. With the following assumptions:

1) Earth's rate is known and constant,

2) time can be measured to an accuracy of one part in 10^{11} per day,

3) errors in observation are due to random factors, such as atmospheric refraction and diffraction variability; and hence may be reduced by averaging,

4) transit events are electro-optically observed;

then, indeed, the scheme permits high accuracy position determinations upon the Earth to be made, and even permits "navigating" the Earth in its orbit to high precision (by reference to other planets).

The difficulty in applying the original concept to navigation of space vehicles (by on-board observations) is centered around the difficulty of establishing, maintaining and measuring the pseudo-diurnal-rates of the relatively low-inertia vehicles. Nevertheless, a manual back-up mode applying the method was provided for in concept (see paragraph 2.2.8 of Section 2).

SECTION 4
RELIABILITY, MAINTENANCE AND SPARES

4.1 The reliability analyses for the entire instrument system have not been completed. A rough estimate of the overall mean time between failures is less than 10,000 hours, so that in-flight repairs would very probably be necessary during a voyage to Mars, involving a stay of some 20,000 hours in space.

Approximately 100 percent spares of electronic plug-in modules, 10 extra vidicons, 10 photo multiplier tubes for transit detection and 6 extra tubes for star tracking are expected to be required. However, these figures have not been firmly established. The ADEPT computer is by its design multiredundant, and repairable in flight with about 50 percent spare modules.

SECTION 5
RECOMMENDATIONS

5.1 THE 2^{23} BIT DYNAGON

The general problem of taking angular readings "on the fly" between rotating members, and to extremely high accuracy, is common to many aerospace system concepts. The fluid bearing dynagon, described in Section 2, could (for example) be applied to a space sextant as well as to the PDT instrument. Therefore, whether the PDT system is to be developed or not, the high resolution dynagon will find many other applications, eventually. It is suggested that this device is worthy of further attention for possible applications to tight servo loops and to an improved space sextant.

5.2 STAR TRACKERS

The "strapped-down" star trackers described in Section 2 have no mechanical parts moving in the optics, consequently, their reliability should be far greater than that for spinning reticle types. The accuracy which apparently can be achieved with these trackers makes them extremely attractive for general, navigational and attitude control use. Development already underway should be encouraged.

5.3 LOCAL VERTICAL

The most significant question in navigation is, "Which way is down?" The planet transit scheme used in this study for finding the "center of a planet" is applicable only to interplanetary mid-course. The more critical problem of finding local vertical and altitude during planetary approach and in earth orbit should receive priority attention.

The conceivable applications of micro-electronic sensor arrays and fiber optics to space navigation planet scanner problems would appear to be worthy of considerable study. Devices and concepts which may come out of such study perhaps will have significant military value in the reconnaissance and surveillance problems.

The manual optical methods (for sighting local vertical) at NASA, Langley and Honeywell, Florida are worthy of being carried into the simulation and human factors evaluation stages, using a planet simulator room.

5.4 TRANSIT DETECTION

The assumptions made in Section 3 regarding the accuracy of transit detection should be experimentally confirmed. An optical system similar to that described in Section 2 could be checked out by use of an inertial component testing rate-table and star simulator.

Similarly, the planet transit concept could be confirmed (or negated) by operation with a planet simulator. Such a simulator also would be required for the general, local-vertical, human-factors experiments, as well as for evaluation of various planet tracker/scanner devices.

5.5 PDT VERSUS SEXTANT

The PDT system potentially offers a 3 to 1 improvement over sextant accuracy, but in any fair comparison the sextant should be given equal advantage by equipping its index arm with a 2^{23} bit dynagon and providing it with comparable electro-optic read-out aids. Very likely the sextant could be improved to the point where lines of position could be established by it to ± 4 arc second; this would permit a geometrically simpler approach to the problem with consequently greater transfer of learning from present navigation methods.

Rather than considering the two methods of sighting as "arch competitors" it might be advisable to strive for design of a single instrument capable of being applied in both modes.

5.6 THE POSITION/VELOCITY FINDER SYSTEM

In similar vein is a further obvious question: If the vehicle stabilization is to be held (or read) for PDT to ±0. 5 arc seconds about each of three axes, and if a simple telescope can have its nominal optical axis referred to the stabilized frame to ±0. 25 arc seconds per axis of freedom, then why not simply use the telescope to sight landmarks or planets, and directly read out lines-of-position versus the "inertially fixed" reference frame? Independent reading might be taken from the dynagons as rapidly as 1, 000 per second while the "read trigger" is depressed by the operator. Statistical averaging of thousands of L. O. P. sightings should permit at least one magnitude improvement over the one shot accuracy. Further, two such telescopes with two operators using the same stabilized reference could permit taking L. O. P.'s to two near bodies simultaneously with a resultant rapid series of position fixes, which perhaps would make it feasible to obtain vector velocity by differencing. Proper optics might permit one operator to track two planets in this manner, but some human factors study will be necessary to confirm this.

Of all the alternative schemes, this one appears to be the simplest and most direct extension of the concepts considered in this study. Coupled with electro-optical aids for the operators, and with the best of the various manual/optical methods for sighting "local vertical," such a scheme might be worked into a standardized space navigation system usable during all phases of any manned mission. The military need for such a capability may become urgent within a decade.

5.7 IN CONCLUSION

The ideas for stabilization error readout to high accuracy (±0.5 arc sec), shaft angle readout to ±0.25 arc second and detection of the center of an airy disk to ±0.2 arc second developed in this study, may permit space navigation thinking to move into a new realm of precision. These intriguing possibilities coupled with the ideas for obtaining local vertical to high accuracy (by manual, electro-optics) may permit major advances to be made in the state of the art of on-board navigation through deep space.

Accordingly it is suggested that the investigation should be continued and should be re-oriented toward the standardized space navigation system concept, rather than the PDT concept, per se.

ATTACHMENT NO. 1

ULTRADEX BROCHURE

Since the mechanization described in Section 2 depends so heavily upon the principle of the Ultradex, the brochure supplied by AA Industries, Inc. is attached hereto for the convenience of those who may not have considered this device previously.

Quite obviously, lighter weight versions of this rather heavy instrument would have to be developed for specific instrument applications, such as PDT. Conversations with AA Industries were encouraging with regard to this point.

HENRY H. SHUFELDT

REPORT ON

PRECISION CELESTIAL NAVIGATION EXPERIMENTS

CONTRACT NO. NONR - 2449 (00)

DATED 15 NOVEMBER 1957

This report covers the results of research conducted at Key West, Florida, and near Newport, Rhode Island, as well as afloat during the period January 1958 through May 1961.

PREPARED BY:

HENRY H. SHUFELDT
P. O. BOX 288
ANNAPOLIS, MARYLAND
JUNE 1961

In connection with Honeywell Aero Document R-ED 24054, Reference 1 is reproduced in its entirety for the convenience of the reader.

REPORT

In accordance with the provisions of O.N.R. CONTRACT NO. Nonr-2449 (00) dated 15 November 1957, celestial observations were obtained at Key West, Florida, during the period 8 February 1958 to 4 March 1958 and subsequently aboard the USS VALLEY FORGE (CVS - 45), the USS TUTUILA (ARG-4), the Yacht PARAMOUR and the Yacht RICKWOOD.

The purpose of this study was to determine what benefits could be derived from the use of high-magnification sextant telescopes (20x and 16x as well as 7x for use with the stars at dusk), what improvements could be made in the marine sextant and its ancillaries, and in the techniques of its employment.

Almost 5,000 celestial observations were made in connection with this program under conditions of visibility, weather, and quality of horizon ranging from excellent to extremely poor. The new instruments designed especially for test during this study, as well as standard sextants and telescopes presently in general use were employed and compared under all manner of conditions such as might be encountered by the working navigator at sea. In fact, the frequency of observations made with a poor horizon was increased when it unexpectedly appeared that the high-powered telescopes under such conditions often gave materially more reliable altitudes than those in general use.

A number of experienced observers participated in this program: Captain P. V. H. Weems, USN (RET.) and CDR. William S. Brown, USN assisted greatly in planning the study, and made many observations. Their advice and assistance are particularly appreciated. Mr. G. D. Dunlap's help in preparing specifications for the Plath sextants and Beck telescopes ordered especially for use in this program is also highly appreciated.

Early in the course of the study it became apparent that the accuracy of observations made with the high-powered telescopes was consistently greater than that obtained with telescopes of 6x magnification, and much greater than that obtained with the 3x telescope in common use, particularly when the horizon was not sharply defined. This gain in accuracy was demonstrated by the plot of observations in each string, compared to the "line of best fit" for the string, as well as by the comparison of intercepts, when observations were made from an accurately known position. The results obtained with the 3x telescope were so obviously inferior to those obtained from a 6x telescope, that early in the study it was decided to use only the 6x glass for purposes of comparison with those of higher power, during daylight and bright twilight conditions. A very considerable number of observations of the stars were made at twilight with a dim horizon to compare the results obtained from the use of the best English and German 2x, 2.5x and 3x star telescopes, as against a 7 x 50 prismatic monocular. Under such conditions of poor horizon illumination, the 7 x 50 monocular was found very considerably superior to any other telescope tested.

It was found that the high-powered telescopes enabled the observer to pick up a number of fixed stars within a few minutes of sunrise or sunset. For example, it was regularly possible with a 20x telescope to complete strings of observations of five or six stars at sunset well before these stars became visible in the field of the conventional sextant telescope. The resultant gain in accuracy, due to very good illumination of the horizon, was marked.

It has been hoped that the high-powered telescopes would permit the observation of some fixed stars between the hours of sunrise and sunset. It was found that this could occasionally be accomplished; however, it did not prove generally possible, due to the loss of brilliance of the star's image caused by the index and horizon mirrors. It seems probable that this difficulty can be overcome by redesigning the marine sextant to permit a direct view of the star, with the horizon becoming the reflected image.

During the study, it became increasingly evident that the great accuracy achieved by the high-powered telescopes in measuring the altitude of a body above the horizon made it most desirable to determine the value of the dip accurately and consistently, which is not possible with the presently available tables. Even when the height of the eye was accurately established, anomalies in the value of the dip were encountered, although full allowance was made for barometric pressure, and air and water temperatures. Two Gavrisheff dip-meters were therefore obtained, one fitted with a 6x telescope, and the other with a 16x. The latter, particularly, gave excellent results, permitting the empirical establishment of the value of the existing dip with an accuracy, when it could be carefully checked, of about 0.1 minute.

It is held that a well designed sextant, manufactured to the close tolerances possible today, and equipped with 20x and 7x telescopes, together with a dip meter, permits a very marked improvement in the accuracy of fixing position afloat by celestial means, as compared to that now possible.

SUMMARY OF FINDINGS

Findings of this study may be summarized as follows:

1. The 20 x 50 mm prismatic monocular of good manufacture, is superior to any sextant telescope of lesser power during daylight hours. It reduces the random errors in a series of observations, as its great magnification permits the greatest nicety in obtaining contact between the body and the horizon. It is entirely satisfactory for use aboard ship except in extremely bad conditions of wind and sea.

2. Similarly the 7 x 50 mm prismatic monocular is superior to any other telescope available for test when the horizon is poorly defined, due to darkness.

3. A dip meter, fitted with a high power telescope, and capable of determining empirically the value of the dip to one-tenth of a minute of arc is an essential adjunct to the sextant, where accuracy is of primary importance.

EQUIPMENT AND TECHNIQUES EMPLOYED

The equipment employed to make the observations for this program falls into three categories: sextants, watches for the measurement of time intervals, and dip meters.

SEXTANTS: To obtain the observations for this study, five sextants were employed. All were of the micrometer drum type. One was a U. S. Navy Mark II Type A (Aluminum) instrument, one was a Hughes "Gothic" and three were manufactured by C. Plath. These latter three sextants were designed and manufactured for use in this study, and incorporated a number of new refinements. They had very large micrometer drums, and vernier scales calibrated to read to one-tenth of a minute; the scales were so large that they could easily be read to five one-hundredths of a minute. They were also fitted with large index and horizon mirrors which matched the full field of the 50 mm objective lenses of the scopes. The error in graduating the arc of the Plath sextants never is as great as 10 seconds; this also was a most desirable feature, as an error of up to 30 seconds is considered acceptable in ordinary U. S. manufacture.

Two of the Plath instruments had special light-weight alloy frames, the third Plath and the Hughes had brass frames, and were somewhat heavier; each weighed about 4 pounds 10 ounces. The alloy frame Plath weighed about 3 pound 12 ounces. The heavier instruments caused no complaints. It was found that they vibrated much less, when being used in a breeze, than did the Navy Mark II sextant.

Interchangeable 6x, 16x and 20x prismatic monoculars were available for the Plath and Hughes sextants, as well as 7x monoculars and 2x and 3x erect telescopes for use with the stars at dusk. All optics were hard-coated to permit maximum light transmission. Rubber eye cups were used with the big power telescopes; their use had a beneficial effect on the results achieved.

The Mark II Navy sextant was fitted with its only telescope, a 3x glass. During the course of the study, it became evident that the two telescopes holding the greatest promise were the 20x, for use at any time when the horizon was well illuminated, and the 7x for use with the stars, when the horizon was dim. Specifications were therefore drawn up with the assistance of Dr. S. G. Hall of the Naval Weapons Plant, Washington, D. C. for a prismatic monocular with a 50 mm objective, and interchangeable eye pieces, giving a magnification of 20x and 7x, respectively. This was manufactured by the firm Beck, of Kassel, Germany, who have had many years of experience designing and producing optics for telescopes. This telescope proved most successful, both on bench tests, and on the sextant.

The new Plath sextants and telescopes employed were found to be excellent and far superior to similiar equipment previously available. In expert hands they are capable of achieving an accuracy in determining the observed altitude considerably greater than that of the computed altitude obtained when the resultant observations are reduced by the Nautical Almanac and the modern navigational tables in common use afloat. These instruments also proved easier to keep in adjustment than the older type sextants.

Overall, with a bright horizon, the 20x telescopes proved slightly superior in accuracy to the 16x, and their greater power enabled the observer to pick up fixed stars somewhat more easily in a bright sky. Unfortunately, they do not have enough light gathering power to make them satisfactory when the horizon is dimly lighted. Under such conditions, the 7x telescope proved superior to any glass previously tested. It is recommended, therefore, that when the utmost accuracy is required, a 20x telescope with 50 mm objective be employed when the horizon lighting is strong, and a similar telescope, but of 7 power, be used when the lighting is weak. The single glass, with a 50 mm objective, and interchangeable 7x and 20x eyepieces proved most satisfactory.

The rapid determination of index error continued to be difficult, particularly during daylight hours, as it had for previous observers. This problem merits further study.

TIMING AND RECORDING: At all times during this program, multi-channel short-wave radio receivers were available, as were two highly reliable second-setting sweep second hand Longines watches. Observations were timed by split-second timers, reading to one-fifth second; these timers could be relied on not to vary one-fifth of a second in forty minutes, regardless of how many times the reading hand was stopped. An experienced observer can start these split-second timers on the WWV time tick with an accuracy of approximately one-tenth second. Whenever possible the timers were started directly from the radio tick, and checked again with the tick at the termination of the observations. When, for any reason, the radio tick was not available, the timers were started from, and checked against the Longines watches. It is believed that the time error in only a few instances exceeded two-fifths of a second; for the great majority of observations, the error did not exceed one-fifth second.

The timing of observations by the recorders who assisted in this program proved entirely satisfactory. The reaction lag of the two recorders who timed the majority of the sights was checked by means of a split second timer equipped to omit a short note when the stop button was pressed. Tests indicated that the lag rarely approached one-half second.

At Key West, a special watch was used, reading in arc to 360 degrees, and adjusted to sidereal time. This watch gave the local hour angle of Aries, and in conjunction with specially prepared altitude and azimuth tables of selected stars, proved most convenient for locating stars before they were readily visible.

Where the utmost accuracy is required, it is recommended that a remote recording chronograph be employed, combined with a remote read-out of the sextant altitude. This will obviate the possibility of error in the recording of time as well as error in the reading or recording of the altitude. In addition, it will greatly speed the taking of a string of sights, and the observer will not be faced with the danger of being unable to relocate a dim star, after reading the altitude; the sextant would be maintained in the observing position for an entire string of observation.

The chronograph should be so arranged that the second beat of the WWV time signal could be indicated on it when desired. It should also be possible to show the half-second beat of a chronometer, reading either Greenwich Mean or Sidereal Time. The altitude reading of the sextant would also be indicated on the tape, at the instant the time of the observation was recorded.

DIP METERS: Early in the program, it became evident that the new telescopes were capable of achieving a degree of accuracy in position fixing not previously obtainable with the marine sextant if the true value of the dip could be established. The depression of the visible horizon, below the horizontal at the eye of observer can readily be calculated. However, the line of sight from the observer to the horizon is affected by terrestrial refraction; the amount of this effect varies with changing weather conditions. Difference in air temperature along the line of sight from the observer to the horizon seem to be the chief cause of this variation. Where the water temperature differs from that of the mass of air above, the air immediately adjacent to the water is cooled or warmed. The effect of this difference is usually particularly strong on a calm windless day.

Many attempts have been made to establish a direct relationship between the difference of temperature of air and sea and the value for the correction for dip, however, the results obtained by different investigators differed so greatly that no definite conclusions can be drawn. It seemed most desirable, therefore, to determine the value of the existing dip empirically at the actual time of making celestial observations. For this purpose, two Gavrisheff dip meters were procured, one fitted with a telescope giving a magnification of 6x, the other with one of 16x. Both instruments proved to be of the greatest value, the one with the 16x telescope giving the better results. In the interests of standardization, a 20x telescope is recommended for future use with the dip meters.

The Gavrisheff dip meter measures the value of the dip by bringing into coincidence two images of the horizon, one erect and one inverted, and 180° apart in azimuth. The instruments employed in this study were fitted with micrometer drums, and vernier scales calibrated to two tenths of a minute of arc. They could be read to one-tenth of a minute by interpolation. An excellent feature of the Gavrisheff design is that index error can be cancelled by rotating the instrument through 180° about the optical axis of the telescope.

These dip meters were not available during the period when observations were made at Key West; they were first used aboard the USS VALLEY FORGE in January 1959 and their value immediately became apparent. Considerable differences between sea and air temperatures were encountered, and had a marked, but by no means consistent effect on the dip. This can be seen from the following tabulation, which presents a portion of the dip measurements made. In each instance the dip by meter represents the average of a number of measurements.

3 Jan. 1600 0-1 level, Air 48°, water injection 60°

Dip by meter, observer S	7.25 minutes
Dip by meter, observer W	7.2 minutes
Dip by tables	5.8 minutes

4 Jan. 1015 0-1 level, Air 53°, water injection 68°

Dip by meter, observer S	6.5 minutes
Dip by meter, observer W	6.45 minutes
Dip by tables	5.8 minutes

In view of the 15° difference between sea and air temperatures, it seemed that the dip as measured by the meter was low in this instance, and it was measured from another level, with the following result:

4 Jan. 1040 0-6 level, Temp. as above

Dip by meter observer S	9.625	minutes
Dip by meter observer W	9.6	minutes
Dip by tables	8.9	minutes

9 Jan. 1100 0-6 level, Air 67°, water injections 70°

Dip by meter observer S	8.7	minutes
Dip by tables	8.9	minutes

Such celestial observations as could be obtained during this period, when corrected for dip as obtained by the dip meter, always resulted in lines of position lying much closer to Loran fixes than those corrected by means of the dip tables.

During this same cruise the USS VALLEY FORGE lay-to some 7 miles from Bermuda, where her position could be accurately fixed by triangulation and radar ranges. Sun sights were obtained, and reduced from the known position. Using the value of the dip as obtained by dip meter in correcting the altitudes, the mean distance from the known position was 0.115 miles; when the tabulated value of the dip, corrected for barometric pressure and temperature was employed, the mean distance was 0.85 miles.

These anomalies in the value of the dip are particularly interesting, as the winds were extremely strong (up to 65 knots) during much of this period, and because many of the dip measurements were made with a height of eye of over 80 feet. Both of these factors are supposed to reduce any anomalies in the dip at sea.

Aboard the USS TUTUILA, in August of 1960, a diurnal variation of one minute was found in the dip by measurement, without appreciable changes in barometric pressure and sea and air temperatures. This variation persisted over several days; the value of the dip being one minute greater in the afternoon and evening than in the morning. That this anomaly existed was also demonstrated by the plot of morning and evening star sights; the lines of position resulting from evening stars each having to be moved one minute towards the respective bodies to cross close about a common point.

There seems little doubt that a dip meter of the quality of the Gavrisheff instruments is of the greatest value in determining the value of the dip, and consequently improves materially the accuracy of celestial navigation.

It cannot correct for error caused by the tilt of the horizon, or for the transitory effect of a wave in the horizon, but it will permit accurate correction of much the greater part of the errors caused by abnormal terrestrial refraction.

It should be noted that the dip meter fitted with the 16x telescope gave materially better results than the one with the 6x telescope. Consecutive readings in each string gave more consistent readings, the magnitude of the random errors being materially reduced.

OBSERVATIONAL TECHNIQUES: It was held that the maximum diameter of the objective lens of any telescope to be used with a sextant would be 50 mm; anything larger than this would be too large and clumsy to handle conveniently aboard ship. As the field of view of a prismatic telescope of normal design with an objective lens of this diameter and giving a magnification of 20x is about three degrees, it was apparent that the Rude Star Finder would not be sufficiently accurate for locating stars, particularly at the higher altitudes, in a brightly lighted sky. Accordingly, for use at Key West, the altitude and azimuth of six morning and seven evening stars were computed for 30 degrees of Local Sidereal Hour Angle. These altitudes and azimuths were precomputed to the nearest minute of arc, spanning the times of sunset and sunrise for the period of field work. In conjunction with the watch mentioned above, reading in Local Sidereal Hour Angle, and an eight inch spherical magnetic compass, these tables greatly facilitated locating the stars against a bright sky. The stars for which the Key West altitude and azimuths were precomputed are as follows:

Morning:		Evening:	
Altair	(Mag. 0.9)	Aldebaran	(Mag. 1.1)
Antares	Mag. 1.2	Betelgeux	Mag. Var.
Arcturus	Mag. 0.2	Capella	Mag. 0.2
Deneb	Mag. 1.3	Pollux	Mag. 1.2
Spica	Mag. 1.2	Procyon	Mag. 0.5
Vega	Mag. 0.1	Rigel	Mag. 0.3
		Sirius	Mag. 1.6

The times of sunrise and sunset of Key West for the period of observations were also computed.

Altitude and azimuth data as outlined above were also prepared for all subsequent twilight star observations, although not to such close tolerances. These were frequently presented in graphic form.

Most of the observers connected with this program were accustomed to using the somewhat heavier Plath and Hughes sextants. The 20x and 16x telescopes with their very considerable magnification, and comparatively small field however, were new to all save one observer, and several hours of practice were required before the users became accustomed to them. The apparent magnification of vibration with these telescopes particularly caused adverse comment at first, and the observers believed that the plots of their strings of sights made with the high powered glasses would be inferior to those made with a 6x monocular. It always proved a source of surprise to the observer to find, when the sights were plotted, that the strings of observations made with the higher powered glasses were considerably smoother than those made with the 6x telescope. Without exception, once the observers became accustomed to the new glasses, they preferred them to those of six power, when accuracy was particularly required.

Tests were made using a bipod to support the sextant, which was mounted at the apex, while the two legs attached to the observer's belt. This device was not found to be practical. Thought was also given to mounting a sextant on a gun stock and also to suspend it from a gallows frame, but no experiments were conducted with such devices.

No unusual techniques were employed in making the observations; every effort was made to take the sights in each string as rapidly as possible. When two observers were working at the same time, simultaneous observations of the same body would frequently be made, one observer using a 20x telescope, and the other a 6x. At the completion of each string of sights telescopes (but not sextants) would be exchanged. When one observer was working alone with a recorder, one string would be completed with one telescope, and the next one with a glass of different power. The value of the high powered telescopes, as compared to those giving a magnification of 6x, for picking up stars during "bright sky" conditions also was studied, and is discussed below.

A string of sights normally consisted of 10 to 13 observations; accuracy tends to deteriorate after about that number, apparently due to eye fatigue. However, a very brief rest for the eye was sufficient to restore accuracy, and frequently 10 to 12 strings would be observed over a period of about an hour. Where the same conditions of lighting and weather persisted, the last string would generally plot as smoothly as the first, provided a brief period had been allowed between strings.

All strings were plotted on graph paper, using a scale of one inch to 10 seconds of time, and when the rate of change of altitude was not too great, one inch to one minute of arc; occasionally, it was desirable to change the latter to one inch to two minutes of arc. The strings were completed in between two minutes 30 seconds and three minutes 30 seconds. Due to the smaller field of the 20x telescope, it was found that overall the time between sights with this glass tended to be slightly longer than when using the 6x. This time difference would not exist if a sextant with remote altitude read-out were employed with the 20x telescope, as the observer would not have to relocate the body after each individual observation.

After the sights were plotted, a line of best fit was drawn through the plots; due to the shortness of the time period involved, a straight line gave a satisfactory representation of the body's path. The rate of change of altitude for the body at the mid time of the string was then calculated, to check for possible anomalies caused by terrestrial refraction, or other causes. Such anomalies, affecting the rate of change of altitude, were occasionally encountered.

When working from a position that could be accurately established, usually by triangulation, the calculated altitudes of the body for the site, corrected for dip, refraction, etc., for the time of the string, would then be plotted on the graph. The deviation of the individual sights from the line of best fit for each string would be measured for comparison purposes, as well as the difference in altitude between lines of calculated altitude and the lines of best fit. As has been stated above, the results achieved with the high powered telescopes were superior almost without exception; the rare exception occurring when observational conditions were deteriorating, and the high powered glass was the last one used.

The magnitude of the random errors about the line of best fit was consistently and materially less for the high powered telescopes, as was the difference between the line of best fit and the line of calculated altitudes. The superiority of the high powered telescopes was particularly great when the horizon was not sharply defined.

Key West was not found to be a very satisfactory locale for the purposes of this program, as the water mass inshore is not homogeneous. A considerable number of surface water temperature measurements were made four to five miles offshore (i. e. at the approximate apparent horizon of the observer on the beach), and the temperature was found to vary as much as 10° F within 200 yards. In this connection it may be noted that some 30 strings of sights were made from aboard a 46 foot sailboat 40 to 60 miles offshore in waves up to six feet in height. The resultant intercepts varied less in magnitude than many of those made from the beach at Key West; apparently the boat was surrounded by water of the same temperature.

Mechanical vibration, as in a destroyer steaming at 20 knots, did not have a markedly adverse effect on the smoothness of the plot of sights. The high powered telescopes produced consistently better results than those with a magnification of 6x, despite the firm conviction of the less experienced observers that the contrary would prove true. High winds, as are often found on the open bridge structure of a carrier, made it extremely difficult to obtain satisfactory observations with any telescope.

EFFECT OF SEAS: Sights were reduced by various methods, as seemed appropriate. Where the utmost accuracy was desired, the Ephemeris was used together with the classical sin-cos altitude and azimuth formulae. Six-place natural functions were used with a computer. Where extreme accuracy was not of prime importance, the Nautical Almanac and the Tables of Computed Altitude and Azimuth (HO 214) were employed. The Delta t and Delta 1 corrections, as well as Delta d, were usually used to reduce the length of the intercepts. All sun sights were corrected for the actual semi-diameter, parallax, and for refraction; the combined semi-annual tables in the Nautical Almanac were not used. For all sights, the table of "Additional Refraction corrections for Non-standard Conditions", published in the Nautical Almanac, was used as required. Before the dip meters became available, the Japanese tables for correcting for the difference between air and sea temperatures (0.11 minutes per degree Fahrenheit) were also employed, and in the majority of cases proved helpful. The corrections they provided in general shortened the intercepts, and in the geographical area of this study, off the eastern seaboard of the United States, were superior to any other similar tables used. However, the accuracy they provided did not equal that achieved with the dip meter.

It had been hoped that the brighter fixed stars, favorably situated in altitude and azimuth relative to the sun, would be visible during the day in the field of a high power telescope, mounted on a marine sextant. This hope was based on a paper entitled "The Visibility of Stars in the Daylight Sky" by Dr. Richard Tousey and E. O. Hulburt, and published in the Journal of the Optical Society of America, Volume 38, No. 10 October 1948. Consultations with Dr. Tousey at the Naval Research Laboratory were held, and confirmed that fixed stars could be observed during daylight hours with a well designed telescope, having a magnification of 16x or 20x, particularly if the telescope were provided with a reticle at infinity to aid the eye in maintaining focus at infinity. It also appeared that it would be desireable to provide a polarizer for the telescope in connection with these observations. The reticles were accordingly installed, and polarizers were provided.

These telescopes did indeed make it possible to pick up stars, when viewed directly, during the daytime. However, no real success can be claimed when the telescopes were mounted on sextants, although Sirius was picked up low to the eastward on two occasions about 15 minutes before sunset and Arcturus was seen at high altitude 25 minutes after sunrise. The light loss in reflecting the star's image through the index and horizon mirrors was too great to make daytime star observations practicable with the sextant in its present form.

The high-powered telescopes did, however, make it possible to observe a very considerable number of fixed stars in bright twilight, when they could not be sighted with a 7x or 6x telescope. These latter glasses, in turn, proved superior in this respect to the 3x telescope as supplied with the Navy Mark II sextant. At Key West, strings of observations totaling between 40 and 60 sights were completed of 4 to 5 stars with a 20x telescope, within 20 minutes of sunset; that is to say, with the 20x telescope, the observations for the evening star fix were completed at about the time a navigator using the ordinary sextant and telescope would start making his observations.

Similarly, on 30 August 1960, in approximately latitude 30° N, Longitude 79° W, 5 stars, as tabulated below, were sighted with a sextant fitted with a 20x telescope within 10 minutes of sunset.

Star	Magnitude	Approx. Elevation	Approx. Azimuth
Altair	0.9	46	110
Antares	1.2	34	194
Arcturus	0.2	50	265
Deneb	1.3	45	054
Vega	0.1	69	160

Stars observed under such conditions, with a well illuminated and sharply defined horizon, yielded far better results than those made in deeper twilight with a 6x or 7x telescope. However, with the sun well below the horizon, the 7 by 50 monocular proved superior in every respect to any other telescope employed. Its large field, excellent light transmission, and sharpness over the entire field, are most desirable features for this use. It should be noted that the 7 by 50 telescope is not satisfactory for daytime use with the sextant, particularly for sun observations. During such observations, there is frequently a very strong and troublesome halation effect.

ACCURACY: The degree of accuracy obtainable with the instruments employed during this program in measuring both time and altitudes and in establishing the value of the dip is so fine, that thought must be given to improving the methods employed in reducing sights. The American Nautical Almanac, under the heading "Accuracy" says in part:

"The largest error that can occur in the G. H. A. or Dec. of any body, other than the Sun or Moon is less than 0. 2 minutes; it may reach 0. 25 minutes for the G. H. A. of the Sun and 0. 3 minutes for that of the Moon.

In practice it may be expected that only one-third of the values of G. H. A. and Dec. taken out will have errors larger than 0. 05 minutes and less than one-tenth will have errors larger than 0. 1 minutes. "

It should be added that these errors in Greenwich Hour Angle and Declination may be cumulative, thus causing a considerable total error. As the time of an observation can be accurately determined to one-fifth of a second, it is most desirable to be able to determine the G. H. A. of the body at the instant of observation with an accuracy of 0. 05 minutes of arc.

The high-power telescopes make Venus available for daylight observation during much of the year, yet corrections to the altitude of this planet for parallax and phase for such observations are not given. The semi-diameter of Venus should also be given, as in the field of a 20x telescope the planet is not a joint of light, but has very appreciable size.

Where accuracy is paramount, it is recommended that observations be reduced from the estimated position, rather than from an assumed position. Smart, "in the Monthly Notices of the Royal Astronomical Society", Volume 79, (May 1919) showed that when the difference between the true and assumed positions is 30 minutes in both latitude and longitude, for an altitude of 75 degrees the error might be 1. 0 miles. At latitude 60 degrees this would diminish to 0. 7 miles; while the probable error would not exceed 0. 3 miles. It can be seen, therefore, that plotting celestial observations from an assumed point may introduce unacceptable error.

The "Tables of Computed Altitude and Azimuth" (H. O. 214) are among the most satisfactory of the so-called "short methods", and are excellent for most navigation. However, they also can cause unacceptable error, even though the Delta T and Delta L corrections are used in addition to that for Delta d. For example:

On page 509 of the 1958 edition of Bowditch (HO9) the calculated altitude 20 degrees 55. 9 minutes is obtained using HO 214 and the corrections Delta d, Delta t and Delta L. The correct answer is 20 degrees 55. 67 minutes.

It is evident therefore that the methods commonly employed at sea for the reduction of celestial observations are not compatible with the degree of accuracy obtainable with the instruments discussed herein. The Ephemeris is available in lieu of the Nautical Almanac and gives the necessary data to a degree of accuracy far beyond that required at sea. However, its presentation is designed for the astronomer, and necessitates long and cumbersome interpolations. Probably the most desirable form for presenting the necessary data would be to state Greenwich Hour Angle, Sidereal Hour Angle, and declination in degrees, minutes, and hundredths of minutes. For some bodies, to facilitate interpolation, it might be well to state Greenwich Hour Angle for some period shorter than an hour.

An electronic computer can, of course, store the necessary data on the celestial bodies to be observed during any program, and can also reduce the observation. Where such a device is not available, the formulae:

$$\text{Sin } h = \sin L \quad \sin d \pm \cos L \quad \cos d \quad \cos t$$

$$\text{and} \qquad \text{Sin } Z = \frac{\cos d \quad \sin t}{\cos h}$$

give excellent results, and a speedy reduction, when used with six or seven place tables of natural functions, and an ordinary desk computer. Precomputation will frequently be most desirable.

A string of observations of each body should be made; this can be done rapidly, particularly where the remote read-out of time and altitude are employed. Such a series of 10 to 15 observations, when plotted, permits the rejection of unsatisfactory individual sights, as well as the detection of transient anomalies, such as waves in the horizon. It also permits an evaluation of the probable accuracy to be derived from the reduction of the selected observation in the string.

The observer should be selected for his visual acuity: he need have no knowledge of navigation. In this connection, it may be noted that some of the best strings of observations obtained during the course of this study, were made by young observers who had never previously used a sextant. They had good vision, and they were also free of prejudice against the high-power telescopes. An expert marksman frequently seems to make an excellent observer.

CONCLUSION

1. It is held that the high powered telescopes, of a magnification of about 20x, when fitted to sextants of the highest quality, and which allow a fine read-out of altitude, permit achieving a markedly superior degree of accuracy than has heretofore been possible with the instruments in general use.

   This improvement in accuracy is derived from three characteristics of the high powered telescopes:

   A. Their increased magnification permits the greatest nicety in making contact between the body and the horizon.

   B. Their high magnification makes fixed stars visible in comparatively bright light, just before sunrise and directly after sunset, when the horizon is sharply defined.

   C. In the great majority of instances, during daylight hours when the horizon is hazy, they permit markedly better determination of the horizon than do the telescopes of 3x and 6x in general use.

   It appears from this study that the accuracy achieved in measuring altitudes with the sextant is in direct proportion to the magnification of the sextant telescope. A 20 by 50 prismatic monocular, of good design, at present seems to be the optimum for use with a bright horizon. The degree of accuracy achieved by such a telescope far outweighs the disadvantages of its comparatively small field and increased weight.

2. At twilight, when the contrast in lighting at the horizon is not strong, the 7 by 50 monocular is found to be superior to any star telescope now in general use.

3. A Gavrisheff dip meter, fitted with a high power telescope can be of the greatest assistance during daylight hours in determining the existing value of the dip of the horizon, and therefore in increasing the accuracy derived from daytime observations of the sun, moon, and planets.

4. Finally, it is held that a ship's position may be fixed at sea, under good observational conditions, to about 0.25 miles, by a round of multiple observations of stars, made just before sunrise or after

sunset, with a high quality sextant, fitted with a 20x telescope. This conclusion is based on the results achieved during the course of this study. Remote read-out of sextant altitude and time should improve this somewhat.

With 6x and 3x star telescopes, it has heretofore been possible under similar conditions to obtain an accuracy of about 0. 4 miles.

THE 20x TELESCOPE FOR STAR SIGHTS

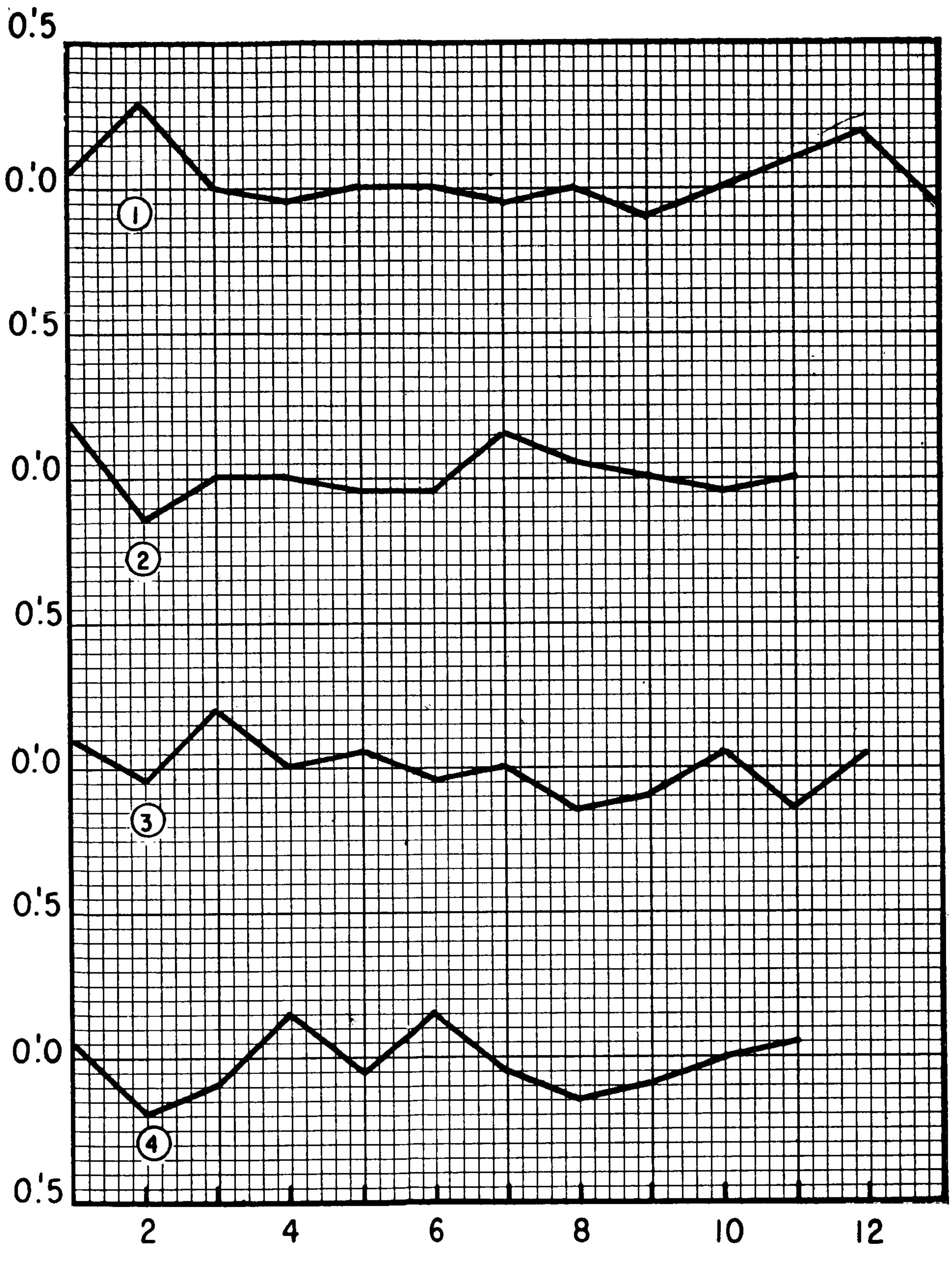
On the opposite page are the plots of four strings of star observations, made with a 20x telescope on the evening of 3 March 1958. The stars, in the order in which they were observed, are Sirius (-1. 6), Aldebaran (1. 1), Betelgeux (Var.), and Rigel (0. 3). The first observation of Sirius was made directly after the sun had set, the last observation of Rigel was completed some 18 minutes later.

No direct comparison between the 20x and the 6x telescopes were possible, as the observer using the 6x glass was unable to pick up the stars concurrently.

The mean aberration from the line of best fit of the Sirius observations is 0. 069 minutes, for Aldebaran 0. 064 minutes, for Betelgeux 0. 079 minutes, and for Rigel 0. 095 minutes.

These observations serve to show the high degree of accuracy that can be expected from star observations, made with a high-powered telescope during the period that the horizon is brightly illuminated.

The time interval between observations is plotted as constant.


0'5
0'0
1
0'5
0'0
2
0'5
0'0
3
0'5
0'0
4
0'5
2
4
6
8
10
12

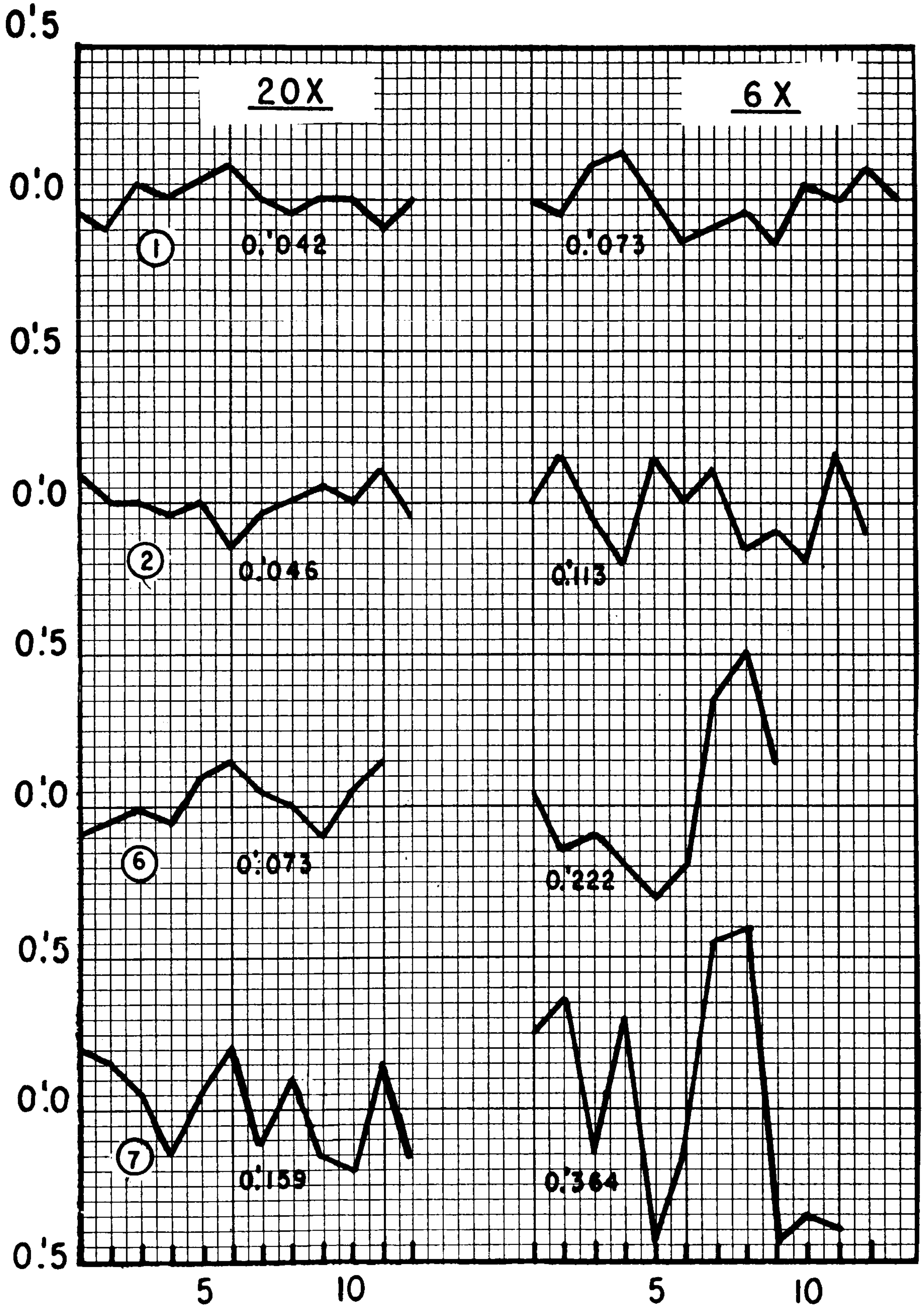
COMPARISON OF OBSERVATIONS OF THE SUN MADE AT SEA WITH 20x AND 6x TELESCOPES

On the facing page appear graphs showing the aberration from the "line of best fit" of individual observations within strings of Sun sights, made at sea with 20x and 6x telescopes. The mean aberration for each string is noted below.

These graphs show the first two and the last two pairs of strings cited in Section A of Appendix I.

The observations in each pair of strings were made by one observer. When the string was completed with one telescope, the latter would be changed as rapidly as possible, and the second string of the pair would be completed.

The time interval between observations is plotted as constant.



0.'5
20X
6X
0.'0
①
0.'042
0.'073
0.'5
0.'0
②
0.'046
0.'113
0.'5
0.'0
⑥
0.'073
0.'222
0.'5
0.'0
⑦
0.'159
0.'364
0.'5
5
10
5
10

COMPARISON OF SIMULTANEOUS OBSERVATIONS OF THE SUN MADE AT SEA WITH 3x AND 20x TELESCOPES

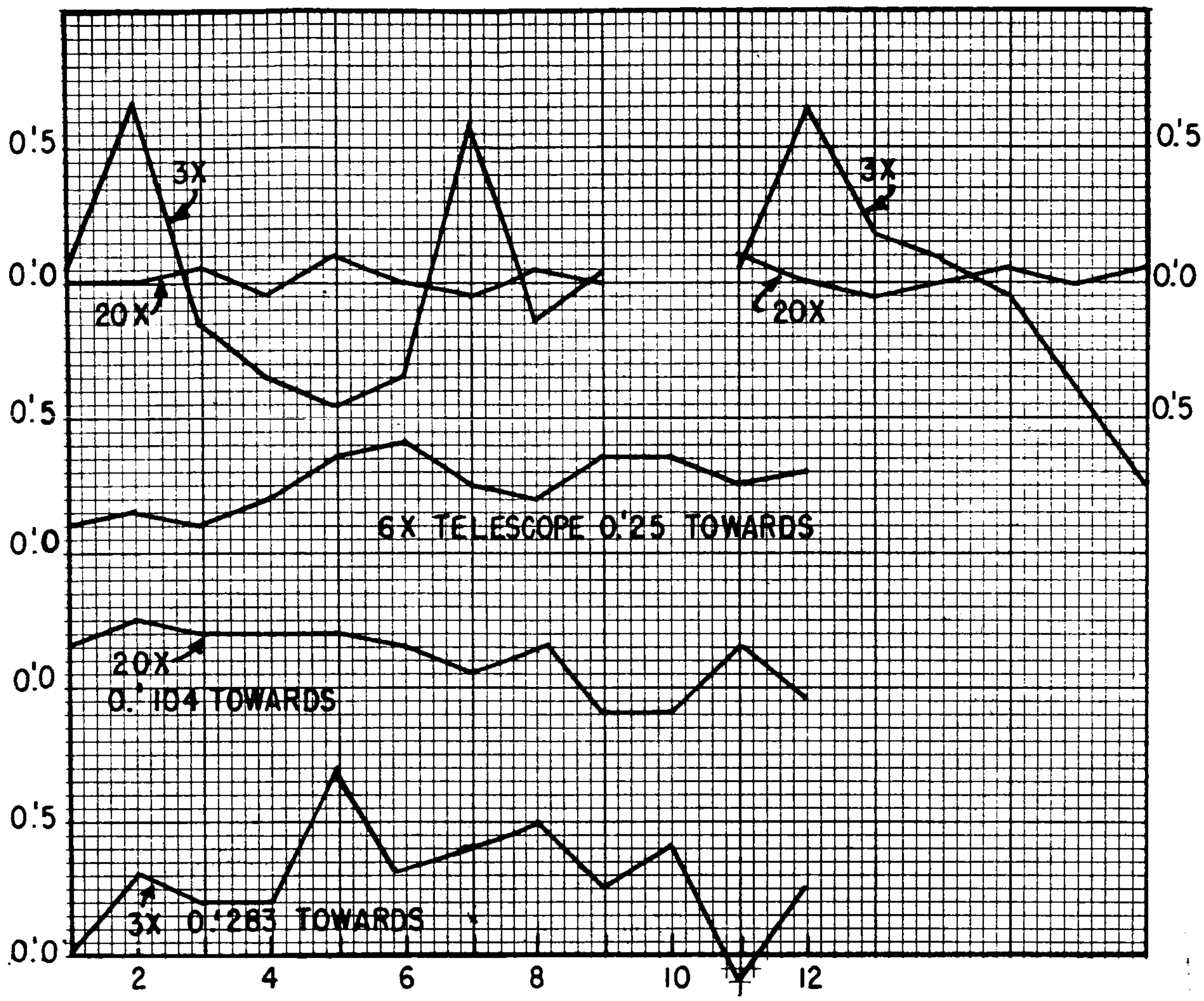
The graphs on the opposite page illustrate results obtained aboard the U. S. S. TUTUILA. These observations were made at about 1030 Zone Plus Five Time on 18 May 1960, some 100 miles S. E. of the Virginia Capes, by two experienced observers. Simultaneous observations were made of the Sun's lower limb, one observer using a Plath sextant fitted with a 20x telescope, the other a U. S. Navy Mark II sextant with a 3x telescope, as supplied with the instrument.

On completing the first string of sights, the observers exchanged sextants, and the second string of observations was made.

There was some overcast, which accounts for the rather short second string. The wind was S. W., Beaufort scale #3, the state of the sea was #3, and with the 3x telescope, the horizon appeared very "fuzzy".

On the lower part of the page appear the plots of three consecutive strings of Sun sights, made with a 3x, 20x, and 6x telescope in that order by the same observer, from a known position. The 3x telescope placed the observer 0. 283 minutes too much towards the sun, the 20x made it 0. 104 towards, and the 6x 0. 250 towards. The horizon on this occasion was not sharply defined.

The time interval between observations is plotted as constant.



0'.5
0'.0
0'.5
0'.0
0'.0
0'.5
0'.0
0'.5
0'.0
0'.5
3X
20X
3X
20X
6X TELESCOPE 0'.25 TOWARDS
20X
0.'104 TOWARDS
3X 0'.283 TOWARDS
2
4
6
8
10
12

APPENDIX I

The following tabulations show in numerical form the smoothness of plot of strings of observations using alternately a 20x and a 6x telescope with the same sextant. It gives an indication of the improvement in accuracy made possible by the high-powered telescope. These observations were made during the daylight hours, by a number of observers. Some were made from the beach, and some at sea; they were made under widely differing observational conditions. The majority were of the sun, although observations of the moon and of Venus are also included.

A string of observations, usually between 10 and 14 in number, would be made with one telescope, which would then be changed for one of different power. The altitudes were plotted, as described in the body of the text, and the "line of best fit" was drawn for each string. The deviation in altitude of each individual observation from the line of best fit was measured to the nearest five-one hundredths of a minute, and these deviations were then added, regardless of sign. The sum of the deviations in each string was then divided by the number of observations in the string, to give the various results listed below.

The results are arranged in two groups, according to whether the observations were made afloat, or from the beach. Many observations were made when the horizon was very poorly defined, in order to obtain data on the performance of the high-power telescopes under such conditions. However, no criteria are available for grading the quality of the horizon for observational purposes. It seemed best, therefore, to make an arbitrary arrangement in descending order of the values of results obtained with the 20x telescopes. The poorest, as well as the best results achieved are included in the tabulation.

Graphs appear elsewhere in this report, which illustrate the distribution of sights within some individual strings listed below, as well as in strings not made for direct comparison purposes between various telescopes.

A. OBSERVATIONS AT SEA

	20x telescope	6x telescope	Body
1.	0.042	0.073	Sun
2.	0.046	0.113	Sun
3.	0.050	0.138	Sun
4.	0.054	0.192	Sun

5.	0.061	0.108	Sun
6.	0.073	0.222	Sun
7.	0.159	0.364	Sun

B. OBSERVATIONS FROM THE BEACH

	20x telescope	6x telescope	Body
1.	0.027	0.060	Sun
2.	0.031	0.089	Venus
3.	0.036	0.078	Sun
4.	0.058	0.054	Moon
5.	0.063	0.127	Sun
6.	0.077	0.227	Sun
7.	0.083	0.123	Sun
8.	0.085	0.323	Sun
9.	0.092	0.273	Sun
10.	0.119	0.716	Sun
11.	0.127	0.196	Sun
12.	0.138	0.200	Sun
13.	0.150	0.231	Sun
14.	0.162	0.265	Sun
15.	0.173	0.327	Sun
16.	0.178	0.112	Sun
17.	0.204	0.373	Sun

APPENDIX II
RESUME OF RECOMMENDATIONS

Various recommendations are made in the body of this report; they are outlined below. Additional recommendations are also included, not necessarily in the order of suggested priority.

I. THE SEXTANT

It is doubtful if the basic design of the sextant can be improved for ordinary marine use. However, it can undoubtedly be improved in detail, and modified for specialized use, in order to gain improvement in accuracy. Few improvements have been made in the sextant during the past three quarters of a century; it is time that a thorough study of the instrument, as a tool for celestial navigation, be initiated.

Set forth below are suggestions as to some of the matters which might be included in such a study.

A. 1. For celestial navigation, particularly where accuracy is important, a sextant with minimal constant or non-adjustable error is required. In the past, the standards of required accuracy have been too low. In this country, a constant error of 30 seconds has been permissible, and an instrument with a maximum error of 15 seconds has been considered superior.

It is recommended that the value of the maximum permissible constant error be redetermined for sextants intended for refined celestial observations. It is suggested that this maximum error be established as 6 seconds. Such a standard is not too high, and can be achieved under modern production methods. For example, the certificate of Plath sextant #39393, used in this program, shows the following constant errors:

Altitude	10	20	30	40	50	60	70	80	90	100	110
Error in Seconds	0	+1	+3	+2	-1	+1	-1	-4	-6	-3	0

2. For celestial navigation, an instrument is required that measures angular elevation to 90 degrees. "Back sights" are now rarely used, and a sextant capable of measuring angles to 145 degrees or thereabouts is not needed. It is possible that an instrument

in the shape of an octant measuring angles to 90 degrees might be more convenient to use. This means study. At the same time, it should be determined whether mechanical accuracy could be improved by increasing the radius of the arc. The design of a sextant, intended for the daylight observation of stars is discussed below.

3. The design of sextant mirrors also warrants study. These should, of course, be of a size compatible with the field of the telescope employed. It would seem that the use of a front coated mirror would be most desirable; particularly for the index mirror. With the conventional "back coated" mirror, particularly at high altitudes, the light from the body is diminished materially by the considerable thickness of glass through which it must pass. The development of such front coated mirrors for use on the sextant, seems most desirable. If front coated mirrors are not employed, the horizon mirror should be positioned in such a manner relative to the index mirror that light rays from a body, observed at high altitude are not at an excessively acute angle to the latter mirror. The more acute this angle is, the more attenuated are the light rays as seen by the observer. It might be well to place the horizon mirror forward of its normal position on the instrument.

4. The clear glass portion of the horizon mirror does not appear to fulfill any useful purpose. It, also, absorbs light, particularly when particles of salt adhere to it. Some experienced navigators have this clear glass portion removed from the mirror, this would seem advantageous.

5. Various prisms have been fitted on sextants, particularly for star observations. Astigmatizers have been employed, which turn the image of the star into a line; these are sometimes helpful when the star is bright, and the horizon is poorly lighted. Under these same conditions a Wolaston prism can also be a help. This gives two images of the star, separated by about 5 minutes of arc in the vertical plane, the true position of the star being half-way between the two images.

   The Van Leeuwen prism has been used with great success for sun observations with fine transits. It seems possible this prism might prove equally satisfactory when used with a sextant. The center of the sun is, in effect, observed on the horizon, and there would be no danger of error due to irradiation effect, and no correction would be required for the sun's semi-diameter.

6. Variable density polarizing shade glasses are at present standard on U. S. Navy sextants. Opinion as to their superiority over a series of neutral tinted shades, graduated in value, is divided. Under certain conditions, as when observing a brilliant star against a dim horizon, a conventional type shade, of rather low light absorption, seems superior to the variable density type; some observers prefer the conventional shades under all conditions. It is recommended that the two types of shades be evaluated under service conditions.

   During a study of terrestrial refraction in Chesapeake Bay made by the Naval Research Laboratory NRL Problem N03-05, Project No. N0284-512, it appeared that a red filter could at times be helpful in making observations of the horizon. The benefit to be derived from using horizon filters of various colors warrants study. During this program, it was found that under certain conditions of low contrast between sky and water, a single polarizing shade, of low light absorption gave a well defined horizon.

   It may be noted that most sextants do not have enough horizon shades to permit proper adjustment for various degrees of horizon lighting.

7. For star observations with the sextant at dusk, it is most desirable that there be a convenient method of adjusting the position of the horizontal axis of the telescope, relative to the frame of the sextant. Moving the telescope out from the frame increases the ratio of light from the horizon, relative to that from the star. Most sextants incorporate some provision for accomplishing this; usually, however, it cannot be done readily. The Hughes "Gothic" sextant has a large thumb screw, so situated that the observer can easily turn it with his thumb, while holding the sextant to his eye. It is recommended that this feature be incorporated in all marine sextants.

8. During daylight hours particularly, it is difficult to establish accurately the sextant index error. It is recommended that this problem be studied.

B. The design of telescopes for use with the sextant warrants study. For daylight use, the 20x telescopes used in this program seemed more satisfactory, on the whole, than those of 16x. However, it may develop that better results can be achieved with a telescope of either greater or lesser magnification that 20x. Whatever the power of such a telescope, it should have as large a field as possible, with sharp focus over the entire field, and excellent light transmission characteristics; the sextant mirrors would, of course, be of an appropriate size.

Similarly, the characteristics of the optimum telescope for use at dusk should be determined. The 7 by 50 telescopes used proved superior to any of the star telescopes generally available, but here again, further improvement is probably possible.

Interchangeable eye pieces for use with the same telescope reduced the amount of equipment required for this program. The possibility of developing sextant telescopes with interchangeable eye pieces should be studied.

All sextant telescopes should be equipped with soft rubber eye cups; they are of great importance with high power telescopes. These enable the observer to steady the sextant against his eye socket, and keep wind and stray light from his eye. The eye cups that are available commercially are not too well suited for this purpose. It is recommended that a cup be designed especially for use with the sextant telescope.

C. The remote read-out of sextant altitude, plotted against time, would be of great advantage. It would remove the chance of human error in noting time and altitude, and would very markedly decrease the time required to complete a string of observations, as the observer would not be required to lose sight of the body, while reading the sextant altitude.

When such a read-out is employed, study is warranted to determine the period of time that should be spent in completing any one string of observations. While anomalies in refraction, lasting for considerable periods of time sometimes occur, transient anomalies causing considerable error in the observed altitude also are encountered. These latter anomalies, lasting usually less than a minute can usually be detected if frequent observations are made over about a two minute period. A string of some six observations, or so made rapidly, can be very misleading, however. While the altitudes may plot smoothly against time, they do not represent the true change of altitude of the body.

D. It is recommended that a sextant be developed for the daytime observation of fixed stars in conjunction with a high-powered telescope. Dr. Richard Tousey, of the Naval Research Laboratory, has shown that stars of the first magnitude, properly situated, can be seen during daylight hours on a clear day (Journal of the Optical Society of America, Volume 38, No. 10, pp. 886-896, October, 1948). On

13 August, stars were observed as follows: at 0840 Sirius (mag. -1.6), 0905 Rigel (mag. 0.3), 1130 Capella (mag. 0.2), 1400 Arcturus (mag. 0.2), 1515 Spica (mag. 1.2), and at 1535 Vega (mag. 0.1). Dr. Tousey states that with a polarizer fitted to the telescope, all these stars were "very easy" to find, with the exception of Vega, which he describes as "easy" to locate. In addition to a polarizer, a telescope used for the daylight observation of stars should be fitted with a reticle at infinity, to assist the eye in maintaining focus at infinity.

During this program, Arcturus was observed at high altitude well after sunrise, and Sirius before sunset with a 20 power telescope fitted to a marine sextant. However, the light absorption in the sextant mirrors is too great to make such observations practical with a sextant in its present form.

The design of the sextant could be modified to make daylight star observations feasible. The star would be observed directly, rather than as a reflected image, and the image of the horizon would be brought up to the star by means of the index mirror. This would require a rearrangement of the mirrors on the sextant. The high power telescope would be fitted with a polarizer and a reticle, as described by Dr. Tousey. In addition, it should have two or three interchangeable prismatic eye-pieces, such as are used with the telescopes of fine transits, to facilitate the observation of stars situated well above the horizon. Such an instrument, intended only for celestial navigation, might well be made as an octant, measuring altitudes only to 90 degrees, the scale being lengthened slightly to give a negative reading or reading "off the scale" to determine index error. The markings on the arc would be reversed from those on the ordinary sextant, the zero mark being at the end of the arc away from the observer.

The fixed mirror would be located in the 45 degrees - 90 degrees sector of the frame; this would necessitate swinging the index arm on the underside of the frame. This mirror could be made rather narrow in the horizontal axis, as the illumination of the horizon during daylight would be strong enough to give a good image; it should probably be without the clear glass portion usually found on the horizon mirrors of conventional sextants.

II. THE DIP METER

The dip meter is considered to be a necessary adjunct to the sextant for refined navigation. The Gavrisheff dip meters used during the latter part of this program proved of great value, and are superior in design to similar instruments encountered. A most desirable feature of the Gavrisheff instrument is that any index error is cancelled by rotating the device through 180 degrees about the optical axis of the telescope, between readings.

It is strongly recommended that the dip meter be studied with a view to improving its optics, and facilitate its use at sea. In the Gavrisheff dip meters presently available, the light absorption on the inverted horizon is sufficient to make observations difficult under certain conditions of light. This should be corrected, if possible.

The placement of the knobs or wheels for bringing the two horizons into coincidence should be so arranged that the dip meter is equally convenient to use in either the erect or inverted position.

The qualities of the telescope or telescopes to be used with the dip meter should be determined. It would seem desirable in the interests of simplicity to design interchangeable telescopes which fit both the dip meter and the sextant if the same telescope is equally satisfactory on both instruments. It should be noted that the highest power telescope (16x) available for use with the dip meter gave the best results.

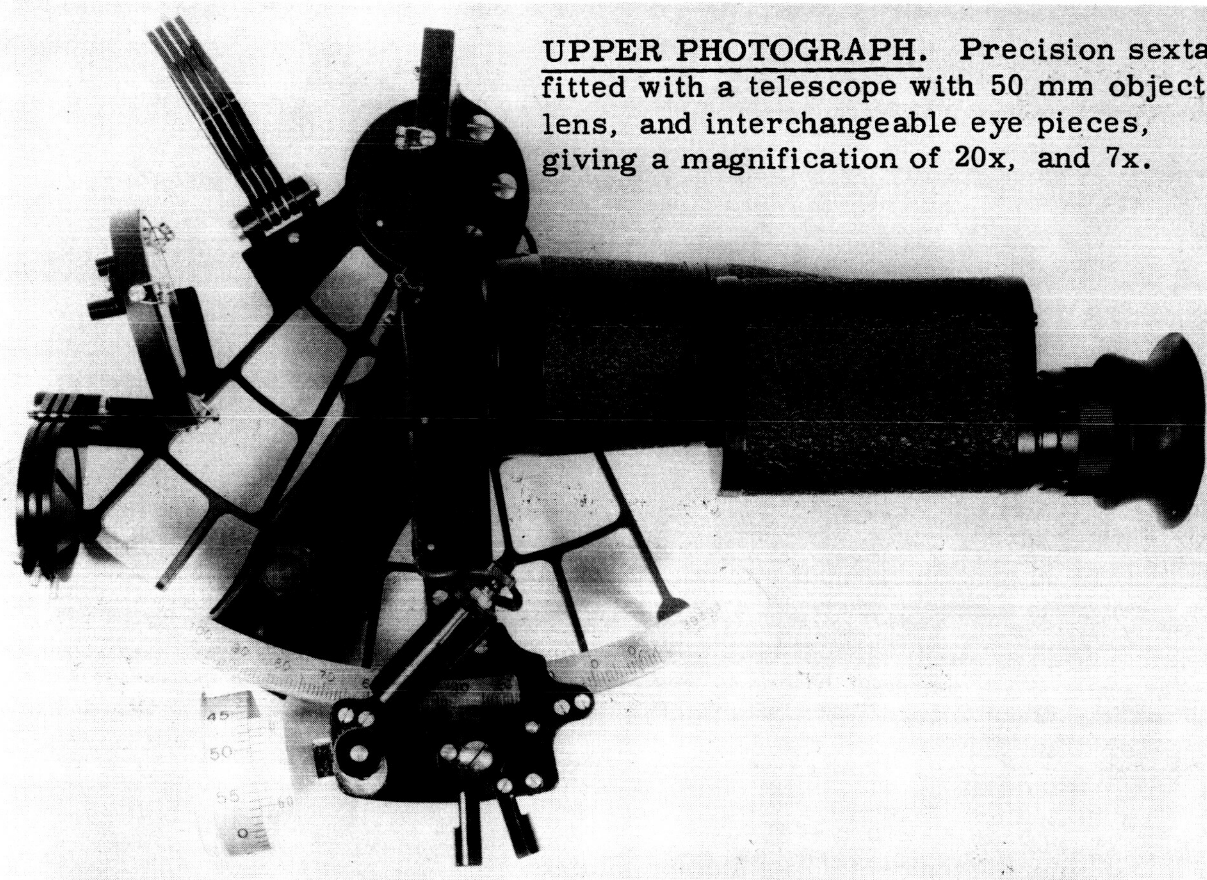
III. SIGHT REDUCTION

During the past 35 years almost all effort in the field of celestial navigation has been directed towards shortening the time required for the reduction of sights; to attain speed, it was felt that some accuracy could well be sacrificed. For general use this was correct; the accuracy obtainable with the "short methods" was sufficient for the requirements of ordinary navigation.

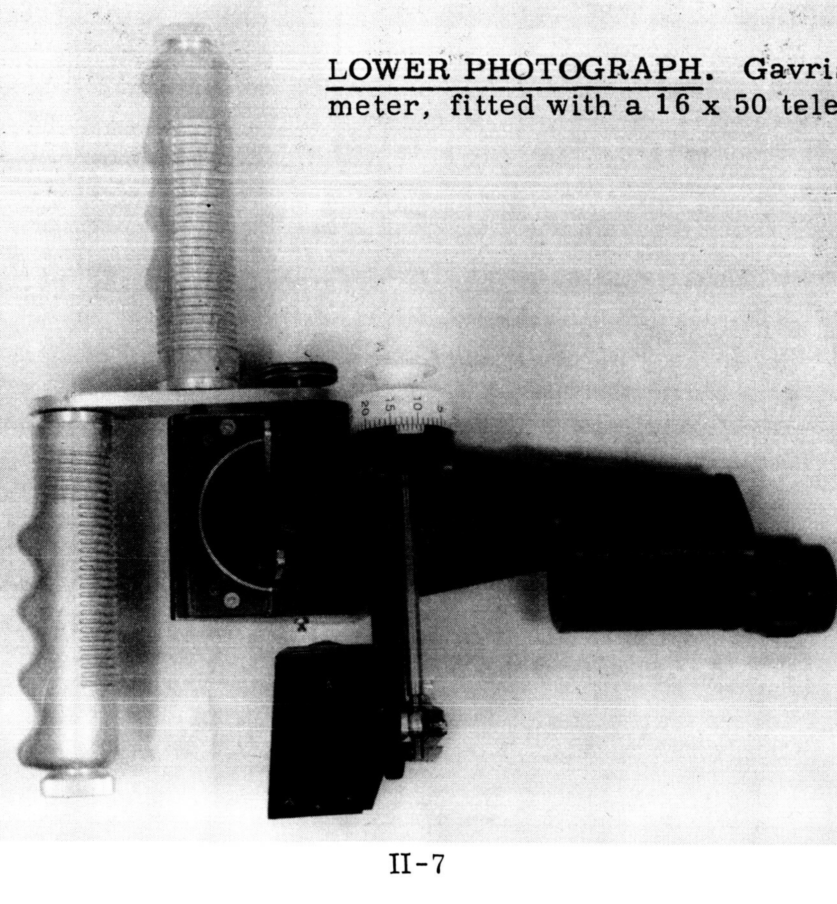
Today, however, there is an urgent need in some specialized fields to obtain the maximum degree of accuracy in position fixing by celestial as well as by other navigational means. For celestial navigation improved sextants and telescopes as well as dip meters are now available, which give greatly improved accuracy, as compared to what could be obtained with similar instruments in the past.

The instruments are available for refined celestial navigation; data for the accurate and rapid reduction of sights should also be made available for those areas and times when such navigation is required. As has been noted in the body of this report, the Nautical Almanac does not give sufficient accuracy when a position must be fixed with an error not exceeding 0.2 miles.

For such use, it is recommended that data on hour angle, declination, etc., be made available in readily usable form, and with an end accuracy of 0.05 minutes of arc.

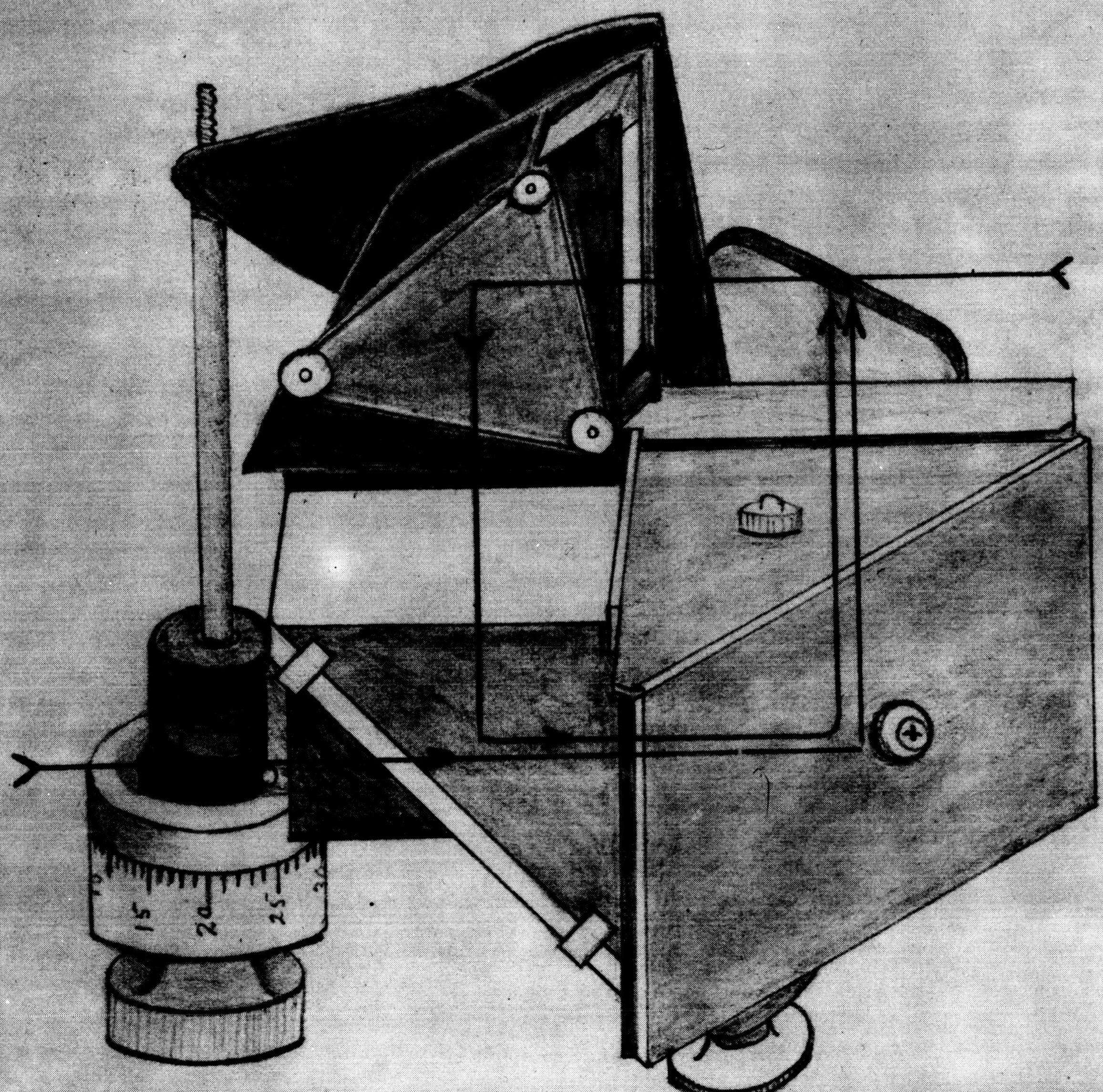


UPPER PHOTOGRAPH. Precision sextant, fitted with a telescope with 50 mm objective lens, and interchangeable eye pieces, giving a magnification of 20x, and 7x.



LOWER PHOTOGRAPH. Gavrisheff dip meter, fitted with a 16 x 50 telescope.

THE PHOTOGRAPH ON THIS PAGE illustrates the optics of the Gavrisheff dip meter.



An electronic computer can be used to store these data, as well as to complete the reduction of sights with speed and accuracy. Lacking such a device, it is recommended that an electric computer be employed to solve the spherical triangle, by means of the standard sin-cosin formulae. It is recommended that natural functions be employed, rather than logarithms. The functions should be stated at least to six decimal places, and for every tenth of a minute of arc. Such a table could readily be prepared from the U. S. Coast and Geodetic Survey, eight place tables, which are tabulated for every second of arc.

IV. SELECTION OF THE OBSERVER

It has been almost universal practice afloat for the ship's navigator to make the great majority of the required celestial observations. The navigator is selected on the basis of his professional knowledge; his visual accuity is rarely considered. This program, as well as similar studies in the past have clearly indicated that skill in the use of the sextant varies greatly between individuals, even those of apparently equally good vision, and equally practiced in the use of the instrument. A limited percentage of individuals have the ability to pick up a star in a bright sky with the sextant long before the majority are able to locate it. In addition only a small number have the ability of judging the instant of contact between the body and the horizon. For refined navigation, the observer should excel in both these skills.

It is urgently recommended that where accuracy in navigation is of paramount importance, the observer be selected solely on the basis of the results he obtains with the sextant; the data he obtains can be processed by others, qualified in that field.